THE ECONOMETRIC SOCIETY MEETING

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THE ECONOMETRIC SOCIETY MEETING

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INTRODUCTION

The Econometric Society was founded on December 29, 1930, in Cleveland, Ohio, by a group of prominent economists, statisticians, and mathematicians from several countries for the purpose of creating a closer union among workers in these fields and thereby aiding in the establishment of economic theory on a more scientific foundation. After a careful consideration of objectives a constitution was adopted and Professor Irving Fisher of Yale University was elected the first president of the Society.

The following extract from the constitution describes the scope of Society:

> The Econometric Society is an international society for the advancement of economic theory in its relation to statistics and mathematics. The Society shall operate as a completely disinterested, scientific organization without political, social, financial or nationalistic bias. Its main object shall be to promote studies that aim at a unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking similar to that which has come to dominate in the natural sciences. Any activity which promises ultimately to further such unification of theoretical and factual studies in economics shall be within the sphere of interest of the Society.

From 16 present at the organization meeting the membership had grown by 1947 to 821, distributed among 42 different countries. Membership is open to all seriously interested in the objectives of the Society, upon proposal by two members, nomination by the Council, and election by the members. Dues, which include subscription to the Society's quarterly journal, *Econometrica*, are $8.00 per year for those residing in the United States and Canada, and $6.00 a year for members residing elsewhere. Application blanks may be obtained from William B. Simpson, Secretary, The Econometric Society, The University of Chicago, Chicago 37, Illinois, U.S.A.

The control of the Society rests in the hands of the Fellows, who numbered 53 in 1947. The first group of Fellows was chosen by the original Council, and additional ones are elected annually by the Fellows from nominations by the Council. To be eligible for nomination as a Fellow a member must have published original contributions to economic theory or to such statistical, mathematical, or accounting analyses as have a definite bearing on problems in economic theory. The Fellows annually elect members to the Council, which manages the affairs of the Society and elects the officers.

The Society holds meetings each year for the presentation and discussion of papers. The principal American meeting is usually held at the end of the year in connection with the meetings of the American Economic Association, the American Statistical Association, and other social-science organizations. Additional meetings are sometimes held with other societies, such as the American Association for the Advancement of Science, the American Mathematical Society, and the Institute of Mathematical Statistics. Because of government limitations on transportation these meetings were held less frequently during the war but were resumed in 1946. Annual meetings were held in Europe from 1931 to 1939 and were resumed in 1948.

The Society's journal, *Econometrica*, has been published quarterly since January, 1933, under the editorship of Professor Ragnar Frisch of the University of Norway. It publishes original papers and also reports of the meetings of the Society with brief abstracts of the paper presented. *Econometrica* goes to all members of the Society and to some 600 other subscribers, including the important university libraries throughout the world. The subscription price to nonmembers is $9.00 per year, and orders for subscriptions may be addressed to Alfred Cowles, Treasurer, The Econometric Society, The University of Chicago, Chicago 37, Illinois, U.S.A.

The Washington Meeting

During 1946 various members of the Society gave thought to possibilities of cooperating with the International Statistical Institute. Fellows of the Society (Messrs. Roos and Tintner) participated in a meeting of statisticians at Lake Junalaska, North Carolina. A resolution accepted at the meeting pointed out that responsibility for problems of collecting economic and social data—hitherto a major field of the Institute's interest—might be assumed by organs of the United Nations; the Institute might therefore extend its activities to advancing statistical methods in various fields, including econometrics. When the United States Arrangements Committee of the International Statistical Institute invited the Econometric Society and a few other internationally organized groups to hold sessions concurrently with those of the Institute in Washington in September, 1947, the Society's officers agreed. They felt that such a meeting would bring its members from the Old and the New World together in a manner not achieved before. The President for 1946, Professor Jacob Marschak, informed the Fellows and the membership at large (*Econometrica*, October, 1946) and appointed a Program Committee consisting of himself

as chairman and Messrs. R. G. D. Allen, Ragnar Frisch, Tjalling C. Koopmans, René Roy, and Arthur Smithies. The Vice-President for 1947, Mr. Charles F. Roos, represented the Society on the Joint Arrangements Committee. It was possible through administrative and financial cooperation of the several interested organizations to secure adequate representation of various countries among the speakers and in the general attendance. This cooperation continued after the close of the conference, in securing the publication of the *Proceedings* of the International Statistical Conferences. The present volume has been edited by Dickson H. Leavens, in cooperation with Miss Joan E. Raup (later succeeded by Mrs. Stuart A. Rice) of the Joint Arrangements Committee.

Space considerations made it necessary to limit the length of papers except in a few instances where excess length was approved by the subcommittee on publication. For the same reason the discussion was not included except for two or three sessions. In some cases speakers were able to supply only brief résumés of their papers; in other cases a résumé is used because the speaker felt that his paper could not be satisfactorily cut to the required length.

STATISTICAL ANALYSIS OF ECONOMIC RELATIONSHIPS: I

Monday, September 8, at 9 : 30 a.m.

CHAIRMAN :

Jerzy Neyman

Director of Statistical Laboratory, University of California (United States)

STATISTICAL ESTIMATION OF ECONOMIC RELATIONSHIPS[1]

by Herman O. A. Wold

Director, Institute of Statistics, University of Uppsala (Sweden)

Being invited to open the discussion on a very broad topic, I thought it appropriate to start with a few rather general comments; I shall then turn to some specific questions of estimation. Let me also explain that in collecting my material, I have been greatly stimulated by J. M. Clark's recent note in *Econometrica.*[2] His plea for more clarity as to the premises underlying econometric theories is certainly to the point. And everybody will agree in his appraisal that econometric research results are very mixed as to importance and quality.

1. My first point will be the striking contrast in rigor and precision between the natural sciences and the social sciences, including economics and econometrics. For illustration, the lower part of Fig. 1 pictures Millikan's classical measurements of the electric charge of the

[1] I am indebted to Mr. S. Malmquist and Mr. R. Bentzel for comments on the manuscript, and to Mr. R. Turvey for help in improving my English.

[2] J. M. Clark. Here and elsewhere footnotes will cite only name of author. For full title and place of publication see list of References at the end of this paper.

electron.[3] In spite of the great experimental difficulties, his measurements are distributed over a range as short as 1.2 percent of the average value.

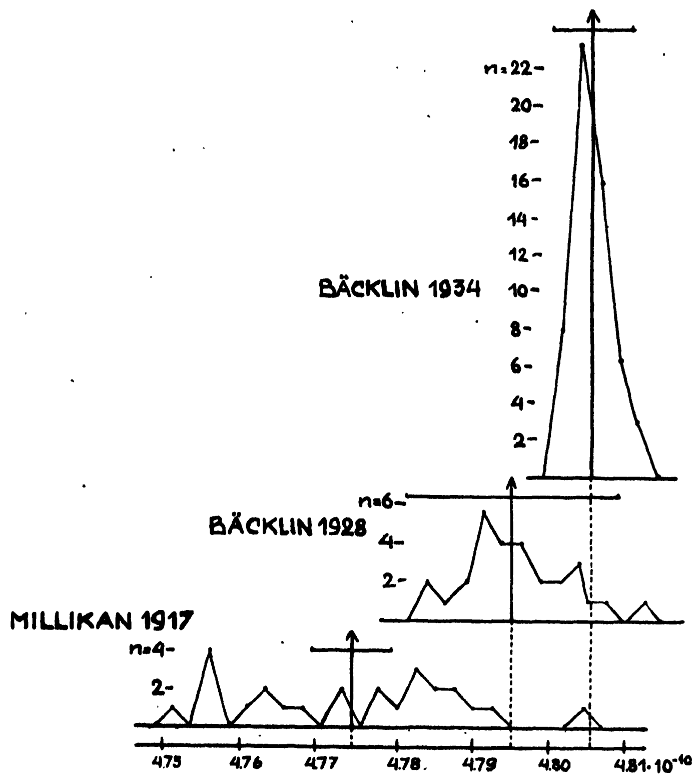

Figure 1. Measurements of the electronic charge.—The observations are given in electrostatic units, multiplied by 10^{-10}, and n is the number of observations within 0.05 percent intervals.

Fig. 2, on the other hand, refers to Henry S c h u l t z's analysis of the demand for corn in U.S. during 1875-95.[4] Estimating price elasticities by the use of multiple regression, he makes two different estimates; one is based on the regression relation where the quantity transacted is taken as dependent variable, the other estimate is calculated with the price as dependent variable. In his final conclusions, Schultz chiefly gives preference to the former estimate, but at the same time refers to the

[3] Quoted from E. Bäcklin.

[4] H. Schultz, Tables II, p. 261, and IV., p. 674. The example is so chosen that the trend effect, which is taken into account by Schultz, is negligible.

two estimates as possible extremes of the elasticity sought. In this particular instance, the two methods give 0.77 and 0.99, the difference thus amounting to about 25 percent of the estimate. This case, furthermore, is by no means exceptional; rather is it chosen so as to illus-

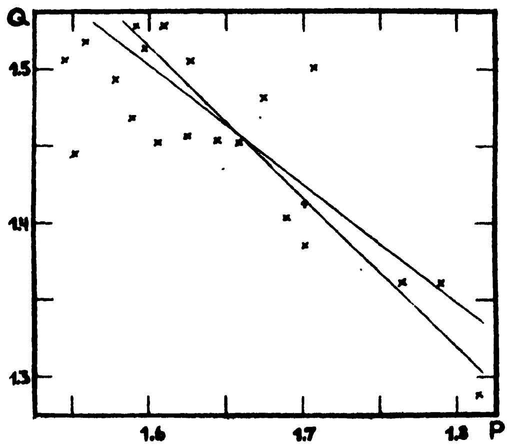

Figure 2. Corn consumption against corn price, U.S., 1875–95, after H. Schultz. Q = logarithm of per capita consumption, P = logarithm of real price.

trate a common state of affairs in Schultz's studies. In passing, I may mention that it has later been shown that one of Schultz's two alternative methods should be rejected, namely the one where price is taken for dependent variable.[5]

No further illustration is needed, I think. In economics we are accustomed to looking upon our quantitative results as relatively vague and uncertain.

2. Let me recall three fundamental obstacles to reaching precise results. Firstly, our field of research is highly variable. Trends in technical, social, and political factors cause essential changes in economic life from one decade to the next. In addition, there are abrupt changes caused by wars and other events producing shocks. In view of this ever changing and often turbulent state of things it is no wonder that quantitative research has difficulty in reaching reliable results.

I should like to point to Fig. 3 as an illustration of economic variability. It refers to Greenland, the Danish colony. For more than a century, a public Danish company has held complete economic control over the native Greenlanders, in all a few thousand souls. All the time, the

[5] H. Wold (2), part III, appendix 2, esp. p. 113. *Cf.* also sections 4 and 12 below.

company has kept detailed statistics, records which have served the Company as material for a series of interesting studies. I quote here a set of curves showing how the demand for coffee has varied with income.[6] There is one curve for each decade. It can be seen that 100 years ago the demand elasticity was high; with increasing income and

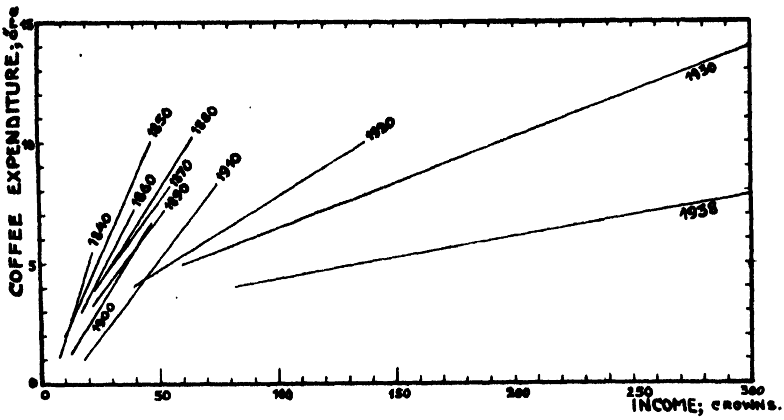

Figure 3. Coffee consumption against income; Greenland 1840–1938 (after P. V. Sveisdrup). Individual budget data; 1 crown = 100 öre.

the rising civilization of the Greenlanders, however, the elasticity has gradually decreased, down to a level near zero. Now if we consider the more common commodities it seems likely that trends of this type are not infrequent in any country. Unfortunately, however, in no country do the budget data available cover the development for more than 30 or 40 years.[7]

Secondly, a serious obstacle in econometric analysis lies in the complicated structure of economic life, in the fact that if we wish to study the interdependence between two economic factors, we should pay regard to a great many other factors which are more or less correlated with the phenomena under analysis. To demonstrate the complications which may arise it is sufficient to recall J. Tinbergen's studies of business cycles; for the United States, his analysis takes into consideration 70 different economic factors and about 50 relations between them.[8]

Thirdly and finally: Experiments are not possible in economic life, that is, experiments in the strict sense, repeated experiments under

[6] Grönlands Styrelse, p. 989. I am indebted to Mr. P. P. Sveistrup, scientific leader of the investigation, for valuable comments on the results.

[7] For an analysis of Swedish data 1913-33, see H. Wold (2).

[8] J. Tinbergen (1).

controlled conditions. While the variability and the multitude of factors make things complex and difficult to analyze, the absence of controlled experiments rather constitutes a handicap for the analysis. Economic research has to work without the aid of the experimental method, the supreme tool of research which has given the natural sciences their undisputed precedence in rigor and precision.

Summing up, I think it is a fair statement that econometrics is a difficult field of research. If the results reached in this field are still far from the perfection of many natural sciences, this fact need not give econometricians an inferiority complex.

The shortcomings of econometrics, however, are obvious, and are frequently pointed out. The question remains whether all is done that could be done to reach more rigorous and precise results. Let us consider this question in the light of two principles which stand out as signposts in our bewildering field of research.

3. Let us first examine the empirical aspect of the question. To begin with, I think there is general agreement that the standard tests of significance are of limited use when dealing with economic data. For instance, the tests available do not cover the cases where we have to take into consideration serial coefficients, and even if such tests were worked out, their verdict could not always be relied upon, owing to the many factors in play and the ever changing state of things. Let it be remarked that this is a type of situation which is met also in the natural sciences, only that the difficulties are often much greater in the social and economic fields. The Millikan experiments, Fig. 1 above, may be taken as illustration. His final result was that the electronic charge lies in the interval from 4.769 to 4.779 scale units. In deducing the range of uncertainty, Millikan calculated the random error of his average result in the ordinary way, and further took into account certain errors of method, estimating their variance to be double the random variance. Now about 15 years later the electronic charge was measured anew by the Swedish physicist Bäcklin, by a fundamentally different method.[9] The value found by Bäcklin lay outside the interval indicated by Millikan; although Bäcklin made liberal allowance for random and methodological errors his result was different, slightly but distinctly. Bäcklin's findings were met with decided scepticism, but later measurements confirmed his results. This example shows that it is not sufficient to make allowance for the ordinary random error; we also have to base our judgment of averages on considerations of nonstatistical character as to imaginable errors of method.

[9] See the upper curves of Fig. 1 above.

It may accordingly be said that in economic-statistical analysis it is difficult to judge the reliability of isolated results. If our statistical observations are to be condensed into reliable relationships, into steady tendencies to regularity, our collection and treatment of statistical material should not be restricted to a narrow sector of the economic field. The larger the range of the variability, and the more of the various factors in play that are covered by the data, the better the perspective that will be attained and the better the position to judge the significance of the statistical findings. For example, considering once more the Greenland coffee-demand curves (Fig. 3 above), their systematic pattern will make us feel much more confident in the numerical findings than if only one or two of them were available.

I have here stressed the view that it is a great advantage if empirical studies in our field are of broad coverage in space and time. Now from this viewpoint the trend of modern economic research is promising, for the tendency of empirical studies seems to be towards broader layout. The tendency is obvious as concerns the large-scale research institutes, but is also visible, I think, in individual research work. In the study of consumers' demand, to take only one example, the work of Allen and Bowley covers all the main items in the individual budget, and material from several countries is dealt with;[10] the standard work of Schultz, with American material for a 60-year period, covers a broad sector of agricultural products.[11]

4. Turning from empirical to theoretical analysis, we find in the econometric field many different theoretical approaches. We find a mosaic of theories, some of which are complementary, while others are competing; we find rather fragments of a theory than one unified theory. Now in view of the many factors in economic life and their great variability, it is but natural that economic theory should be fragmentary. An all-embracing theory would no doubt be too complicated to be useful. But are the existing fragments all right? Are there not too often too many alternative approaches and suggested solutions to the same problem? J. M. Clark has touched upon this question in the note already referred to, and has also pointed out the weak point of many of these theories, the questionable realism of the underlying premises.

This lack of realism can to some extent be explained by the main handicap of economic analysis: economic theory cannot be tested by experiments. Now thus handicapped we should be all the more careful in selecting our premises and assiduous in testing them by nonexperimental means. But I fear that the opposite is in fact more true: the

[10] R. G. D. Allen and A. L. Bowley.

[11] H. Schultz.

less the theories can be tested experimentally, the more freely they seem to be put forward. I think, accordingly, that much remains to be done towards rejecting the less realistic approaches so that we can concentrate on the more realistic ones.

This task is by no means easy. I should like to stress one particular difficulty. We are often concerned with approaches where the basic assumptions seem rather plausible, while the theory erected on these premises leads more and more out into the blue. The explanation is that the incomplete realism of the assumptions becomes more and more apparent as their logical implications are developed; the premises have an ultimate breaking stress, and the theory should not be carried beyond this limit. Now our field of research being complex and difficult, we can never expect that the premises should be very close to reality, and so the danger pointed out is always present or around the corner. We may further realize that once we have started to build a theory on a new set of assumptions, it is not easy to make a halt in deducing new theorems. To continue theorizing is the more tempting since it is so difficult to confront the results obtained with statistical observations, observations that are seldom sufficiently detailed and accurate to enable a decisive test.

By way of illustration let us consider the equilibrium theories of demand and supply. The basic hypothesis is that demand and supply are functions of the price of the commodity considered, say

$$(1) \qquad d = D(p), \qquad s = S(p).$$

a. *Cournot's theory.*[12] The equilibrium price is determined by the condition

$$(2) \qquad d = s.$$

The interpretation is that if price is other than the equilibrium price, a gap between demand and supply will arise (*cf.* Fig. 4). Under the influence of free competition, price adjustments will take place until equilibrium is reached.

b. *The cobweb theory*[13] specifies the path of the price adjustments: A number of consecutive time periods are considered, say, ..., $t-1$, t, $t+1$, ... In the simplest case it is assumed that demand during period

[12] A. Cournot, Chapter VIII, section 50.

[13] For a survey of the cobweb theory, see M. Ezekiel (1).

t is a function of price during the same period, and that supply during t is a function of price during $t-1$; in symbols,

$$(3) \qquad d_t = D(p_t), \qquad s_t = S(p_{t-1}).$$

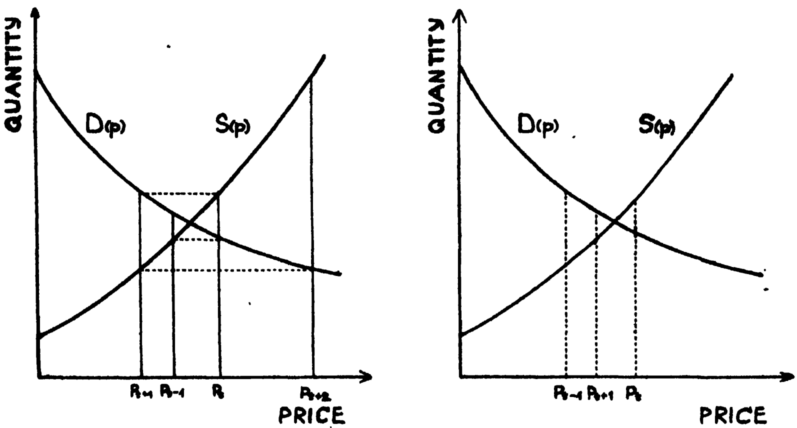

Figure 4. Price paths according to (*i*) the cobweb theory, and (*ii*) the price system (3), (5).

It is further assumed that within each period the price will be adjusted so that demand equals supply:

$$(4) \qquad d_t = s_t \, .$$

Taking price as given arbitrarily for the period $t-1$, the equations (3)—(4) form a system which allows us to deduce the subsequent price development p_t, p_{t+1}, p_{t+2}, ... (*cf.* Fig. 4,*i*). Three different cases may be distinguished:

α. The price will converge towards the Cournot equilibrium;

β. The price will oscillate stationarily around the Cournot equilibrium;

γ. The price will diverge from the Cournot equilibrium.

Which case will occur depends on the slopes of the demand and supply curves. Fig. 4,*i* illustrates the case γ.

Certain implications of the cobweb theory have always aroused my suspicion. The relation (4) is unrealistic if the commodity allows storage, or if it can go further in the production process (for instance, unconsumed milk being used to make cheese). And for the relatively few perishable commodities, there is no empirical evidence that some particular demand and supply curves will give rise to the divergent case γ.

The explanation seems to be that in the cobweb theory the implications of the premises have been developed too far. While the assumptions (3)—(4) may seem plausible in themselves, they are not sufficiently realistic to allow the conclusion γ. An equilibrium theory on the elementary lines of (1)—(4) cannot, I think, be pushed much further than Cournot's original conclusion: Under stable conditions and free competition, price will tend towards an equilibrium.

c. The point in question may be brought out in terms of another system of equations: We retain (3), and replace (4) by

$$(5) \qquad p_t = p_{t-1} + c(d_{t-1} - s_{t-1}) \qquad (c > 0)$$

The system could be taken as referring to a retail market. Demand is brought into contact with supply by merchants, who keep stocks and act as price regulators. If demand is greater (less) than supply, merchants will set a higher (lower) price during the next period. We may call (5) a *price-formation relation*; by way of a first approximation it indicates how merchants adjust prices under free competition. Now let us consider two demand and supply curves (1), with rising and falling slopes respectively, but otherwise arbitrarily defined. Provided that the constant c in (5) is sufficiently small, p_t as determined by (3) and (5) will then converge towards Cournot's equilibrium price (*cf.* Fig. 4, *ii*). We have thus been able to avoid the divergent case γ.[14]

We note that (4) is formally equivalent with

$$p_t = D^{-1}\,[S(p_{t-1})],$$

where $D^{-1}[x]$ stands for the inverse function of $D(x)$. This shows that (4) may be interpreted as a price-formation relation, of a very special kind.

5. I shall now turn to more technical aspects of the problem of estimating economic relationships. While my comments up to now have referred to empirical and theoretical analysis as two rather independent lines of research, the statistical data will now be regarded as the material for estimating the parameters involved in a postulated economic relation. My comments will be confined to linear relations.

First a remark on the statistical nature of the relationships in the economic field. Economic relations can seldom, if ever, be expected to fit the observations with perfect exactness. The postulated relations, accordingly, cannot be written as *functional* relations, we have to give

[14] If we wish to maintain the stationary case β, we may in (5) introduce an erratic term (*cf.* section 8,b below), and choose for c a value near to its upper "critical limit."

them the form of *stochastic* relations. We have here to distinguish between two different situations:

α. Let us by way of illustration consider the case of two variables, say x_1 and x_2. A formal representation of a stochastic relation between x_1 and x_2 is, in any situation, provided by their joint distribution function, say

$$F(u_1, u_2) = \text{Prob}\ (x_1 \leqslant u_1, x_2 \leqslant u_2\ ). \tag{6}$$

If we try to express the relationship (6) in terms of a regression relation, we encounter the difficulty that in general we have no basis for choosing between alternative regressions, for instance the regression of x_1 on x_2, the regression of x_2 on x_1, or perhaps the orthogonal or the diagonal regressions. To be able to choose between these alternatives we must have at our disposal some further information as to the nature of the relationship between x_1 and x_2. The information required is of non-statistical nature.

β. It is a frequent situation that our experience allows us to point out one of the variables, say x_2, as causally dependent on the other, x_1. The term causal is here used in the common-sense meaning; the situation may also be described by saying that x_2 appears as dependent on x_1 in the same way as in a controlled experiment, say a dosage-response experiment, x_1 being the dosage variable and x_2 the response (effect) variable. As soon as such information is available, the relationship may formally be expressed as a regression relation, *viz.*, the regression of x_2 on x_1, say

$$x_2 = c_0 + c_1 x_1 + y. \tag{7}$$

Here y is the regression residual, which is interpreted as the effect on x_2 of other variables than x_1.

Stochastic relations like (7) that allow a causal interpretation are sometimes called "causal" or "cause-and-effect" relations.[15] Since it seems desirable to avoid the term causal, such relations will here be called *explanatory*.[16] It needs no comment that the notion of an explanatory relation extends to the case of several variables, say

$$x_n = c_0 + c_1 x_1 + \dots + c_{n-1}\, x_{n-1} + y, \tag{8}$$

the left-hand variable being regarded as causally dependent on the right-hand variables.

[15] In the terminology of M. Ezekiel (2), p. 50, the variable x_1 is "causal," while x_2 is "resultant."

[16] This is in agreement with the current practice of calling the right-hand variables "explaining."

Formally, (8) has the same appearance as a regression relation as used to *estimate* the left-hand variable for given values of the right-hand variables. Such estimation, however, need not involve any information or assumption as to causal dependence; in fact, given a set of variables, an estimation relation may be formed for each variable, taking this for left-hand variable. The notion of an *explanatory* relation, let this be stated explicitly, should thus be carefully kept apart from the notion of an *estimation* relation.

6. The relations usually employed in economic analysis are (apart from estimation relations) of the type here called explanatory; relations of the less specified type (6) are not often used. In what follows, we shall exclusively confine our attention to explanatory relations. Our object is to discuss the methods of estimating the parameters of such relations.

Let us consider an explanatory relation of the familiar type

$$(9)\quad x_n(t) = c + \sum_{j=1}^{n-1} c_j^{(0)} x_j(t) + \sum_{j=1}^{n} c_j^{(1)} x_j(t-1) + \dots + \sum_{j=1}^{n} c_j^{(s)} x_j(t-s) + y(t).$$

This relation explains the variable $x_n(t)$ as a linear function of the other nonlagged variables $x_1(t), \dots, x_n(t)$, and of lagged values of all variables $x_1(t), \dots, x_{n-1}(t)$. The maximal lag is denoted by s. The residual $y(t)$ is interpreted as the effect of variables not included in the analysis.

The traditional method of estimating the parameters c and $c_j^{(k)}$ in (9) is to apply multiple regression analysis. This is done by taking $x_n(t)$ as the dependent variable and the right-hand variables as independent. This gives us the $c_j^{(k)}$ as the regression coefficients, and $y(t)$ as the regression residual.

All of us know that this traditional method must in practice be handled with care and caution. For one thing, the resulting equation must not be used as a functional relation; *i.e.*, the terms may not be freely rearranged. For instance, it cannot in general be used if we wish to obtain some other variable than $x_n(t)$, say $x_1(t)$, in terms of the other variables involved. This remark applies to (9) both as an explanatory relation and as an estimation relation. In fact, as a causal relation (9) says nothing about the causal explanation of $x_1(t)$. And as is well known, in order to estimate $x_1(t)$ and $x_n(t)$ in terms of the other variables involved we must in general use two numerically different relations.[17]

All of us also know that the traditional method has been subjected to several points of criticism. One of these has been particularly alarming;

[17] For further comments on this point see H. Wold (4), section 6.

I think of the argument set forth a few years ago by T. Haavelmo and which is the starting point for several later investigations.[18] Let me briefly recall Haavelmo's argument. An economic relation, say (9), is but one of a whole system of relations which describe the economic structure. The same variables will appear in several equations of the system, and if we follow Haavelmo in considering systems of a certain type, it turns out that consistent estimates of the parameters cannot be obtained by the traditional method, by treating the equations one by one. All parameters should instead be estimated simultaneously, which under general conditions may be done by the maximum-likelihood method of R. A. Fisher. The following statement may be quoted as typical for the conclusions reached on this line: "*That is, it is impossible to derive statistically the demand functions from market data without specification of the supply functions involved.*"[19] The bias which in such cases is inherent in the traditional method, also called the single-equation method, will here be called the *Haavelmo bias.*

According to Haavelmo's argument, it might well be that all earlier results obtained by the traditional, single-equation method were more or less biased. An even more alarming consequence would lie in the resulting complications of the estimation procedure. For if the bias is present in the single-equation method, it might still be present if the simultaneous analysis were confined to five equations, or ten—we should have to work with a system covering the whole economic field. Having seen that the most ambitious attempts to describe the total economic structure have worked with up to fifty relations (*cf.* section 2 above), the estimation problems would become quite a serious affair. For me personally, the question was very relevant, for two years ago, when Haavelmo's work reached Scandinavia, I had nearly finished the statistical side of an investigation of consumers' expenditures. All the numerical results of this study might be subject to the Haavelmo bias.

In this situation, I undertook a critical examination of the traditional method and its logical foundations.[20] It is my intention to give here a brief report of the results, as far as they concern the Haavelmo bias. Let me state at once the conclusion that the traditional method, when applied to a certain general class of models, is free from Haavelmo's bias. The class in question is wide enough to cover most, if not all, dynamic models used in econometric research up to 1940.

[18] T. Haavelmo. *Cf.* M. A. Girshick and T. Haavelmo, also for further references.

[19] M. A. Girschick and T. Haavelmo, p. 83. The italics are from the original.

[20] Part of the study was carried out as joint work; see R. Bentzel and H. Wold. See also H. Wold (3), (4).

7. In econometric research during the thirties, much attention was paid to the approach known as *model sequence analysis*, or *process analysis*.[21] It is sufficient to recall the pioneer work done by J. Tinbergen in applying process analysis to statistical data. Now it is precisely this type of dynamic systems, properly specified, which goes free from the Haavelmo bias.

It should be observed that from a mathematical viewpoint this type of model is a special case of the more general systems considered by Haavelmo. Now the less specified a model is, the less specified are the conclusions that may be reached on the basis of the model. As may accordingly be worth while, we shall consider in some detail the approach of model sequence analysis.

The methodology of the model sequence approach has developed only gradually, and still involves controversial or untouched questions. It will therefore serve our purpose to outline the model sequence approach with respect to its characteristic features. We shall have to consider two aspects of the approach: its formal structure, and the principles of its specification. As before, we consider linear relations only.

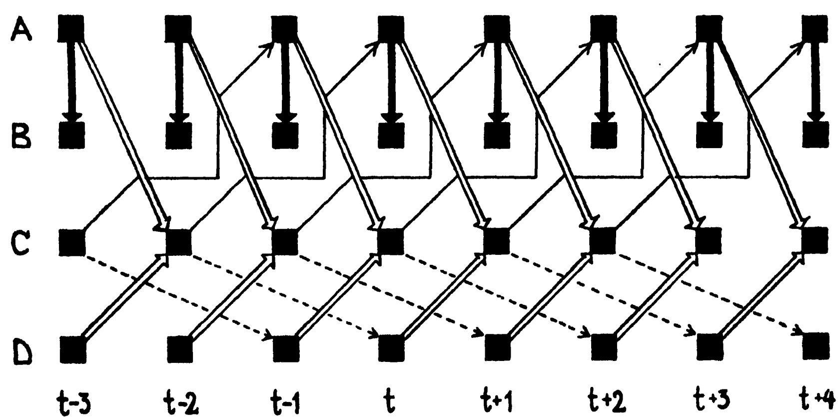

Figure 5. Diagrammatic illustration of model sequence analysis: The "arrow scheme" of J. Tinbergen.

8. *The formal properties of model sequence systems.* The basic feature of the model sequence approach is that the economic phenomena considered are supposed to be governed by a system of explanatory relations. These relations, called the hypothetical model, form an equation system with the following properties:

a. Each relation refers to one variable, this variable being explained as a function that may involve lagged values of the same variable,

[21] For a survey of this approach see J. Tinbergen (1), (2). *Cf.* also R. Bentzel and H. Wold.

lagged or nonlagged values of other variables, but no leading (forward) value of any variable.

b. Each relation may involve an erratic term, called the residual, which accounts for the effect of variables not included in the analysis.

c. The system may or may not contain relations for all variables considered. The variables explained are called "endogenous," the other ones "exogenous." If all variables are explained, the system is called "complete."

d. There is just one relation (not two or more) referring to each endogenous variable; *i.e.*, the explanation of each endogenous variable is unique.

e. The system is *recursive* with respect to *time*, in the following sense: If we know the past of all variables, if the future of the exogenous variables is assumed to be known and if the future disturbances are disregarded, the system gives us recursively the future development of the endogenous variables.

f. In the recursive deduction, the variables are obtained consecutively, one by one, from the explicit relations referring to the endogenous variables. In other words, the system is *recursive* also with respect to the *variables*.

Systems with the above properties may be called *sequence analysis systems*. To stress the properties e-f, they may alternatively—in view of an obvious matrix interpretation—be called *triangular systems*.

Tinbergen's "arrow scheme" provides a diagrammatic description of recursive systems (*cf.* Fig. 5). The definition of linear recursive systems may also be given in symbols: their general pattern is[22]

$$(10)\quad x_i(t) = c_i + \sum_{j=1}^{i-1} c_{ij}^{(0)} x_j(t) + \sum_{j=1}^{n} c_{ij}^{(1)} x_j(t-1) + \dots + \sum_{j=1}^{n} c_{ij}^{(s)} x_j(t-s) + y_i(t),$$

where $i = m+1, \dots, n$. The variables $x_1, \dots, x_m$ are exogenous, $x_{m+1}, \dots, x_n$, are endogenous, while y_i is the residual. The system is complete if $m = 0$. It is seen that the recursive calculation e-f gives the endogenous variables in the order $x_{m+1}, \dots, x_n$.

9. *Principles for the specification of sequence analysis systems.* Coming now to problems of application, we must in the first place specify the economic content of an equation of type (10), *i.e.*, make clear what kind of relation between the variables involved the equation is supposed to express.

[22] R. Bentzel and H. Wold.

The relations in a model sequence system (10) are usually referred to as explanatory, as allowing a cause-and-effect interpretation in the common-sense meaning already indicated in connection with (8). More precisely, we take each relation to be behavioristic in the sense that it would refer to a specific unit in the economic sphere, indicating how this unit reacts to changes in the right-hand variables. For instance, in the retail market model presented in section 4,c above, there are three equations; the units to which these refer are the consumers, the producers, and the merchants respectively.[23] In this connection we may mention that it would be natural to require that the equations of the system should be *autonomous; i.e.*, if the patterns of behavior described by one equation were assumed to change, this change should not affect the other patterns.[24] This requirement is of importance if the system (10) is used for studying the effect of "structural changes." For instance, suppose that the free market described by the system of section 4,c is subjected to price control. If the three equations are autonomous, the effect of the control on demand and supply could be found out simply by replacing the third relation (the price-formation relation) by the price curve as determined by the price-control authority.[25]

With the qualifications indicated above, the system (10) thus forms a general pattern of specification. Being very general, its application to an actual problem usually involves further specification. For instance, we may make $m = n - 1$, so that the system reduces to the single dynamic relation (9). Or it may be assumed that certain parameters are known *a priori*, for instance, that some of the parameters $c_{ij}^{(k)}$ are zero. Such further specification may be based on different kinds of arguments: theoretical considerations, statistical analysis, common sense, and perhaps other kinds of arguments.

As to the residuals, $y_i(t)$, two principles seem to be universally accepted:

α. they should be *small*,

β. they should be *irregular*.

Clearly, the smaller the residuals, the less does our system leave to be explained, and the more satisfactory is our theory. On the other hand,

[23] As stressed by J. Marschak (1), certain relations in the economic sphere are technical or legal. In the above specification, the terms "unit in the economic sphere" and "behavioristic relation" should be taken in a wide sense so as to include such types of relation.

[24] I am indebted to Mr. J. R. N. Stone for discussion on this point.

[25] *Cf.* also the discussion of J. Marschak's lecture at these Conferences; J. Marschak (2).

any trace of interdependence among the residuals may be regarded as something regular still to be explained; in other words, the more irregular the residuals the better. The ideal would be that all residuals could be regarded as independent in the sense of the theory of probability.

The situation may also be regarded from the viewpoint of forecasting: the smaller the residuals, the more efficient the forecasts given by the system, and any regularity among the residuals is liable to reduce the efficiency of the forecasts.

10. *The "resolving" power of recursive systems.* Having set forth the recursive system (10) and the principles that guide its specification, it might be asked whether such systems are sufficiently general for their purpose, sufficiently general to serve as hypothetical models of the economic structure. In a certain sense the answer is in the affirmative, as seen from the following theorem, where for any $z(t)$ the mean value symbol M is defined by

$$M[z(t)] = \lim \frac{1}{t_2 - t_1 + 1} \sum_{t=t_1}^{t_2} z(t) \qquad \begin{pmatrix} t_1 \to -\infty \\ t_2 \to +\infty \end{pmatrix}.$$

THEOREM:[26] *Let $x_1(t), \ldots, x_n(t)$ be a set of time series such that*

$$(11) \qquad M[x_i(t)] \quad \textit{and} \quad M[x_i(t),\, x_j(t+k)] \qquad \begin{pmatrix} i, j = 1, \ldots, n \\ k = 0, \pm 1, \pm 2, \ldots \end{pmatrix}$$

exist, and let m $(0 \leqslant m < n)$ and $\varepsilon(>0)$ be arbitrarily given. Then the maximal lag s and the parameters c and $c_{ij}^{(k)}$ may be so chosen that the residuals $y(t)$ as defined by (10) will satisfy

$$(12) \qquad M[y_i(t)] = 0, \qquad\qquad -\varepsilon < M[y_i(t),\, y_j(t+k)] < \varepsilon,$$

for $i, j = 1, \ldots, n$; $k = \pm 1, \pm 2, \ldots$ and for $i \neq j$; $k = 0$.

It will be observed that the theorem applies to any time series satisfying the stationarity conditions (11), irrespective of the genetic structure of the series. For instance, the genetic structure may involve squares and other nonlinear functions of the variables, periodical components, or fractional or infinitesimal time lags, or it may be that in the genetic structure the variables appear in an order which differs from the

[26] For the case where $n=1$, see H. Wold (5), theorem 1; the above theorem may be proved by straightforward generalization. There is, of course, a similar theorem for multidimensional random processes $\{\xi_1(t), \ldots, \xi_n(t)\}$, in which case M is to be replaced by the mathematical expectation E. We also note that therhea a corresponding theorem for systems (10) of the special type where all rigdindse variables are lagged (the "reduced form" in the Haavelmo approachhhul ti-t);esar variances will then generally be larger, of course.

order in which they are numbered. Whatever their structure, the series may be interpreted in terms of the linear system (10). This being so, the representation (10) will not in general correspond with the genetic structure. This fact is brought out by the properties of the residuals: As indicated by (12) these will not in general be independent, but only uncorrelated, and that only within an ε margin. In statistical analysis as based on recursive systems (10) it is accordingly of great relevance to examine the residuals with respect to their mutual independence (*cf.* also section 9,β above).

Let it be remarked that there are in general several sets of parameters $c_{ij}^{(k)}$ that will give residuals satisfying (12); in other words, there is no uniqueness theorem for the parameters. Carrying the analysis further, however, it would be possible to obtain a unique representation of each variable, namely as a linear function of lagged and nonlagged residuals plus a certain well-defined component which is deterministic in the sense that it can be "almost exactly" forecast[27] on the basis of past values of the variables considered.[28]

11. We now turn to the estimation of the parameters $c_{ij}^{(k)}$ of the system (10), and in particular to the question of the Haavelmo bias. As is usually done, we shall confine ourselves to stationary systems.

We assume that all roots of the characteristic equation of the system (10) are smaller than 1 in absolute value.[29] In view of the theorem of section 10 it is but natural to assume that we are concerned with a system (10) where *the residuals have zero means and are mutually uncorrelated.* Our object is then to compare two estimates for the parameters, the one being given by the maximum-likelihood method, the other by the single-equation method, *i.e.*, the traditional least-squares method. As usual when dealing with the maximum-likelihood method, we assume that the residuals are random variables which are normally distributed. Under these assumptions the maximum-likelihood method is consistent, *i.e.*, asymptotically unbiased.[30] It can further be shown that the two methods give the same estimates; in other words, the recursive system

[27] *I.e.*, forecast with an error whose variance is zero.

[28] *Cf.* H. Wold (5), theorem 2, which covers the special case $n=1$. For a corresponding theorem on stationary processes, see H. Wold (1), theorem 7, and V. Zasuhin.

[29] H. Wold (1), p. 53; H. B. Mann and A. Wald, p. 192.

[30] This can be shown by a straightforward application of the methods used by H. B. Mann and A. Wald, part II.

will be free from the Haavelmo bias.[31] In fact, in easily understood symbols, the maximum-likelihood function will in this case reduce to the simple form

$$C \exp \left(- \tfrac{1}{2} \sum_i \sum_t [y_i(t)]^2 / \sigma_i^2\right)$$

(where C is independent of the coefficients of the equations), an expression which is maximized by the same parameter values that make each of the square sums $\sum_t [y_i(t)]^2$ a minimum.

We note that if one or more of the parameters $c_{ij}^{(k)}$ are assumed to be known *a priori*, the traditional method will still be free from the Haavelmo bias.[31] We also note that the independence between any two residuals $z_i(t)$, $z_j(u)$ (in particular, for $i \neq j$, $t = u$) is an essential condition for the absence of the bias[32]. Thus the risk of a Haavelmo bias gives us an additional reason for testing whether the residuals resulting from the traditional method are independent (*cf.* section 9, β and section 10 above).

12. *On the criticism of the single-equation method.* As already mentioned, the linear systems where Haavelmo found his bias are of a more general type than (10). This being so, we are led to ask under what circumstances one needs more general systems than (10), specifically when one needs the type of system considered by Haavelmo.

One argument that calls for more general systems than (10) was pointed out by Bentzel and me in the paper already referred to. If theoretical considerations lead to a system of type (10), with some specified time unit, say a quarter of a year, it may happen that statistical observations are available only for longer time periods, say one year. The system (10) may be transformed so that the left-hand members correspond with the statistical data, simply by summing over a certain number of time periods. But this device will in general carry us away from the class of recursive systems, to systems where a bias similar to Haavelmo's is present. This argument clearly loses in strength if the lags to be considered are large. Now in social sciences, including economics, we are concerned with changes that are the result of human reactions, and all of us know that human behavior is largely governed by habits, habits that are surprisingly regular and that as a rule change only slowly. Accordingly while there might be a restricted group of econometric applications where our argument leads us to modify the traditional method, I think it would carry the argument too far to take it as a basis for a sweeping criticism of the traditional method.

[31] R. Bentzel and H. Wold, pp. 106 and 109. Haavelmo (*l.c.*, p. 10) has given general conditions for an equation system of more general type than (10) to go free from the bias. Our theorem may be obtained as a corollary to Haavelmo's, his conditions being readily found to be fulfilled by the recursive systems (10).

[32] *Cf.* H. B. Mann and A. Wald, esp. pp. 218-219.

A second limitation of the approach (10) lies in the requirement, stated in section 8, that each equation should be explanatory, *i.e.*, express a cause-and-effect relation, indicate a unilateral dependence. In order to form such a relation, certain *a priori* evidence is required. Now such evidence is sometimes lacking, and it may even happen that there is evidence to the effect that the dependence among the variables considered is mutual, *i.e.*, bilateral instead of unilateral. For instance, while the typical retail market may be described in terms of unilateral relations of the type indicated in section 4,c, the situation in wholesale markets is often more complicated. In the wholesale market the buyer does not always passively react to the price, he may sometimes be in a position to influence it by his buying offers.[33]

Bi– or multi-lateral interdependence requires an approach of a more general type than (10). It would carry us too far, however, to enter here upon questions belonging to this order of ideas.

Finally, I should like to comment on one point where the systems considered by T. Haavelmo differ from the system (10); *viz.*, the point stated in section 8,d: Haavelmo allows more than one explanatory relation for each variable.[34] For instance, in one of his numerical illustrations Haavelmo writes down a demand relation and a supply relation, and assumes that both relations apply to the quantities transacted. We note that part of his analysis stands and falls with this identification of demand and supply, notably his proposition that a demand relation cannot be consistently treated by the traditional, single-equation method. We further note that it is by no means logically necessary to identify demand and supply. In the model in section 4,c above, for instance, it would be legitimate to treat the demand relation by the traditional method.[35]

The point in question may be brought out in general terms. If two variables $x_1(t)$ and $x_2(t)$ are made equal for every t, apart from the erratic components, this would mean that an equilibrium is maintained through the adjustment of another variable, or perhaps a group of other variables. While this device clearly amounts to introducing a static element in the dynamic approach, complete dynamization would require a separate relation for the equilibrating variable (*cf.* the system section 4,c *versus* the system section 4,b, where price is the equilibrating variable).

[33] See *e.g.* C. F. Roos, esp. Chart XXX. *Cf.* also R. Bentzel and H. Wold, pp. 103 and 105.

[34] M. A. Girschick and T. Haavelmo. In earlier papers, Haavelmo even considers systems which are not recursive in the sense of section 7,e (T. Haavelmo, *l.c.*; *cf.* the comments by R. Bentzel and H. Wold).

[35] *Cf.* also R. Bentzel and H. Wold, section 9.

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Résumé

Après quelques observations sur les difficultés des recherches statistiques en sciences sociales par comparaison aux sciences expérimentales, suivent quelques remarques sur l'analyse statistique basées sur le type général suivant de modèle économétrique:

$$(1) \qquad x_i(t) = c_i + \sum_{j=1}^{i-1} c_{ij}^{(0)} x_j(t) + \sum_{j=1}^{n} c_{ij}^{(1)} x_j(t-1) + \dots$$

$$+ \sum_{j=1}^{n} c_{ij}^{(s)} x_j(t-s) + z_i(t) \qquad (i = m+1, m+2, \dots, n).$$

Les x_j sont les variables à étudier, les c_i et les $c_{ij}^{(k)}$ certaines constantes interprétées comme des perturbations dues à des facteurs non inclus dans le système. On dit que x_j est endogène si $i > m$, exogène si $i \leqslant m$. Si $m = 0$, le système est dit complet.

Les systèmes du type (1) sont précisément ceux qu'on utilise dans la méthode économique connue sous le nom d'analyse de *modèles—séquences*. Ceci est un cadre méthodologique assez large. Comme il est bien connu, J. Tinbergen a fait oeuvre de pionier dans l'application des modèles-séquences à des données statistiques.

Théorème sur la capacité résolvante des modèles—séquences: Si $\{x_i(t)\}$ est une suite stationnaire d'événements aléatoires à dispersion finie, et si m $(0 \leqslant m < n)$ et ε (> 0) sont donnés arbitrairement, alors l'écart maximal s et les constantes c peuvent être choisis de manière que les résidus $z_i(t)$ définis par (1) satisfassent aux relations suivantes

$$E[z_i(t)] = 0 \text{ pour tout } i \text{ et } t,$$

$$|E[z_i(t)\, z_j(u)]| < \varepsilon \text{ pour deux } z_i(t),\ z_j(u) \text{ quelconques.}$$

La méthode traditionelle d'adaptation des systèmes du type (1) à des données statistiques est de traiter chacune des équations par l'analyse

de régression multiple, en considérant la variable du premier membre comme variable dépendante. En partant de systèmes plus généraux que (1), T. Haavelmo a fait la remarque importante que les coefficients c donnés par la méthode traditionnelle présentent un biais et que des estimations sans biais peuvent être obtenues par la méthode du maximum de vraisemblance de R. A. Fisher.

Comme il a été remarqué par R. Bentzel et H. Wold, le biais signalé par Haavelmo ne se rencontre pas dans les systèmes du type (1) si tous les résidus sont indépendants entre eux, donc

$$E[z_i(t)\, z_j(u)] = 0 \text{ pour deux } z_i(t),\ z_j(u) \text{ quelconques.}$$

En combinaison avec le théorème ci-dessus notre remarque sert à renforcer le fondement logique de l'analyse traditionnelle de régression multiple. Les mêmes auteurs ont indiqué que les unités de temps des données statistiques ne sont pas toujours aussi petites que les unités du modèle hypothétique, et qu'en formant un système modèle d'équations avec des premiers membres correspondant aux données statistiques, un biais similaire à celui signalé par Haavelmo s'introduit.

Mr. Wold's Paper was discussed by Mr. Tjalling C. Koopmans.

ERRORS AND SHOCKS IN ECONOMIC RELATIONSHIPS

by Theodore W. Anderson
Instructor in Mathematical Statistics, Columbia University (United States)

and Leonid Hurwicz
Associate Professor of Economics, Iowa State College (United States)

Résumé

The present paper is devoted to the study of certain properties of what are called shock-error models.

Such models contain stochastic elements (disturbances) of two types: those associated with specific variables ("errors") and those associated with certain equations ("shocks").

Systems of the type considered can be written in matrix form as

$$\alpha\xi_t = u_t, \tag{1.1}$$

$$x_t = \xi_t + v_t. \tag{1.2}$$

Here the x_t is the observed, ξ_t the true value of the variable vector at the time t. u_t and v_t are respectively the shock and error vectors at time t. α is the structural coefficients' matrix.

When endogenous variables (η, y) are distinguished from the exogenous ones (ζ, z), instead of (1.1) we write more explicitly

$$\sum_{\tau=0}^{\tau^*} \beta_\tau \eta_{t-\tau} + \gamma\xi_t = u_t, \tag{2.1}$$

$$y_t = \eta_t + v_t^y, \tag{2.21}$$

$$z_t = \zeta_t + v_t^z. \tag{2.22}$$

The covariance matrices of u and v are written Σ^u, Σ^v respectively. It is assumed that the u's are not correlated with the v's.

B. Special models. The two important special models treated in the past are the *shock model* (where $v = 0$) and the *error model* (where $u = 0$). Both have been used in economics. The error model is also of importance in psychology (factor analysis).

Main Problems studied

(1) *Identification.* Even in the special models it is necessary to impose restrictions on some parameters to estimate the others. In the shock-error model the possibilities of indeterminacy are even stronger. The problem analyzed is under what conditions it is possible to identify (*i.e.*, determine from observations, apart from sampling fluctuations) the structural coefficients' matrix α and the covariance matrices Σ^u, Σ^v.

(2) *Estimation.* Some methods of estimating α, Σ^u, Σ^v are provided (maximum likelihood, moments).

(3) *Specification Bias.* What bias arises when, say, data produced by a shock model are treated as though they came from an error model? Some special cases of this problem have been examined.

One of the objectives of the paper is to integrate the work already done by a great many writers on various special cases of shock and, especially, error models.

Résumé

La présente étude concerne certaines propriétés des modèles economiques dites "chocs-erreurs." De tels modèles contiennent des éléments stochastiques (perturbations) de deux types: ceux se rapportant à des variables spécifiques ("erreurs") et ceux se rapportant à certaines équations ("chocs").

On peut écrire les systèmes de ce type sous forme de matrices; voir, à cet effet, les formules (1.1) et (1.2) dans le texte anglais ci-dessus, où x_t est la valeur observée, ξ_t la valeur réelle du vecteur variable au moment t. u_t et v_t sont les vecteurs représentant respectivement les chocs et les erreurs au moment t. α est la matrice des coefficients structurels.

En cas que l'on distingue entre les variables endogènes (η, y) et les variables exogènes (ζ, z) on emploie les formules (2) plus explicites.

Les matrices de covariance de u et de v s'écrivent respectivement Σu et Σv. On suppose qu'il n'y a pas de corrélation entre les u et les v.

Modèles spéciaux. Les deux modèles spéciaux importants étudiés dans le passé sont celui des chocs (le cas $v = 0$) et celui des erreurs (le cas $u = 0$). L'un et l'autre ont été utilisés dans la théorie économique. Celui des erreurs a de l'importance aussi en psychologie (analyse factorielle).

Principaux problèmes étudiés

(1) *Identification.* Même dans les modèles spéciaux il est nécessaire d'imposer des restrictions à certains paramètres afin de pouvoir estimer les autres. Dans le cas général, les possibilités d'indétermination sont même plus grandes. Les auteurs étudient le problème des conditions sous lesquelles il est possible d'identifier (c'est-à-dire déterminer sur la base des observations, abstraction faite des fluctuations aléatoires) la matrice α des coefficients structurels et les matrices de covariance Σu et Σv.

(2) *Estimation.* Sont exposées, quelques méthodes d'estimation de α, Σu, et Σv (maximum de vraisemblance, moments).

(3) *Erreurs systématiques de spécification.* Quelles sont les erreurs systématiques quand, disons, des données obtenues sur la base d'un modèle à chocs sont considérées comme ayant été fournies par un modèle à erreurs? Quelques cas spéciaux de ce problème sont examinés.

L'un des buts de la présente étude est de systématiser les recherches déjà faites par un grand nombre d'auteurs sur plusieurs cas spéciaux des modèles en question.

Messrs. Anderson and Hurwicz's paper was discussed by Mr. R. G. D. Allen.

SAMPLING ASPECTS OF THE PROBLEM OF RELATIONSHIP FROM THE ERROR-IN-VARIABLE APPROACH*

by R. C. Geary

Statistics Branch, Department of Commerce and Industry (Ireland)

Summary

1. There are two main divisions in the error-in-variable approach:

(i) that in which the disturbance variance is assumed known;

(ii) that in which the disturbance variance is not assumed known.

The present paper deals with both divisions.

2. As to (i), the point of departure is Gerhard Tintner's research (1945, 1946) into the determination of the coefficients of linear equations between the systematic parts of variables and the ascertainment of the number, r, of equations. The intimate relationship between the Tintner theory and the Harold Hotelling principal-component theory is examined. It is shown that the incidental estimates of the systematic part of the ith variable is given by

$$\sum_{j=1}^{p-r} c_{ij} F_{jt},$$

where p is the number of variables and c_{ij} and F_{jt} are the coefficients and factors, respectively, in the principal-component theory. A simple proof is given of the P. L. Hsu (1941) result (utilized by Tintner) that

$N \sum_{v=1}^{r} \lambda_v$, where the λ_v are the smallest latent roots, is distributed

as χ^2 when sample size N is large.

3. By reference to the case of two variables the large-sample efficiency of the estimates of the coefficients, using the Tintner and other methods, is examined.

4. A new method of estimating linear relationships (which should be applicable in most cases in which the Tintner method would be effective) is developed and is shown to furnish more efficient estimates of the coefficients than does the Tintner method. In the large-sample case a χ^2-test for number of relations is found.

5. Certain practical results based on U.S. economic data for the years 1922-1938 are presented by way of illustration. It is shown that after

* Author was not present to read this paper.

fitting curves to the data, which take up over 90 per cent of the variances, the residuals remain very highly correlated, indicating that disturbances must be very small in these data.

6. The principal result in division (ii) is the determination of the exact sampling distribution, in the normal hypothesis, of

$$k = m_{13}/m_{23},$$

where m is the covariance of the ith and jth variables as observed and where k is a consistent estimate of the rates of the coefficients in the simplest case of a single relation between two variables. The application of the theory to the many-variable case is considered. It is shown that this theory yields less efficient estimates of the coefficients than does the theory referred to in paragraph 4.

Résumé

1. Il y a deux divisions principales dans la théorie des relations statistiques, dans le cas de variables sujettes à erreurs. I) celle où la variance de la perturbation est supposée connue; II) celle où la variance de la perturbation n'est pas connue. La présente étude traite l'une et l'autre.

2. En ce qui concerne le I) on prend comme point de départ les recherches de Gerhard Tintner (1945, 1946) sur le calcul des coefficients d'equations linéaires entre les parties systématiques des variables et la détermination du nombre r d'équations. L'auteur examine le rapport étroit entre la théorie de Tintner et la théorie des composants principaux de Harold Hotelling. Il montre que les estimations incidentelles de la partie systématique du i-ième variable sont donnèes par

$$\sum_{j=1}^{p-r} c_{ij} F_{jt},$$

où p représente le nombre de variables et c_{ij} et F_{jt} sont respectivement les coefficients et facteurs dans la théorie des composants principaux. Une simple preuve est donnée du résultat de P. L. Hsu (1941) (utilisé par Tintner) suivant lequel $N\sum_{v=1}^{r}\lambda_v$ où les λ_v sont les plus petites racines latentes, est distribué comme χ^2 quand l'effectif N de l'échantillon est grand,

3. En se référant au cas de deux variables, l'auteur examine l'efficacité, pour les échantillons importants des estimations des coefficients, en utilisant la méthode de Tintner et autres méthodes.

4. Une nouvelle méthode d'estimation de relations linéaires (qui devrait être applicable dans le plus grand nombre des cas où la méthode de Tintner serait effective) est développée et il est démontré qu'elle fournit des estimations plus efficaces des coefficients que ne le fait la méthode de Tintner. Pour le cas d'échantillons importants, un test χ^2 a eté déduit pour le nombre de relations.

5. A titre d'illustration, quelques exemples pratiques ont été présentés, basés sur des séries statistiques économiques des Etats-Unis pour les années 1922-1938. Il est démontré qu'après avoir ajusté des courbes aux données, qui expliquent 90% des variances, les résidus demeurent en corrélation étroite, ce qui signifie que les perturbations dans ces données doivent être très petites.

6. Le résultat principal dans la division II est la détermination de la distribution exacte, dans le cas de la normalité, de $k = m_{13}/m_{23}$, où m_{ij} est la covariance des valeurs observées des i-ième et j-ième variables et où k est une estimation correcte des rapports des coefficients dans le cas le plus simple d'une seule relation entre deux variables. L'auteur traite l'application de la théorie au cas de plusieurs variables. Il démontre que cette théorie fournit des estimations moins efficaces des coefficients, que ne le fait la théorie mentionnée au paragraphe 4.

The papers at this session were further discussed by Messrs. Herman O. A. Wold, Jacob Marschak, Gerhard Tintner, Tjalling C. Koopmans, Leonids Hurwicz, G. Rasch, and Jerzy Neyman.

STATISTICAL ANALYSIS OF ECONOMIC RELATIONSHIPS : II

Tuesday, September 9, at 9:30 a.m.

CHAIRMAN :

G. Darmois

Member from France, Statistical Commission of the Economic and Social Council of the United Nations; Professor of Statistics at the University of Paris (France)

PREDICTION FROM AUTOREGRESSIVE SCHEMES AND LINEAR STOCHASTIC DIFFERENCE SYSTEMS

by J. R. N. Stone

Director, Department of Applied Economics, University of Cambridge (United Kingdom)

It is assumed here that the changes in a set of interacting variables have been in the past and will continue to be in the future capable of representation by a given system of linear stochastic difference equations. Predetermined variables other than lagged endogenous variables will be ignored. There are as many relationships as there are endogenous variables.

Every system of the above kind can be put into the *autoregressive form*.[1] This means that the current value of each variable is expressed in terms of a certain number of its own past values plus a linear combination of the disturbances in the original equations with or without time lags. This is *not* equivalent to the *reduced form* from the standpoint of the estimation of the parameters by the method of least squares.

[1] This is the form obtained by carrying out a process of elimination as Tinbergen has done in his econometric business-cycle studies.

In the autoregressive form there will be the same number of lagged terms in the systematic part of the expression for each variable and the coefficients of the term with any given lag will in each case be the same. This property depends essentially on the fact that a determinant (in this case a determinant of operator polynomials) is the same whichever way you expand it. Ignoring the stochastic element, which does not affect the position in this respect, we may write a set of k linear lag equations between variables $x_1, x_2, \ldots, x_k$,

$$(1) \qquad \beta_{i1}x_1 + \beta_{i2}x_2 + \ldots + \beta_{ik}x_k = 0, \qquad i = 1, 2, \ldots, k,$$

where each β_{rs} is a constant or polynomial in D, the operator delaying by unit time the variable on which it acts. Let B_{rs} be the cofactor of β_{rs} in $|\beta_{rs}|$. Both B_{rs} and $|\beta|$ are polynomials in D. Operating on the i-th equation with B_{ij} and summing for i gives $|\beta|x_j = 0, j = 1, 2, \ldots, k$, *i.e.*, each x_j satisfies the same autoregressive equation, $|\beta|x = 0$.

The variance of a prediction from an autoregressive equation depends on two factors. The first is the variance of the disturbance or error component and the second is a coefficient depending, at least if the error components are independent over time, simply on the autoregressive coefficients. If σ_p^2 is written for the variance of a prediction and if σ_ε^2 is written for the variance of the error component then it is intuitively obvious that $\sigma_p^2 \geqslant \sigma_\varepsilon^2$. In the simple case of one lag term where

$$(2) \qquad x_t + ax_{t-1} = \epsilon_t$$

it is evident that in the limit $\sigma_p^2/\sigma_\varepsilon^2 = 1$ if $a = 0$, the case of random deviation, and $= \infty$ if $a = 1$, the case of simple Brownian motion.

The first step in deriving the predictive power of any system is to find the limiting value of $\sigma_p^2 = \mu_0$ in the following notation. This will give the predictive power at infinity. This as will shortly appear is of only limited value in practice since the predictive power of autoregressive schemes is extremely low for periods far in the future. Next it will be useful to derive the value of σ_p^2 for each successive time period since in some cases the approach to the limiting value may be slow. Finally it is possible to obtain an efficient method of estimating the predictive power of a linear stochastic difference system by first estimating the regression coefficients and the variances of the disturbances in the reduced form and then inserting these in the autoregressive form.

In transforming a set of linear stochastic difference equations into the autoregressive form it does not usually happen that the disturbance term is uncorrelated in successive time periods. It is useful therefore to allow for the serial correlation of the disturbance term from the outset.

Let the autoregressive series be

$$(3) \qquad a_0 x_t + a_1 x_{t-1} + \dots + a_k x_{t-k} = \epsilon_t,$$

with $a_0 = 1$.

Squaring both sides and taking means we obtain

$$(4) \quad (a_0^2 + a_1^2 + \dots + a_k^2)\mu_0 + 2(a_0 a_1 + a_1 a_2 + \dots + a_{k-1}a_k)\mu_1$$
$$+ 2(a_0 a_2 + a_1 a_3 + \dots + a_{k-2}a_k)\mu_2 + \dots + 2a_0 a_k \mu_k = \sigma_\varepsilon^2$$

where $\mu_i = E(x_t x_{t-i}) = E(x_{t+i}\, x_t)$ so that $\mu_0 = E(x_t^2)$ is the function which is required in terms of σ_ε^2.

Let $E(x_{t-i}\epsilon_t) = \rho_i$, $i > 0$. If as may sometimes happen the ϵ_t are independent then the $\rho_i = 0$. If we multiply equation (3) by x_{t-i} and average, we obtain

$$(5) \quad a_0\mu_i + a_1\mu_{i-1} + \dots + a_i\mu_0 + a_{i+1}\mu_1 + a_{i+2}\mu_2 + \dots + a_k\mu_{k-i} = \rho_i$$

and as i takes values 1, 2, ..., k this gives k equations for finding $\mu_1, \dots, \mu_k$ in terms of μ_0, the a_j, and the ρ_i. By substituting in (4) the value of μ_0 is found without difficulty.

This procedure will be illustrated by a simple example. Suppose $k = 2$ and $\rho_i = 0$, $i > 0$. Then remembering that $a_0 = 1$, equation (4) becomes

$$(6) \qquad (1 + a_1^2 + a_2^2)\mu_0 + 2(a_1 + a_1 a_2)\mu_1 + 2a_2\mu_2 = \sigma_\varepsilon^2$$

and equations (5) become

$$(7) \qquad \begin{aligned} \mu_1 + a_1\mu_0 + a_2\mu_1 &= 0, \\ \mu_2 + a_1\mu_1 + a_2\mu_0 &= 0, \end{aligned}$$

whence

$$(8) \qquad \frac{\mu_0}{\sigma_\varepsilon^2} = \frac{(1 + a_2)}{(1 - a_2)[(1 + a_2)^2 - a_1^2]}.$$

Figure 1 illustrates the ratio $\phi = \mu_0/\sigma_\varepsilon^2$, the ratio of the variance of the prediction at infinity to the variance of the error component, for different values of a_1 and a_2. The value of a_1 is shown along the abscissa while that of a_2 is shown along the ordinate. The contour lines show combinations of a_1 and a_2 which yield a constant value of ϕ. $\phi = 1$ at the center point of the diagram where $a_1 = a_2 = 0$. The diagram is

enclosed in a triangle with vertices $a_1 = -1$, $a_2 = 1$; $a_1 = 1$, $a_2 = 1$ $a_1 = 0$, $a_2 = -1$ along the edges of which $\phi = \infty$.

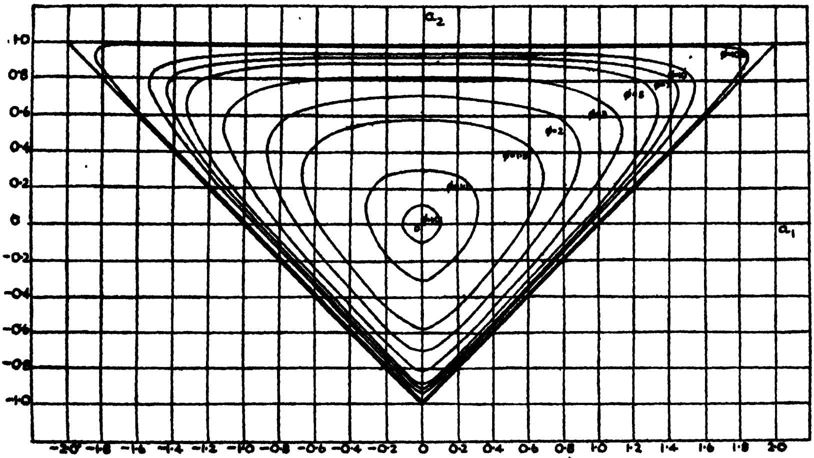

Figure 1. Limiting value of the disturbance magnification factor, ϕ, for different combinations of the autoregressive coefficients a_1 and a_2 in the sense of a linear autoregressive scheme with two time lags and unautocorrelated disturbances.

$$x_t + a_1x_{t-1} + a_2x_{t-2} =$$

$$\phi = \frac{\mu_1}{\sigma_\varepsilon^2} = \frac{1 + a_2}{(1 - a_2)\,[(1 + a_2)^2 - a_1^2]}$$

The expression given above for μ_0 is in terms of σ_ε^2 and the original autoregressive coefficients, a_i. It is always possible,[2] however, to express x_t as a linear function of the disturbances up to and including time t. Thus we may write

$$(9) \qquad x_t = b_0\epsilon_t + b_1\epsilon_{t-1} + \dots + b_n\epsilon_{t-n}, \qquad n \to \infty,$$

where the b's are functions of the a's. If (9) defines a stationary process the sum of the b's is convergent. If furthermore the ϵ's are independent then the variance of x_t, say σ_x^2, is simply

$$\sum_{i=0}^{n} b_i^2\sigma_\varepsilon^2.$$

But it is evident that an exactly similar expression is obtained for μ_0, the variance of the prediction at infinity. Thus

$$(10) \qquad \mu_0 = \sigma_x^2,$$

[2] This development has been worked out in detail by H.O.A. Wold in *A Study in the Analysis of Stationary Time Series* (1938), especially chapter III, section 23.

that is, the variance of a prediction relating to the distant future is equal to the variance of the series from the distant past to the present. This shows the extreme inefficiency of long-term predictions from autoregressive schemes; in the long run the variance of a prediction rises to the variance of the series itself. Thus series properly describable by autoregressive schemes are sharply divided from those in which the errors do not influence their future course and which accordingly have a long-term prediction error no greater than their short-term prediction error.

The b's may readily be obtained from the a's by an iterative process as may be seen from an example. Consider the scheme

$$x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \epsilon_t, \tag{11}$$

where $\alpha_1 = -a_1$, and $\alpha_2 = -a_2$, and suppose

$$\begin{aligned} x_{t-2} &= \epsilon_{t-2}, \\ x_{t-1} &= \epsilon_{t-1}. \end{aligned} \tag{12}$$

A matrix of the b's may be constructed such that successive columns contain the b's appropriate to successive ϵ's, while successive rows give the set of b's for successive time periods. Obviously the matrix for the first three time periods can immediately be written down and this when appropriately multiplied will yield the b's for ϵ_{t-2}, ϵ_{t-1}, and ϵ_t in the equation for x_{t+1}. Thus, if we write ξ_{t+1} for $x_{t+1} - \epsilon_{t+1}$,

$$\begin{pmatrix} x_{t-1} \\ x_t \\ \xi_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \alpha_2 & \alpha_1 \end{pmatrix} \begin{pmatrix} x_{t-2} \\ x_{t-1} \\ x_t \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & \alpha_2 & \alpha_1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha_2 & \alpha_1 & 1 \end{pmatrix} \begin{pmatrix} \epsilon_{t-2} \\ \epsilon_{t-1} \\ \epsilon_t \end{pmatrix},$$

and the product of the first two matrices in the last member will yield in its third row the b's for ϵ_{t-2}, ϵ_{t-1}, ϵ_t in the equation for x_{t+1}, while that for ϵ_{t+1} is 1. This process can obviously be continued indefinitely to give all the b's required, and can be adapted to an autoregressive scheme with a large number of lags.

The variance of a prediction is obtained from this scheme simply by squaring the coefficients of the unknown ϵ's and multiplying their sum

by σ_ε^2. Thus in the above example if ϵ_{t-2} and ϵ_{t-1} are given, it can be shown that the variances of prediction $\div\ \sigma_\varepsilon^2$ for times t, $t+1$, etc. are respectively 1, (α_1^2+1), $[(\alpha_2+\alpha_1^2)^2+\alpha_1^2+1]$, etc.

An alternative approach, which readily permits serial correlations in the disturbances to be taken into account, may be set out as follows. The equation for time $t+p$, $p = 1, 2, \ldots$, may be written

$$(14) \qquad a_0x_{t+p} + a_1x_{t+p-1} + \ldots + a_{p-1}x_{t+1} + k_{pt} = \epsilon_{t+p},$$

where the k_{pt} depend only on x_t and previous values, and are known. The sampling model consists in regarding the x_{t-r} $(r \geqslant 0)$ and hence the k_{pt} as the same from sample to sample while the ϵ_{t+s} $(s > 0)$ are random elements; all with variance σ_ε^2. To deal with the case in which the ϵ's are correlated we write

$$(15) \qquad E(x_{t+r}\epsilon_{t+s}) = \rho_{rs}, \qquad s > r > 0.$$

We may eliminate at once all the k_{pt} from equations (14) by setting down the mean of the *p-th* equation thus

$$(16) \qquad a_0\bar{x}_{t+p} + a_1\bar{x}_{t+p-1} + \ldots + a_{p-1}\bar{x}_{t+1} + k_{pt} = 0.$$

Subtracting from (14) gives:

$$(17) \qquad a_0y_{t+p} + a_1y_{t+p-1} + \ldots + a_{p-1}y_{t+1} = \epsilon_{t+p},$$

where

$$(18) \qquad y_{t+p} = x_{t+p} - \bar{x}_{t+p}.$$

The means $\bar{x}_{t+p}$ are conditional on the k_{pt} being given. Then squaring each equation of (17) in turn and averaging gives a set of equations of the form:

$$(19a) \quad a_0^2\mu_{pp} + a_1^2\mu_{p-1,\,p-1} + \ldots + a_{p-1}^2\mu_{11} + \sum_{i=0}^{p-1}\sum_{j=0}^{p-1} a_ia_j\mu_{p-i,\,p-j} \qquad (i \neq j),$$

where

$$\mu_{sr} = \mu_{rs} = E(y_{t+r}y_{t+s}).$$

Also multiplying equations (17) by y_{t+r} and averaging gives a set of equations of the form

$$(19b) \qquad a_0\mu_{rp} + a_1\mu_{r,\,p-1} + \ldots + a_{p-1}\mu_{r1} = \rho_{rp}.$$

Then putting $p = 1$ in (19a) gives μ_{11}. This gives in succession $\mu_{12}, \mu_{13}, \ldots, \mu_{1r}$ by putting $r = 1$ in (19b) and letting p take the values 2, 3, etc. Similarly putting $p = 2$ in (19a) and substituting for μ_{11} and

μ_{12} gives μ_{22}, and so on. There does not appear to be a more simple method of finding the variances μ_{rr} that are required.

We come now to the question of the predictive power of a system of identifiable equations with respect to each of the variables which it contains. The simplest procedure seems to be first to determine the coefficients and error variances from the reduced form, a method that yields efficient and consistent estimates, and then from these to derive the coefficients and error variances in the autoregressive form. The methods of this note may then be applied. This procedure is most easily seen from an example.

Let x stand for consumption, y for additions to wealth, and z for total income, and suppose these three variables connected by the following simple relationships:

$$\begin{aligned} x_t &= az_{t-1} + \delta_t, \\ y_t &= bx_t - cx_{t-1} + \epsilon_t, \\ z_t &= x_t + y_t, \end{aligned} \tag{20}$$

where δ_t and ϵ_t are error terms. We may at once eliminate y and concentrate on x and z. The equations for these variables written in the the reduced form are

$$x_t = az_{t-1} + \delta_t, \tag{21}$$

$$z_t = a(1 + b)z_{t-1} - cx_{t-1} + [(1 + b)\delta_t + \epsilon_t]. \tag{22}$$

From these equations we may obtain consistent and efficient estimates of a, b, c, σ_δ^2, and σ_ϵ^2.

The autoregressive forms of (21) and (22) are

$$x_t - a(1 + b)x_{t-1} + acx_{t-2} = \delta_t + a\epsilon_{t-1} = \eta_t, \tag{23}$$

$$z_t - a(1 + b)z_{t-1} + acz_{t-2} = (1 + b)\delta_t - c\delta_{t-1} + \epsilon_t = \zeta_t. \tag{24}$$

Proceeding as before we have in respect of (23)

$$[1 + a^2(1 + b)^2 + a^2c^2]\mu_0 - 2[a(1 + b) + a^2(1+b)c]\mu_1 + 2ac\mu_2 = \sigma_\eta^2, \tag{25}$$

$$\mu_1 - a(1 + b)\mu_0 + ac\mu_1 = \rho_1 = 0, \tag{26}$$

$$\mu_2 - a(1 + b)\mu_1 + ac\mu_0 = \rho_2 = 0. \tag{27}$$

From (26) and (27)

$$\mu_1 = \frac{a(1 + b)}{(1 + ac)}\mu_0, \tag{28}$$

$$\mu_2 = \left\{ \frac{a^2(1 + b)^2}{(1 + ac)} - ac \right\} \mu_0, \tag{29}$$

whence, substituting in (25), we obtain

$$\mu_0 = \frac{(1 + ac)(\sigma_\delta^2 + a^2\sigma_\varepsilon^2)}{(1 - ac)[(1 + ac)^2 - a^2(1 + b)^2]} \tag{30}$$

On the other hand in respect of (24) we have, writing ν in place of the previous μ,

$$[1 + a^2(1 + b)^2 + a^2c^2]\nu_0 - 2[a(1 + b) + a^2 (1 + b) c]\nu_1 + 2ac\nu_2 = \sigma_\varepsilon^2, \tag{31}$$

$$\nu_1 - a(1 + b)\nu_0 + ac\nu_1 = -(1 + b)c\sigma_\delta^2, \tag{32}$$

$$\nu_2 - a(1 + b)\nu_1 + ac\nu_0 = 0. \tag{33}$$

From (32) and (33)

$$\nu_1 = \frac{a(1 + b)\nu_0 - (1 + b)c\sigma_\delta^2}{(1 + ac)}, \tag{34}$$

$$\nu_2 = \frac{a^2(1 + b)^2\nu_0 - a(1 + b)^2c\sigma_\delta^2}{(1 + ac)} - ac\nu_0 \tag{35}$$

Whence, substituting in (31), we obtain

$$\nu_0 = \frac{(1 + ac)(c^2\sigma_\delta^2 + \sigma_\varepsilon^2) + (1 + ac)(1 + b)^2\sigma_\delta^2}{(1 - ac)[(1 + ac)^2 - a^2(1 + b)^2]}. \tag{36}$$

Given the variance of the long-range predictions of x and z it is easy to derive a similar variance for y.

The main conclusion of this note is that prediction from linear stochastic difference systems, unless it be restricted to very short periods ahead, requires that the disturbance term be reduced by a suitable elaboration of the system to very small proportions. In practice of course this may not be possible. This general conclusion has been stressed by several writers and particularly by Yule and Kendall who have done so much to increase our knowledge of the properties of autoregressive schemes. In his classic paper[3] Yule refers to the fact that "disturbances occurring in every interval imply an element of unpredictability very rapidly increasing with the time." Kendall in his application

[3] G. U. Yule, "On a Method of Investigating Periodicities in Disturbed Series, with Special Reference to Wolfer's Sunspot Numbers," *Philosophical Transactions of the Royal Society*, Series A, vol. 226, 1927, pp. 267–298.

of autoregressive schemes to English agricultural series considers[4] the question of using his results for purposes of prediction and comes to the conclusion that on the whole this cannot be done. He draws attention to the wide margins of error involved and suggests that "one could predict just as well by an intelligent consideration of all the circumstances and those somewhat arbitrary decisions which are called intuitive." In both these investigations it proved impossible to reduce the value of σ_ε^2 perceptibly by the addition of more than two lag terms.

Résumé

On suppose dans la présente étude que les changements dans une série de variables interdépendantes peuvent, dans le passé comme à l'avenir, être représentés par un certain système linéaire stochastique d'équations aux différences. Des variables prédéterminées autre que les variables endogènes décalées seront ignorées. Il y a autant de relations qu'il y a de variables endogènes. Un tel système peut toujours être mis sous une forme autorégressive.

Soit l'équation (3) du texte anglais de ce mémoire la série autorégressive. En élevant au carré les deux membres de l'équation (3) et en prenant les moyennes, nous obtenons l'équation (4), où $\mu_i = E(x_t x_{t-i}) = E(x_{t+i} x_t)$ de sorte que $\mu_0 = E(x_t{}^2)$ soit la fonction cherchée en termes de $\sigma_\varepsilon{}^2$.

Soit $E(x_{t-i}\epsilon_t) = \rho_i$, $i>0$. Si, comme il peut arriver, les ϵ_t sont indépendants, les $\rho_i = 0$. Si nous multiplions l'équation (3) par x_{t-i}, $i=1, 2, \ldots, k$, et si nous prenons la moyenne, nous obtenons les équations (5) qui constituent k équations pour trouver $\mu_1, \mu_2, \ldots, \mu_k$ en termes de μ_0, des a_j et des ρ_i. En substituant ensuite en (4) on trouve sans difficulté la valeur de μ_0.

L'expression précitée pour μ_0 est en termes de $\sigma_\varepsilon{}^2$ et les coefficients autorégressifs originaux a_i. Il est toujours possible, pourtant, d'exprimer x_t comme une fonction linéaire des perturbations jusqu'au temps t. Nous pouvons donc écrire l'équation (9) où les b sont des fonctions des a. Si l'équation (9) détermine un procès stationnaire, la somme des b est convergente. Si en outre les ϵ sont indépendants, la variance

[4] M. G. Kendall "Oscillatory Movements in English Agriculture," *Journal of the Royal Statistical Society*, vol. CVI, pt. II, 1943, pp. 91–124.

de x_t, σ_x^2, est simplement $\sum_{i=0}^{n} b_i^2\sigma_\varepsilon^2$. Mais évidemment on obtient la même expression pour μ_0, la variance de la prédiction à l'infini. Ainsi $\mu_0 = \sigma_x^2$, c'est-à-dire la variance d'une prédiction qui se rapporte à un avenir très éloigné est égal à la variance de la série du passé éloigné jusqu'au présent.

En pratique il est important d'obtenir les variances des prédictions ayant trait à l'avenir tout proche, puisque, comme on a pu le voir, un système autorégressif est peu efficace pour des predictions très éloignées. Même au cas où les ε sont en corrélation de série on trouve sans difficulté méthode pour obtenir la variance d'une prédiction par périodes successives, pas à pas.

Pour obtenir le "pouvoir de prédiction" d'un système d'équations linéaires stochastiques par rapport à chacune des variables qu'il contient, il semble que le procédé le plus simple est de déterminer d'abord les coefficients et les variances des erreurs partant de la forme réduite, et en dériver ensuite les coefficients et les variances des erreurs dans la forme autorégressive. Après cela on peut appliquer les méthodes exposées dans la présente note.

Mr. Stone's paper was discussed by Messrs. Gerhard Tintner, Olav Reiersol, Jacob Marschak, Herman O. A. Wold, Herman Rubin, J. R. N. Stone, and G. Darmois.

FURTHER CONTRIBUTION TO THE SCATTER ANALYSIS*

by Niculae Georgescu-Roegen

Professor, Institute of Statistics, Actuarial Studies, and Calculus, Faculty of Sciences, Bucharest.

1. A scatter in a p-dimensional space can be the statistical image of a single point as well as the statistical image of a $(d < p)$-dimensional variety. If the variates are the height of the parents and that of the offspring, the scatter is the image of a single point that represents the normal couple of heights. In this case, there is no true sense in speaking of a mathematical relation between the two variates. But if the scatter is formed by the observed prices and quantities under conditions of constant supply, we can logically regard the scatter as the image of the supply curve. It seems that the distinction between these different types of scatters has not so far been clearly pointed out. We are thus led to call a scatter that is the image of a d-dimensional variety, a *scatter of the dth order.*[1]

2. We shall assume that the scatter is formed by N groups of observations and that it involves $n+m$ variates $(X^1, X^2, \ldots, X^n, Y^1, Y^2, \ldots, Y^m)$, of which only the variates X are subject to sampling errors. The scatter will also be assumed to be the image of a linear variety.

3. At the beginning, we shall also assume that the deviations $\xi^i = x^i - X^i$ are distributed normally and that each ξ_k^i may be correlated to every other ξ_l^j, but that the distribution reflects the iteration of the successive deviations, *i.e.*, $b_{ij}^{kl} = E(\xi_i^k \xi_j^l) = E(\xi^k_{i+\alpha}\, \xi^l_{j+\alpha}) = b_{i+\alpha,\, j+\alpha}^{kl}$, for all values of α, $E(x)$ being the probable value of x.

If the scatter is of the $(n+m-1)$ order, *i.e.*, the theoretical variety is given by $A_k X^k + B_k Y^k + C = 0$, where $A_k X^k$ stands for $\Sigma_k A_k X_k$, it can be proved that the criterion of maximum likelihood leads to the system

$$\frac{\partial H_0}{\partial A_k} = 0, \qquad \frac{\partial H_0}{\partial B_k} = 0, \qquad \frac{\partial H_0}{\partial C} = 0, \tag{1}$$

* Speaker was not present to read this paper.

[1] Some of the results obtained along this line of thought are to be found in the author's thesis "Le problème de la recherche des composantes cycliques," *Journal de la Société de Statistique de Paris*, October, 1930.

where

$$H_0(t_i) = -\frac{1}{|g_{ij}|}\begin{vmatrix} 0 & t_i \\ t_j & g_{ij} \end{vmatrix},$$

$$(2) \qquad t_i = A_k x_i^k + B_k y_i^k + C, \qquad g_{ij} = g_{i\text{-}j} = g_{j\text{-}i} = b_{ij}^{kl} A_k A_l.$$

If we further assume that there is no chain correlation, the first relation (2) becomes

$$(3) \qquad H_0 = \frac{\sum_i (A_k x_i^k + B_k y_i^k + C)^2}{k(A_1, A_2, \ldots, A_n)},$$

where $e^{k(t_1, t_2 \cdots t_n)/2}$ is the characteristic function of the distribution of $(\xi^1, \xi^2, \ldots, \xi^n)$. From the last equation (1) we obtain $C = 0$, if the origin is taken at the statistical mean values of X and Y.

The minimum of H_0 is easily shown to be equal to the smallest root λ_1 of the equation

$$(4) \qquad \left| \sum_i x_i^k x_i^l - \lambda s^{kl} \right| = 0,$$

where $s^{kl} = E(\xi_i^k \xi_i^l)$. The solution of the system (1) follows immediately.

4. For practical purposes, the preceding results can be applied only when we know the s^{kl} or, at least, their mutual ratios. When all s^{kl} are known, the χ^2 test can be applied to the fitting of the variety thus determined. If only the mutual ratios of the s^{kl} are known, we are able to calculate R by the formula

$$(5) \quad 1 - R^2 = \frac{n\lambda_1}{\lambda_1 + \lambda_2 + \ldots + \lambda_n} = \frac{n\lambda_1}{\Sigma_i h(x_i^1, x_i^2, \ldots, x_i^n)},$$

where $h(\xi^1, \xi^2, \ldots, \xi^n)$ is the reciprocal quadratic form of $k(t_1, t_2, \ldots, t_n)$ and λ_i are the roots of (4). The coefficient R will be called the *rigidity coefficient*. It varies from -1 to $+1$ and, *ceteris paribus*, the greater its absolute value, the better is the fit of the variety calculated. The rigidity coefficient coincides with the known total-correlation coefficient in the case when the scatter is of classical form.

5. When the equation (4) has d roots equal to the smallest one, it can be shown that the scatter is of the $(n+m-d)$ order; that means that the theoretical observed points $M_i(X_i^1, X_i^2, \ldots, X_i^n, Y_i^1, \ldots, Y_i^m)$ lie on an $(n+m-d)$-dimensional linear variety. This result gives us a very important tool for deciding how many variates from a group

of variates are really connected mutually. If $d \neq 1$, by a suitable substitution, we can establish a single linear relation between $(n+m-d+1)$ homogeneous linear forms containing groups of the original variates. This relation will be the mathematical formulation of the *law*.

6. The same results concerning the equation (4) and the system (1) in the case where H_0 is given by (3) are obtained by the method of moments. It can be proved that this method enables us to extend the results to the following far more general conditions, namely:

a) the distribution of ξ is independent of the position of observed points M_i;

b) the distribution of ξ reflects the iteration of the successive deviations;

c) the moments of the distribution of ξ and that of the points M_i in the theoretical variety are finite up to the order required by the proofs.

7. If we further assume that:

d) The distribution of ξ fulfils the ergodic condition, we can work out formulae for the moments of the statistical coefficients involved in the previous method. Unfortunately these formulae are rather complicated, for which reason we avoid giving them in this abstract.

The problem of finding the variation range of the difference between the smallest roots of the equation (4) is most important. We have so far succeeded in working out the formula for the case when $n = 2$ and N_1 is very large. This criterion will allow us to decide whether a two-dimensional scatter represents a two-variate distribution or a true connection between these variates.

8. In actual problems we very seldom know the mutual ratios of the s^{kl}. The preceding results can, however, be immediately applied to the analysis of time series. If $y = f(t)+\eta_t$ represents a time series involving only p sinusoidal components, it is known that $f(t)$ satisfies a finite-difference equation

$$(6) \qquad (\Delta^{(2p)}_2 + A_1\Delta^{(2p-2)} + \ldots + A_p)f = 0.$$

The roots of the corresponding characteristic equation

$$(7) \qquad z^p + A_1 z^{p-1} + \ldots + A_p = 0$$

will determine the lengths of the periods. The even finite differences of y up to the $(2p)$th order will therefore form a scatter of the pth order. From this scatter, by the methods presented above, the variety (6) can be easily obtained, and hence we can deduce the length of the periods.

Résumé

1. Un *scatter* dans un espace à p dimensions peut être aussi bien l'image statistique d'un point que celle d'une variété à $d < p$ dimensions. La distinction entre ces deux types de scatters n'a pas été relevée d'une manière claire. Nous dirons qu'un scatter est de l'ordre d s'il est l'image statistique d'une variété à d dimensions.

2. Nous allons supposer le scatter formé par N groupes d'observations concernant $n+m$ variables $(X^1, X^2, \dots, X^n;\ Y^1, Y^2, \dots, Y^m)$ dont seules les variables X peuvent être entachées d'erreurs d'observation. Le scatter sera supposé être l'image d'une variété linéaire.

3. Supposons que les déviations $\xi^i = x^i - X^i$ sont distribuées selon la loi normale et que la distribution mettra en évidence l'itération des déviations succesives: $b^{kl}_{ij} = E(\xi^k_i\ \xi^l_j) = E(\xi^k_{i+\alpha}\xi^l_{j+\alpha}) = b^{kl}_{i+\alpha,\ j+\alpha}$ pour toutes les valeurs de α, $E(X)$ signifiant la valeur probable de x. Si le scatter est de l'ordre $n+m-1$, la variété théorique sera représentée par $A_kX^k + B_kY^k + C = 0$, où on a noté par A_kX^k la somme $\Sigma_k A_k X^k$. Les valeurs le plus probables de A, B, C seront données par le système (1) où les notations sont celles des relations (2).[1]

Si maintenant nous supposons que les ξ ne sont plus liées en chaîne, la relation (2) devient (3), où $e^{\frac{1}{2}k(t_1, t_2, \dots, t_n)}$ represent la fonction caractéristique de la distribution des ξ. La dernière équation (1) nous montre qu'on peut prendre $C = 0$. Le minimum de H_0 sera donné par la plus petite racine λ_1 de (4), ou $S^{kl} = E(\xi^k_i\ \xi^l_i)$.

4. Les résultats précédents ne peuvent être appliqués en pratique que si on connait au moins les rapports mutuels des s^{kl}. Si tous les s^{kl} sont connus, on peut appliquer le critère χ^2. Si seuls les rapports mutuels des s^{kl} sont connus, on peut calculer R par la formule (5) où $h(\xi^1, \xi^2, \dots, \xi^n)$ est la forme quadratique adjointe à $k(t_1, t_2, \dots, t_n)$ et les λ représentent les racines de (4). R est un coefficient de rigidité qui devient égal au coefficient de correlation totale si le schéma du scatter est classique.

5. Si l'équation (4) a d racines égales à la plus petite des racines, on peut montrer que le scatter est de l'ordre $n+m-d$ et on peut donc déterminer de cette façon combien de variables du groupe observé sont liées par une rélation rigide.

6. Les estimations de A, B, C par les relations précédentes peuvent être justifiées dans des conditions moins restrictives, par la méthode des moments.

[1] Les relations numérotées se trouvent dans la version anglaise.

7. Si la distribution des ξ satisfait le principe ergodique, on peut même calculer les premiers moments des coefficients employés dans la présente méthode.

8. Soit une série dans le temps $y = f(t) + \eta_t$ où $f(t)$ est une somme de p composantes sinusoïdales. Celle-ci satisfait l'équation aux différences finies (6) dont l'équation caractéristique (7) nous donnera par ses racines les périodes cyclique de y. En effet, les différences d'ordre pair de y forment un scatter de l'ordre p pour lequel on connait les rapports mutuels des s^{kl}.

THE ESTIMATION OF PARAMETERS IN LINEAR AUTOREGRESSIVE TIME SERIES

by Maurice G. Kendall

Statistician, Chamber of Shipping (United Kingdom)

1. The problem is to estimate the values of the constants α in a linear autoregressive series of type

$$\alpha_0 u_{t+k} + \alpha_1 u_{t+k-1} + \cdots + \alpha_k u_t = \varepsilon_{t+k} \tag{1}$$

given a number n of consecutive terms of the series. The ε's are values of a random variable and successive values may or may not, in general, be dependent. In the major part of this paper I take them as independent.

Particular cases of (1) of importance are the scheme generating a Markoff chain

$$\alpha_0 u_{t+1} + \alpha_1 u_t = \varepsilon_{t+1} \tag{2}$$

and Yule's equation

$$\alpha_0 u_{t+2} + \alpha_1 u_{t+1} + \alpha_2 u_t = \varepsilon_{t+2}. \tag{3}$$

In all cases I shall take $\alpha_0 = 1$ but it is useful to continue to write α_0 in theoretical expressions.

2. Certain functions of the constants α are continually recurring in investigations of the type here considered and I proceed to discuss them. Let

$$\left(\frac{1}{\sum\limits_{j=0}^{k} \alpha_j t^j} = \sum_{0}^{\infty} \beta_j t^j \right) \tag{4}$$

Thus if T denote the operation of lowering by unity the suffix of a term, *e.g.*, $Tu_{t+1} = u_t$, (1) may be written

$$\left(\sum_{j=0}^{k} \alpha_j T^j \right) u_{t+k} = \varepsilon_{t+k}.$$

A particular integral of the equation is then

$$u_t = \frac{1}{\sum \alpha_j T^j} \varepsilon_t$$

$$= \sum_{j=0}^{\infty} \beta_j T^j \varepsilon_t$$

$$= \sum_{j=0}^{\infty} \beta_j \varepsilon_{t-j}. \tag{5}$$

We assume that expressions giving the complementary function of the solution damp out, *i.e.*, that the roots of

$$\sum_{j=0}^{k} \alpha_j x^j = 0$$

have modulus less than unity. The complete solution of (1) is then given by (5). It will be convenient to regard α_j as defined but zero outside the range 0 to k, β_j as defined but zero for negative j. We may then write our summations as from $-\infty$ to ∞ and in the majority of cases can omit these limits.

3. Let

(6) $$\zeta_j = \operatorname{cov}(\varepsilon_t, \varepsilon_{t+j}).$$

Then, multiplying (1) by u_{t-l} and summing over values of j we find, if the expected values of ε and hence of u are zero,

$$\begin{aligned}(\Sigma\alpha_j\rho_{k+l-j}) \operatorname{var} u &= \Sigma u_{t-l}\varepsilon_{t+k} \\ &= \Sigma\varepsilon_{t+k}\,\Sigma\beta_j\varepsilon_{t-j-l} \\ &= \Sigma\beta_j\zeta_{k+j+l}, \qquad l > -k, \end{aligned} \tag{7}$$

where the ρ's are autocorrelations of the series. If the autocorrelations of the ε's are denoted by τ we have

(8) $$\Sigma\alpha_j\rho_{k+l-j} = \frac{\operatorname{var} \varepsilon}{\operatorname{var} u}\Sigma\beta_j\tau_{k+l-j}.$$

In particular, if successive values of ε are independent we get the Yule-Walker equations:

(9) $$\Sigma\alpha_j\rho_{k+l-j} = 0, \qquad l > -k.$$

4. Multiplying (1) by u_{t+k-l} and summing over u we find

(10) $$(\Sigma\alpha_j\rho_{l+j}) \operatorname{var} u = \Sigma\beta_j\zeta_{l+j}, \qquad l > 0.$$

In particular, if the ε's are independent we have Wold's equations

(11) $$\Sigma\alpha_j\rho_{l+j} = \beta_l\frac{\operatorname{var} \varepsilon}{\operatorname{var} u}.$$

5. Let $A_i = A_{-i}$ be the coefficient of t^i or of t^{-i} in

(12) $$(\Sigma\alpha_j t^j)(\Sigma\alpha_j t^{-j}) = \Sigma A_i t^i.$$

Then

(13) $$A_i = \Sigma\alpha_j\alpha_{j+i}.$$

Then also

$$\begin{aligned}\zeta_l &= \Sigma\varepsilon_t\varepsilon_{t+l} \\ &= \sum_t\left(\sum_i\alpha_i u_{t-i}\right)\left(\sum_j\alpha_j u_{t+l-j}\right) \\ &= \sum_{i,j}\alpha_i\alpha_j\rho_{i+l-j} \operatorname{var} u \\ &= \left(\sum_i A_i\rho_{l-i}\right)\operatorname{var} u. \end{aligned} \tag{14}$$

If the ε's are uncorrelated we then have

$$\text{(15)} \qquad \begin{cases} \Sigma A_i \rho_{l-i} = 0, & l \neq 0, \\ \qquad\quad = \dfrac{\text{var } \varepsilon}{\text{var } u}, & l = 0. \end{cases}$$

These equations are derivable directly from (9) on multiplying by α_l and summing.

We may note in passing that when the ε's are independent

$$\frac{1}{\Sigma A_i\, t^i} = \Sigma \beta_j t^j\, \Sigma \beta_j t^{-j}$$

$$= \Sigma \beta_i \beta_{i+j} t^j$$

$$\text{(16)} \qquad = (\Sigma \rho_j t^j)\, \frac{\text{var } \varepsilon}{\text{var } u},$$

since

$$(\text{var } u)\, \rho_j = \Sigma u_t u_{t+j}$$

$$= \Sigma_t (\Sigma \beta_i \varepsilon_{t-i})\, (\Sigma_l\, \beta_l \varepsilon_{t+j-l})$$

$$= \beta_i \beta_l \zeta_{j+i-l}$$

$$= \beta_i \beta_{i+j} \text{ var } \varepsilon.$$

The expansion of $1/\Sigma A_i t^i$ is to be regarded as a formal one since $\Sigma \beta_j t^j$ and $\Sigma \beta_j t^{-j}$ will not both converge unless perhaps when $t = 1$. The actual expansion must be regarded as a product of $1/\Sigma \alpha_j t^j$ and $1/\Sigma \alpha_j t^{-j}$, the former expanded in ascending powers of t, the latter in descending powers of t. The expression (16) then gives us a generating function for the autocorrelations of the series. (See Quenouille, 1947)

6. Now define

$$\text{(17)} \qquad (\Sigma \alpha_j t^j)^2 = \Sigma B_j t^j,$$

$$\text{(18)} \qquad \frac{\Sigma \alpha_j t^j}{\Sigma \alpha_j t^{-j}} = \Sigma C_j t^j.$$

We may derive the equations

$$\text{(19)} \qquad B_j = \Sigma \alpha_i \alpha_{j-i},$$

$$\text{(20)} \qquad \alpha_j = \Sigma \beta_j B_{i-j},$$

and since

$$\Sigma B_j t^j = (\Sigma A_j t^j)(\Sigma C_j t^j),$$

$$\text{(21)} \qquad B_j = \Sigma C_i A_{j-i}.$$

Furthermore, since

$$\frac{\Sigma \alpha_j t^{-j}}{\Sigma \alpha_j\, t^j} = \Sigma C_j t^{-j},$$

we have, on multiplying by (18)

$$\text{(22)}\qquad \begin{cases} \Sigma C_i C_j = 0, & i \neq j, \\ \qquad\quad = 1, & i = j. \end{cases}$$

Quenouille (1948) has considered functions which, in the parental form, may be written

$$\text{(23)}\qquad \omega_j = \Sigma B_i \rho_{j-i}, \qquad j > k,$$

and has shown that for large samples, w_j, the sample value of ω_j expressed by

$$\text{(24)}\qquad w_j = \Sigma B_i \tau_{j-i}$$

is normally distributed independently of τ_j and the other w's. Since from (14)

$$\zeta_l = (\Sigma A_i \rho_{l-i}) \text{ var } u,$$

we have

$$\sum_j C_j \zeta_{l-j} = \left(\sum_{i,j} C_j A_i \rho_{l-j-i} \right) \text{var } u$$

$$= (\Sigma B_i \rho_{j-i}) \text{ var } u$$

$$= \omega_j \text{ var } u$$

or, equivalently

$$\text{(25)}\qquad \omega_j = \frac{\text{var } \varepsilon}{\text{var } u} \Sigma C_i \tau_{l-j},$$

which exhibits w as a weighted average of the autocorrelations of the ε's.

7. In the case of the Markoff chain

$$u_{t+1} + \alpha u_t = \varepsilon_{t+1}$$

we have

$$A = (1 + \alpha t)(1 + \alpha t^{-1})$$

whence

$$\text{(26)}\qquad A_0 = 1 + \alpha^2, \ A_{-1} = A_1 = \alpha$$

whence

$$B = (1 + \alpha t)^2$$

whence

$$B_0 = 1, \ B_1 = 2\alpha, \ B_2 = \alpha^2$$

$$\text{(27)}\qquad C = (1 + \alpha t)/(1 + \alpha t^{-1})$$

whence

$$\text{(28)}\quad C_1 = \alpha, \ C_0 = 1 - \alpha^2, \ C_{-1} = -\alpha(1 - \alpha^2), \ C_{-2} = \alpha^2(1 - \alpha^2), \text{ etc.}$$

For Yule's equation

$$u_{t+2} + \alpha_1 u_{t+1} + \alpha_2 u_t = \varepsilon_{t+2}$$

we find

$$\text{(29)}\qquad \begin{cases} A_0 = 1 + \alpha_1^2 + \alpha_2^2, \\ A_{-1} = A_1 = \alpha_1(1 + \alpha_2), \\ A_{-2} = A_2 = \alpha_2; \end{cases}$$

$$(30)\quad \begin{cases} B_0 = 1, \; B_1 = 2\alpha_1, \; B_2 = \alpha_1^2 + 2\alpha_2, \\ B_3 = 2\alpha_1\alpha_2, \; B_4 = \alpha_2^2; \end{cases}$$

$$(31)\quad \begin{cases} C_2 = 1 - \alpha_1^2 - \alpha_2^2 + \alpha_2\alpha_1^2, \\ C_1 = \alpha_1(1 - \alpha_2), \\ C_0 = \alpha_2, \\ C_{-1} = -\alpha_1(1 + \alpha_2 + \alpha_1^2 - 2\alpha_2^2 + \alpha_1^2\alpha_2), \\ C_{-2} = -\alpha_2 + \alpha_1^2 + 2\alpha_1^2\alpha_2 - \alpha_1^4 + \alpha_2^3 - 3\alpha_1^2\alpha_2^2 + \alpha_2\alpha_1^4 \text{ etc.} \end{cases}$$

8. I proceed to consider a number of methods for estimating the α's. The theoretical results will be illustrated by data from the following experimental series:

(a) *Series 1.* (*cf.* Kendall, 1946) 480 terms constructed according to the formula

$$u_{t+2} - 1.1u_{t+1} + 0.5u_t = \varepsilon_{t+2}$$

where ε is a rectangular random variable; this series has been split into eight sets of 60 terms each which will be referred to as 1*a* to 1*h*.

(b) *Series 5.* 1600 terms constructed according to the same formula, split into four sets of 400 terms referred to as 5*a*, 5*b*, 5*c*, and 5*d*.

(c) *Series 3.* (Kendall, 1946) 240 terms constructed according to the formula

$$u_{t+2} - 1.1u_{t+1} + 0.8u_t = \varepsilon_{t+2}$$

split into four sets of 60 referred to as 3*a* to 3*d*.

(d) *Series 6.* 1600 terms constructed according to the same formula split into four sets of 400 referred to as 6*a* to 6*d*.

(e) *Series 2.* (Kendall, 1946) 240 terms according to

$$u_{t+2} - 1.2u_{t+1} + 0.4u_t = \varepsilon_{t+2}$$

split into four sets of 60, 2*a* to 2*d*.

(f) *Series 4.* (Kendall, 1946) 240 terms according to

$$u_{t+2} + u_{t+1} + 0.5u_t = \varepsilon_{t+2}$$

split into four sets of 60, 4*a* to 4*d*.

In all cases the variable ε was taken from the *Tables of Random Sampling Numbers* by Babington Smith and myself. The observed serial correlations for series 5 and 6 were calculated according to the formula

$$(32) \qquad r_k = \frac{\Sigma u_i\, u_{i+k}}{\Sigma u_i^2}$$

that is to say, about the true mean zero, not the sample mean. To make the number of pairs contributing to $\Sigma u_i u_{i+k}$ exactly 500 in the subseries the set was taken as circular, *e.g.*, $\Sigma u_i u_{i+2}$ contained as its last two terms $u_{499}u_1$ and $u_{500}u_2$. The values of the Σ's for the full series of 1600 were taken as the sum of those for the four subseries. For series of the length here considered these approximations make practically no difference, and in any case there is a theoretical reason for preferring (32) to

$$r_k = \frac{\text{cov}\,(u_i, u_{i+k})}{\sqrt{\{\text{var}\; u_i \;\text{var}\; u_{i+k}\}}}.$$

See para. 17 (d) below.

For my original series 1 to 4 I did not use circular sets, but calculated according to the formula for n terms

$$r_k = \frac{\sum_{i=1}^{n-k} u_i\, u_{i+k}}{\sqrt{\left\{\sum_{i=1}^{n-k} u^2_i \quad \sum_{i=1}^{n-k} u^2_{i+k}\right\}}}.$$

METHOD 1

9. The method originally used by Yule (1927) was to regard (1) as a regression equation. This led, via the usual minimization of sums of squares, to the first k of equations (9), with sample values r instead of ρ. In the particular case of Yule's equation this leads to

$$r_1 + a_1 + r_1 a_2 = 0,$$

$$r_2 + a_1 r_1 + a_2 = 0,$$

or

$$(33) \qquad \begin{cases} a_1 = -\dfrac{r_1(1 - r_2)}{1 - r_1^2}, \\[2ex] a_2 = \dfrac{r_1^2 - r_2}{1 - r_1^2}, \end{cases}$$

where we denote estimators of the α's by Roman letters. Table 1 gives the results of applying these formulae to the experimental series.

TABLE 1

Series	a_1	α_1	a_2	α_2	Series	a_1	α_1	a_2	α_2
1a	−0.891	−1.1	0.149	0.5	3a	−1.093	−1.1	0.710	0.8
1b	−1.047	,,	0.649	,,	3b	−1.049	,,	0.706	,,
1c	−0.939	,,	0.292	,,	3c	−1.109	,,	0.819	,,
1d	−1.194	,,	0.646	,,	3d	−1.142	,,	0.842	,,
1e	−1.102	,,	0.495	,,					
1f	−1.196	,,	0.574	,,	Mean of 3a-3d	−1.098	,,	0.769	,,
1g	−1.580	,,	0.785	,,					
1h	−1.219	,,	0.474	,,	3 (as a whole)	−1.125	,,	0.806	,,
Mean of 1a-1h	−1.146	,,	0.508	,,	6a	−1.085	,,	0.821	,,
					6b	−1.076	,,	0.793	,,
1 (as a whole)	−1.132	,,	0.485	,,	6c	−1.111	,,	0.808	,,
					6d	−1.096	,,	0.733	,,
5a	−1.111	,,	0.468	,,					
5b	−1.117	,,	0.524	,,	Mean of 6a-6d	−1.092	,,	0.789	,,
5c	−1.067	,,	0.546	,,					
5d	−1.065	,,	0.500	,,	6 (as a whole)	−1.092	,,	0.790	,,
Mean of 5a-5d	−1.090	,,	0.509	,,	4a	−1.016	1.0	0.453	0.5
5 (as a whole)	−1.094	,,	0.510	,,	4b	−1.022	,,	0.373	,,
					4c	−1.080	,,	0.600	,,
2a	−1.087	−1.2	0.404	0.4	4d	−1.079	,,	0.670	,,
2b	−1.474	,,	0.565	,,					
2c	−1.194	,,	0.485	,,	Mean of 4a-4d	−1.049	,,	0.524	,,
2d	−1.178	,,	0.405	,,					
Mean of 2a-2d	−1.233	,,	0.465	,,	4 (as a whole)	−1.062	,,	0.552	,,
2 (as a whole)	−1.207	,,	0.420	,,					

The results are fairly good. There is no evidence of bias, so far as I can see, and for series of 240 terms or more the estimates are reasonably close to the true values.

10. The reasons for searching for anything better are these:

(a) In the general case when ε is not a normal variate there is no particular reason for regarding (1) as a regression equation, at least so far as concerns the least-squares solution. In the normal case the frequency function of the ε's is proportional to

$$\exp\left(-\frac{\Sigma\varepsilon^2}{2\operatorname{var}\varepsilon}\right) \propto \operatorname{Exp}\left\{\frac{-1}{2\operatorname{var}\varepsilon}\sum_k{}'\left(\sum_j \alpha_j u_{t+k-j}\right)^2\right\}.$$

The maximum-likelihood estimators of the α's are then the values which minimize $\Sigma_k(\Sigma_j\,\alpha_j\,u_{t+k-j})^2$ which leads back to Yule's method. But if the ε's are rectangular (as for instance in all my experimental series) no maximum-likelihood estimators exist except in a rather trivial sense and the validity of the method must rest on other grounds;

(b) the shortness of the series available in many fields makes it necessary to squeeze the maximum amount of information out of observed data even at the expense of considerable arithmetical inconvenience;

(c) Yule's method only employs the first k serial correlations and a method that draws on more may be more accurate. It is far from certain that this must be so, because the lower serials may constitute a sufficient set, but the possibility is worth examining.

METHOD 2

11. Consider then the Yule-Walker equations (9). These are valid whatever the distribution of ε, provided that successive values are independent. If we estimate the autocorrelations by the observed serials r we have for the estimates

$$\Sigma a_j r_{k+l-j} = 0, \qquad l > k, \tag{34}$$

For instance, with Yule's equation

$$\begin{aligned} r_1 + a_1 \quad + a_2 r_1 &= 0, \\ r_2 + a_1 r_1 + a_2 \quad &= 0, \\ r_3 + a_1 r_2 + a_2 r_1 &= 0, \\ \text{etc.} \end{aligned} \tag{35}$$

The solution of the first two gives us the results of Method 1. Suppose we find a least-squares solution of the first m of these equations, *i.e.*, minimize

$$\sum_{l=1}^{m} \left(\sum_{j=0}^{p} a_j r_{k+l-j} \right)^2.$$

This leads to the k equations

$$\sum_{p=1}^{m} r_{k+l-p} \quad \sum_{j=0}^{k} a_j r_{k+l-j} = 0, \qquad p = 1, \ldots, k. \tag{36}$$

For instance, with the first three equations of (35) we have, writing $r_0 = 1$ to preserve the symmetry of the notation

$$\begin{aligned} (r_0 r_1 + r_1 r_2 + r_2 r_3) + a_1(r_0^2 + r_1^2 + r_2^2) + a_2(r_0 r_{-1} + r_1 r_0 + r_2 r_1) &= 0, \\ (r_{-1} r_1 + r_0 r_2 + r_1 r_3) + a_1(r_{-1} r_0 + r_0 r_1 + r_1 r_2) + a_2(r_{-1}^2 + r_0^2 + r_1^2) &= 0. \end{aligned} \tag{37}$$

TABLE 2(a) (See text)

Number of Equations used	Series 1 ($\alpha_1 = -1.1$, $\alpha_2 = 0.5$)		Series 2 ($\alpha_1 = -1.2$, $\alpha_2 = 0.4$)		Series 3 ($\alpha_1 = -1.1$, $\alpha_2 = 0.8$)		Series 4 ($\alpha_1 = 1.0$, $\alpha_2 = 0.5$)	
(m)	$-a_1$	a_2	$-a_1$	a_2	$-a_1$	a_2	a_1	a_2
2	1.132	0.486	1.207	0.420	1.125	0.807	1.062	0.552
3	1.104	0.454	1.255	0.472	1.116	0.796	1.128	0.627
4	1.089	0.439	1.294	0.511	1.116	0.796	1.137	0.634
5	1.075	0.427	1.328	0.543	1.116	0.796	1.137	0.634
6	1.073	0.427	1.325	0.540	1.113	0.791	1.136	0.626
7	1.073	0.427	1.328	0.543	1.103	0.782	1.129	0.615
8	1.075	0.428	1.330	0.544	1.104	0.782	1.117	0.602
9	1.074	0.427	1.329	0.544	1.103	0.777	1.115	0.601
10	1.074	0.428	1.329	0.543	1.103	0.777	1.120	0.605

TABLE 2(b) (see text)

Number of Equations used (m)	Series 3		Series 4	
	$-a_1$	a_2	a_1	a_2
11	1.105	0.778	1.122	0.606
12	1.108	0.777	1.121	0.606
13	1.108	0.774	1.121	0.606
14	1.109	0.777	1.117	0.602
15	1.114	0.780	1.118	0.603
16	1.118	0.780	1.118	0.603
17	1.118	0.779	1.117	0.603
18	1.119	0.780	1.120	0.606
19	1.120	0.781	1.122	0.608
20	1.120	0.781	1.122	0.608
21	1.120	0.782	1.122	0.609
22	1.122	0.785	1.124	0.610
23	1.126	0.787	1.128	0.613
24	1.126	0.787	1.132	0.615
25	1.127	0.790	1.131	0.615
26	1.131	0.794	1.136	0.624
27	1.134	0.795	1.152	0.637
28	1.135	0.795	1.159	0.641
29	1.135	0.796	1.156	0.642
30	1.138	0.798	1.158	0.644

The results of applying this method to Series 1 to 4 up to $m = 10$ are shown in Table 2(a) and to Series 3 and 4 up to $m = 30$ in Table 2(b). Evidently the method is no improvement on Method 1. In Table 2(a) the resulting values appear to converge (illusorily) with increasing m, but to values which are in some cases further from the truth than the results of Method 1; in Table 2(b) the values for $m = 30$ are usually further away from the truth than for $m = 2$.

12. One might have expected (if one had not learnt to expect nothing in experiments with time series) that where sufficient estimators do not exist the use of additional information provided by further serial coefficients would have resulted in increased precision. The explanation is, I think, that the serials of higher order are so much affected relatively by sampling effects, even in long series, that any gain from the use of additional coefficients is more than offset by the consequent importation of sampling unreliability.

METHOD 3

13. The third method I tried was to determine the α's by equating the first k covariances of ε to zero, *i.e.*, by using equations (14) with $r_l = \rho_l$. In the particular case of Yule's equation this leads for $l = 1, 2$ to,

$$A_{-2}r_3 + A_{-1}r_2 + A_0r_1 + A_1r_0 + A_2r_{-1} = 0,$$
$$A_{-2}r_4 + A_{-1}r_3 + A_0r_2 + A_1r_1 + A_2r_0 = 0.$$

Using the values of (29) with a for α we find

$$(38)\qquad a_2\left\{\frac{r_1+r_3}{1+r_2}-\frac{1+r_4}{r_1+r_3}\right\}+(1+a_2{}^2)\left\{\frac{r_1}{1+r_2}-\frac{r_2}{r_1+r_3}\right\}$$

$$+\frac{a_2{}^2}{(1+a_2)^2}\left\{\frac{r_1}{1+r_2}-\frac{r_2}{r_1+r_3}\right\}\frac{\left(\frac{r_1+r_3}{r_1}-\frac{1+r_4}{r_2}\right)^2}{\left(\frac{1+r_2}{r_2}+\frac{r_1+r_3}{r_2}\right)^2}=0$$

which gives a_2, whereupon we can ascertain a_1 from

$$(39)\qquad a_1(1+a_2)\left\{\frac{1+r_2}{r_1}-\frac{r_1+r_3}{r_2}\right\}+a_2\left\{\frac{r_1+r_3}{r_2}-\frac{1+r_4}{r_2}\right\}=0.$$

Equation (38) can be easily solved by successive approximation, and this method was used for the experimental results given in Table 3. It was, however, pointed out to me by Miss Ayling, who did the computing, that the quartic (38) is of a special type, which can be reduced to a quadratic. Writing (38) in the form

$$a+p(1+a^2)+q\,\frac{a^2}{(1+a)^2}=0\quad ;$$

we have

$$p(1+a^4)+(1+2p)(a^3+a)+(2+2p+q)a^2=0,$$

or

$$p\left(a^2+\frac{1}{a^2}\right)+(1+2p)\left(a+\frac{1}{a}\right)+2+2p+q=0,$$

whence, if

$$b=a+\frac{1}{a}$$

$$(40)\qquad pb^2+(1+2p)b+(2+q)=0.$$

TABLE 3 (See text)

Series	α_1	a_1	α_2	a_2
1	-1.1	-1.178	0.5	0.572
5a	-1.1	-1.112	0.5	0.450
5b	-1.1	-1.104	0.5	0.526
5c	-1.1	-1.117	0.5	0.626
5d	-1.1	-1.028	0.5	0.467
(in total)	-1.1	-1.094	0.5	0.522
2	-1.2	-1.647	0.4	0.501
3	-1.0	n.a.	0.8	n.a.
6a	-1.1	n.a.	0.8	n.a.
6b	-1.1	-1.158	0.8	0.906
6c	-1.1	-1.135	0.8	0.806
6d	-1.1	-1.094	0.8	0.768
(in total)	-1.1	-1.143	0.8	0.874
4	1.0	1.041	0.5	0.375

Table 3 shows the results of applying this method to Series 1 to 4 and to the constituent series and the whole of Series 5 and 6. In two cases the method failed to give an acceptable result, for Series 3 because a_2 became greater than unity and for Series 6*a* because a_1 and a_2 became imaginary. This alone would be enough to condemn the method. A comparison of this Table with Table 1 will show that the present method gives in nearly every case worse results than that of Method 1.

METHOD 4

14. A further method is to use equations (24) with w equal to zero, its expected value when the successive values of ε are independent [as is seen from (25)]. This leads to

$$\Sigma B_1 r_{j-1} = 0, \qquad j > k. \tag{41}$$

For Yule's equation this leads to

$$\left\{\begin{array}{l} r_3 + 2a_1r_2 + (a_1^2 + 2a_2)r_1 + 2a_1a_2r_0 + a_2^2r_1 = 0, \\ r_4 + 2a_1r_3 + (a_1^2 + 2a_2)r_2 + 2a_1a_2r_1 + a_2^2 = 0, \end{array}\right. \tag{42}$$

etc. Taking the first two of these equations as written in full in (42), one can solve for a_1 and a_2 to obtain new estimators.

Equations (42) may be solved by eliminating one of a_1, a_2, giving a quartic in the other. The method of taking trial values and successively approximating I found unsatisfactory because of multiple roots which straddle the real values.

15. This method proved even more unsatisfactory than Methods 2 and 3. For the four series 5*a* to 5*d* I found the following pairs of values for α_1 (−1.1);

5*a*	−1.489,	−0.744,
5*b*	−1.148,	−1.087,
5*c*	−1.265,	−0.799,
5*d*	−1.110,	−0.967;

and the following for α_2 (0.5)

5*a*	0.591,	0.320,
5*b*	0.674,	0.368,
5*c*	0.812,	0.353,
5*d*	0.664,	0.346.

Some values for the other series were worked out, but they are hardly worth recording.

16. The result of the experiments, then, was to confirm that even when the random component is not normal, the original method of Yule seems to give the most satisfactory results. I have not arrived

at any convincing theoretical reason why this should always be so. Possibly, even for rectangular variation such as was used for my experimental series, the distribution of ε is "near enough" to normal to make the normal theory applicable. Certainly, it seems to me, the use of higher serials is dangerous on sampling grounds, for in damped series they are *relatively* more affected by sampling fluctuation than the lower serials and sample estimates are more unreliable.

17. Two comments may be added on some theoretical points:

(a) it would be possible to work out standard errors for the estimators considered above, although the work would become intolerably tedious for methods 2, 3 and 4. (T.W. Anderson, 1947, following Mann and Wald, 1943, has recently worked out the standard errors for method 1 applied to Series 1, 2, 3, and 4). However, such standard errors themselves depend on the autocorrelations for which we should have to substitute the observed serials; and this would involve further error when observed values may differ very markedly from true values. I did not think the labor would be worth while at this stage;

(b) if the Yule principle of estimation is regarded as one of least squares, there may be some general result to the effect that the estimators have minimal variance. The case does not appear to be covered by known results but it would be worth examining;

(c) the use of maximum-likelihood solution, even when the random element is normal, may lead to estimators that are slightly biassed. As this bias is of the order $1/n$ I do not think it is important for series of the length here considered. For shorter series it may become more serious;

(d) the use of the formula

$$r_k = \frac{\sum_{i=1}^{n-k} x_i x_{i+k}}{\sum_{i=1}^{n} x_i^2} \quad \frac{n}{n-k}$$

or

$$r_k = \frac{\sum_{i=1}^{n-k} x_i x_{i+k}}{\sqrt{\left\{ \sum_{i=1}^{n-k} x_2^2 \sum_{i=1}^{n-k} x_{i+k}^2 \right\}}}$$

is better than those for which r_k is worked out as an ordinary correlation about the sample mean. This was possible in my experimental series, but when no theoretical mean is known, the point is rather troublesome, though I do not think that it is important for series as long as the experimental ones considered above.

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Résumé

Dans la théorie des séries statistiques ordonnées dans le temps on est amené à l'équation, dite autorégressive,

$$u_{t+k} + \alpha_1 u_{t+k-1} + \dots + \alpha_k u_t = \varepsilon_{t+k} \tag{1}$$

où les quantités u sont les termes de la série, ε est une variable aléatoire et les α sont des paramètres du système. Etant donnée une série observée de ce type on cherche à estimer les α sous l'hypothèse que les valeurs successives des ε sont indépendantes.

On déduit aisément de l'équation (1) les relations

$$\rho_t + \alpha_1 \rho_{t-1} + \dots + \alpha_k \rho_{t-k} = 0, \quad t > -k, \tag{2}$$

où les ρ sont les coefficients d'autocorrélation de la série. Si l'on met $\rho_i = r_i$ (r_i étant le coefficient de corrélation sériale d'ordre i de la série observée), dans les k premières équations du type (2), on peut déterminer les α. C'est là le procédé de Y u l e, auquel il est arrivé en considérant (1) comme une équation de régression. On peut justifier cette méthode quand les ε suivent la loi de Laplace-Gauss (la loi normale). Dans le cas contraire la justification est plus difficile. Pour cette raison, j'ai cherché des méthodes nouvelles pour estimer les α.

Si l'on prend plus de k équations du type (2) on peut déduire une solution par la méthode des moindres carrés. Dans les cas expérimentaux que j'ai considérés cette méthode ne donne pas de meilleurs résultats que celle de Yule.

L'on peut aussi déduire des équations, linéaires en ρ, pour la covariance de ε_i et ε_{i+k} et en mettant $\rho_i = r_i$ on arrive à des équations, différentes des (2), qui admettent d'être résolues pour les α. Cette méthode paraît pire que celle de Yule dans un nombre de cas expérimentaux.

Quenouille et Orcutt ont récemment considéré des fonctions, également lineaires en ρ, qui sont effectivement des moyennes des coefficients de corrélation sériale des ε. Ces quantités fournissent une quatrième méthode pour estimer les α. Les équations ne sont pas facile à résoudre dans le cas général.

Je présenterai des résultats expérimentaux sur une série artificielle de 1600 termes.

Mr. Kendall's paper was discussed by Messrs. Gerhard Tintner, Leonid Hurwicz, Jerzy Neyman, Herman Rubin, Felix Bernstein, G. Rasch, Olav Reiersol, and Maurice G. Kendall.

ECONOMETRICS AND PRIVATE BUSINESS

Tuesday, September 9, at 4:00 p.m.

CHAIRMAN :

Jacques Dumontier

Administrateur de l'Institut de Conjoncture (France)

LAWS OF PRODUCTION AND COST

by Ivar Jantzen

Civil Engineer, Dr. techn. h.c. (Denmark)

The aim of business economics is not only to give a description and a causal analysis of methods, but also especially to investigate the possibilities for:

I. Minimum cost for a given quantity of production (theory of production, cost, and rationalization);

II. Maximum profit (price policy).

Both problems involve an analysis of principles and methods. It must be added that the attainment of point II also involves the attainment of point I for a given quantity of production.

This paper is mainly confined to the theory of (1) production cost and (2) rationalization of industrial plants with constant marginal cost in the sectors of production.

The first of these theories illustrates the necessity of utilizing to the greatest possible extent all given capacity (maximum production per unit of time), if necessary by successive expansion of productive sectors forming bottlenecks for the utilization of other sectors.

The second problem is for each sector to find the technique (plant, method of working, organization, etc.) best adapted to give the lowest possible cost for the quantity planned to be produced.

My object for these technico-economic investigations is the *sector* (a working unit with its own accounting). The sector is a relative term. It may be a man, a bureau, a workshop, a department, or an aggregate

of these and other elements, a business, a trade, a community, etc. We can consider a series of economic sectors, where many of the smaller sectors constitute a larger sector, *e.g.*, many sectors a business, many businesses a trade, many trades a trade group, and many trade groups the national production. We may speak about business economics for every one of these sectors. It will often be possible to look at a higher sector as if it were one of the lower sectors, *e.g.*, to consider the community as if it were a business, etc. Conversely we may consider a business as a small community, a sector as a small business, etc.

When investigating one of these sectors or parts of them, it will always be profitable to have a survey of the production cost and other items in tables or diagrams showing the variations of these items as functions of quantities produced, hours worked, or values produced.

I shall confine most of this paper to the analysis of an *industrially working sector*, *e.g.*, in manufacturing industry, where production in certain intervals has a constant marginal cost.

For all *biological production*, *e.g.*, agriculture, and always where biological growth is the decisive factor, we must take into account a certain element of fatigue necessitating a corresponding increase of the marginal cost with increasing effect.[1] As the *market* is also a living being, we have the same case, and therefore the selling cost must increase with increasing effect of the sales department in order to conquer such fatigue.

Fig. 1 is a figurative sketch of an industrial working sector. The way or technique of making a product requires a plant of different elements, which are available only in certain indivisible "quanta" or blocks paid for time elapsed and not for quantities of products produced.

The fixed costs per week, the *overhead*, for these different quanta are also indivisible quanta of dollars, which we will call A_1, A_2, ... The aggregate of the quanta we will call the *nucleus* of the sector consisting of *columns* of uniform *quanta*.

(Even if a nucleus is at present in the best possible technique, a spark in the brain of an inventor can blow up one of the columns or all of them. We have often seen that the consequence has been a chain reaction all over the world.)

When a quantum A is working at full capacity k we shall have the minimum cost per unit $A : k = a$, for this quantum, *i.e.*, $A = ak$. The area of the nucleus, $= \Sigma ak$, represents the overhead of the sector, and the area bz represents the current cost, where b = marginal cost and

[1] Effect = production per time unit; maximum effect = capacity; production = number of units produced.

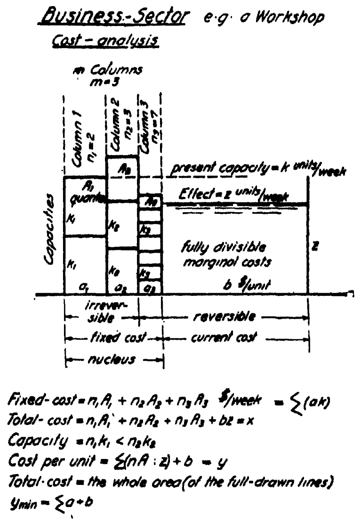

Figure 1

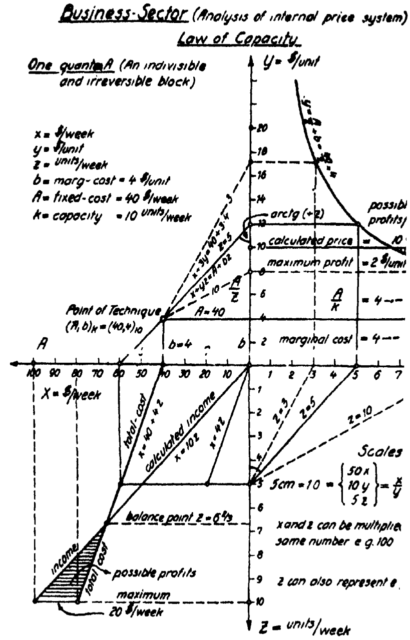

Figure 2

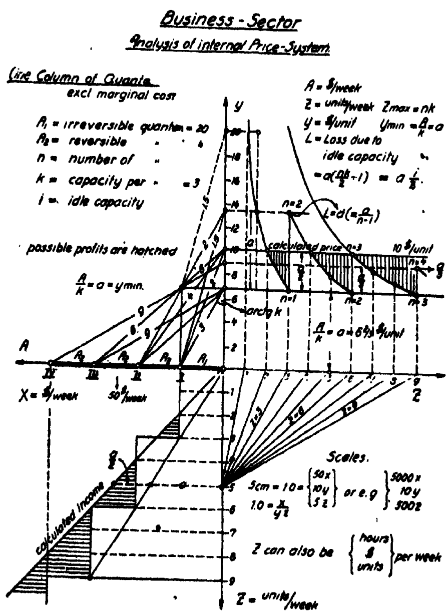

Figure 3

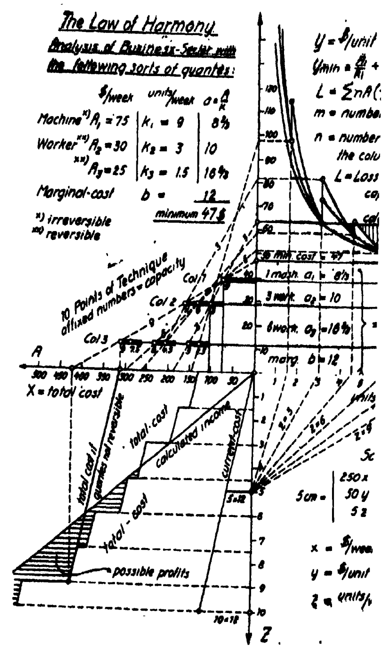

Figure 4

z = units per week produced. If b were not constant the area of divisible costs should widen at the top.

The whole minimum cost per unit $= \Sigma a+b$. All other costs per unit are due to idle capacity and are losses. When the columns do not have the same height and are not fully utilized at the same time, there must always be *costs due to idle capacity* above the necessary minimum cost per unit.

If for a single quantum L = loss per unit due to idle capacity and i = idle capacity at an effect $= z$, we have: $i = k-z$, $A = (L+a)z = ak$, *i.e.*, $L = ai/z$.

In the next figures we shall see first a *quantum* (or a sector regarded as an indivisible block), next a *column of quanta*, and then a *whole nucleus*, and for each of them and corresponding marginal costs we shall see the development of the cost per unit $= A{:}z+b = y$, and of the total cost $= A+bz = x$, *i.e.*, we shall see the costs as functions of the quantities z produced per week.

The quanta can be *reversible*, *i.e.*, they can be dismissed or discharged when not needed, or they can be *irreversible*, *i.e.*, they cannot be removed from the sector.

Fig. 2 shows a *single quantum* or a business sector regarded as an indivisible block, and this time represented in one point, the *point of technique*, which determines the whole figure, having fixed cost A and marginal cost b as seen in the (A, b) system of coordinates on the left-hand side. The y-axis at the top is in dollars per unit, the z-axis on the right is in units per week.

As \$/unit $\times$ units/week = \$/week, which we have on the x-axis at the left, we must have in the figure as in reality $x : yz = 1.0$. This quantity 1.0 is represented on all the axes: on the x-axis by 50 \$/week, on the y-axis by 10 \$/unit, and on the z-axis by 5 units/week, because $\frac{50}{10\times 5} = 1.0$.

Downwards we also have a z-axis, and when we draw lines from 1.0 on this z-axis to the different values of z on the horizontal z-axis, these lines will make with the vertical axis an angle whose tangent is $1/z$.

A line through the point of technique and parallel to the z-line will have the equation $x+yz = A+bz$, and will intersect the y-axis at the level $y = A{:}z+b$ and the x-axis at a distance $x = A+bz$, *i.e.*, on the y-axis the cost/unit and on the x-axis the total cost, because the upper angle with the y-axis has a tangent $= z$.

Each point of the line, taken as a point of technique for a plant, would of course give the same results if there were possible plants corresponding to these points. We shall deal with these problems later on.

A line revolving around the point of technique showing the different units/week will indicate the graphs for cost/unit in the yz-system (hyperbola) and the cost/week (total cost = straight line $x = A+bz$) in the xz-system as seen in the figure. The development of these graphs is called the *Law of Capacity* for the plant given by the point of technique.

We estimate an *internal price* per unit giving *balance point* of 2/3 of the capacity, 10 units/week. In the hatched areas we see the possible *internal profits*. If $z_0 = \frac{2}{3}k$, $i_0 = \frac{1}{3}k$, we have:

Maximum profit $= L_0k = aki_0/z_0 = \frac{1}{2}A$, and maximum profit per unit $= L_0 = \frac{1}{2}a$.

Fig. 3 shows a column of 4 uniform quanta, one of them irreversible. They have a capacity of 3 units/week each, an overhead cost of 20 \$/week, *i.e.*, a = minimum cost/unit $= 6\frac{2}{3}$ \$. We assume no marginal cost here, and therefore we see the 4 points of technique on the x-axis itself.

Rotating a line around point I from $z = 0$ to $z = 3$, we can draw the first hyperbola for cost/unit and the first step for total cost. For the next points we rotate from 3 to 6 and 6 to 9, etc., putting in new quanta when the preceding one has exhausted its capacity.

It is easily seen that the risk (L) by putting in a new quantum and not using it will be an increased cost/unit, $L = a{:}\,(n-1)$, when n is the number of the new quanta, as $L = ai/z$, where $i = k$ and $z = (n-1)k$; *i.e.*, for quanta Nos. 2, 3, 4, 5, ..., the maximum loss due to idle capacity is a, $a/2$, $a/3$, $a/4$, ...

Fig. 4 shows a whole plant with 3 columns of quanta, which must all have some idle capacity at the same time for the plant to be able to increase its effect. If a column has exhausted its capacity, we shall have a bottleneck and must add a new quantum to the column, taking the above-mentioned risk of new losses due to idle capacity.

The quanta in this example have capacities in the first column of 9 units/week, in the second column 3 units/week, and in the third column 1.5 units/week. (It can be a machine with 3 shifts: 3 machine workers and for each of them 2 finishing workers. For simplicity the same wage is assumed for all shifts.)

The minimum cost/unit for each quantum is seen to give the sum of 47 \$/week $= a$, which is marked out also in the yz-system, and the columns will arrange their technical points in the Ab-system at a vertical reciprocal distance $= a = A{:}k$.

When we draw up the cost lines in exactly the same way as in the previous graphs, we shall see the graphs for each of the 10 points of technique, and see that column 2 is above column 1 and column 3 above column 2. For column 1 we have the minimum at 9 units/week, for column 2 at 3 and 6 units/week, and for column 3 at 1.5, 3.0, 4.5, 6.0,

7.5, and 9.0 units/week. As 9 is a common multiple of the capacities of all the quanta, we have here an absolute minimum cost/unit. We say that the capacities are in *harmony.*

Taking the estimated *internal price* that gives a balance point at 2/3 of the capacity 9, we see the different *possible profits* in the hatched areas.

This development of cost curves for a complicated plant consisting of elements of different capacities will be called the *Law of Harmony*, which is the complicated Law of Capacity. We *harmonize* the plant by adding new quanta to the columns that otherwise would make bottlenecks for the other columns. When there is no harmony (as here at point 9) we shall always have costs that are losses due to idle capacity. If the quanta are not reversible, we see in the xz-system for total costs the risk when not working at full capacity.

Fig. 5 is the same example as Fig. 4, but shown numerically. On the right-hand side we see the variation in the cost/unit ending in the absolute minimum of 47 $/unit, and then immediately increasing to 61.4 when we put in 3 new quanta. The middle series shows the losses due to idle capacity for Column 2, where minimum cost = 10. After the first minimum we have 10, after the next we have 5, and after the third 3.3 = 10:3, following the formula $L = a:(n-1)$, where n is the number of the next quantum.

In the previous graphs we have seen the development of the graphs for total cost and cost/unit for given quanta and column is in the nucleus, *i.e.*, in a given technique for an existing plant, and we have seen the effect of adding new quanta to the columns of the nucleus. In Figs. 6-8 we assume that we are going to *choose* the best technique to produce a planned quantity z per week, whether it be a whole plant to be built or only a column of an existing nucleus to be rationalized.

We assume different empirical possibilities for rationalization or planning, and for each of these we mark out a point of technique (A, b) in the Ab-system of coordinates.

Shifting the direction z toward this group of points we mark out the first point we meet that has $k \geqslant z$; this point will give minimum cost/unit for a production of z units/per week.

When a line rotates around this point we can draw the hyperbola for cost/unit and the straight line for total cost for the given technique.

We can employ the same method with regard to other values of z and mark the corresponding best points of technique. All these resulting lines, meeting the first point of the group, will form an *envelope* (here a polygon) $A = f(b)$ = *The Law of Technique* for producing the product.

When a z-line rolls on this envelope we obtain on the y-axis the smallest possible unit costs, and on the z-axis the smallest possible total cost

Indices	1	2	3	
A = quantum	75	30	25	$/week
k = Capacity	9	3	1.5	units/week
a = A/k	8⅓ + 10 + 16⅔			+ 12 (b) = 47

	number of quanta: A = n, Indices 1	2	3	n · k, Indices 1	2	3	$a(\frac{nk}{z}-1)$, Indices 1	2	3	Loss due to idle capacity = 1 + 2 + 3	cost/unit = 47 + Loss
	1	1	1	9	3	1.5	66.5	20	8.2	94.7	141.7
5	1	1	1	9	3	1.5	41.6	10	0	51.6	98.6
5	1	1	2	9	3	3	41.6	10	16.3	67.9	114.9
	1	1	2	9	3	3	29.1	5	8.2	42.3	89.3
	1	1	2	9	3	3	16.6	0	0	16.6	63.6
	1	2	3	9	6	4.5	16.6	10	8.2	34.8	81.8
	1	2	3	9	6	4.5	10.4	5	2	17.4	64.4
.5	1	2	3	9	6	4.5	8.3	3	0	11.3	58.3
.5	1	2	4	9	6	6	8.3	3	5.4	16.7	63.7
[illegible]	1	2	4	9	6	6	6.7	2	3.3	12	59
[illegible]	1	2	4	9	6	6	4.2	0	0	4.2	51.2
[illegible]	1	3	5	9	9	7.5	4.2	5	4.1	13.3	60.3
7	1	3	5	9	9	7.5	2.5	2.9	1.2	6.6	53.6
7½	1	3	5	9	9	7.5	1.7	2	0	3.7	50.7
7½	1	3	6	9	9	9	1.7	2	3.3	7.0	54
8	1	3	6	9	9	9	0.9	1.2	2	4.1	51.1
9	1	3	6	9	9	9	0	0	0	0	47
9	2	4	7	18	12	10.5	8.3	3.3	2.8	14.4	61.4
10	2	4	7	18	12	10.5	6.7	2	0.8	9.5	56.5

Figure 5

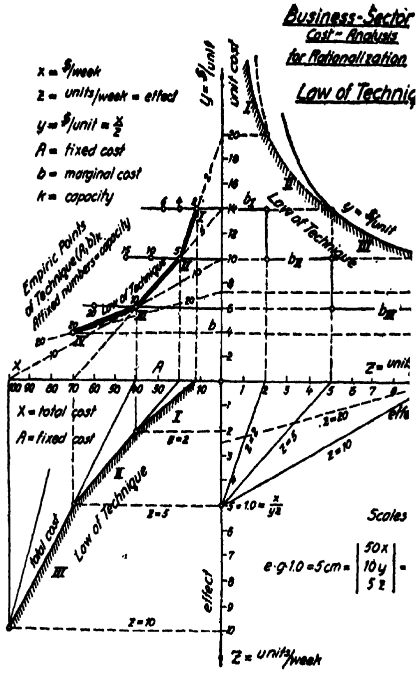

Figure 6

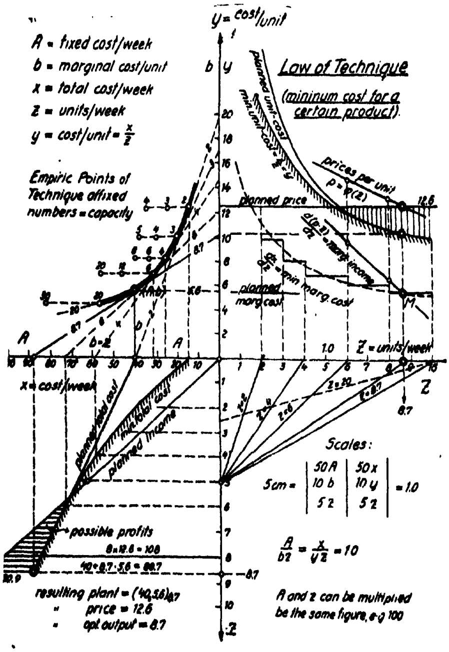

Figure 7

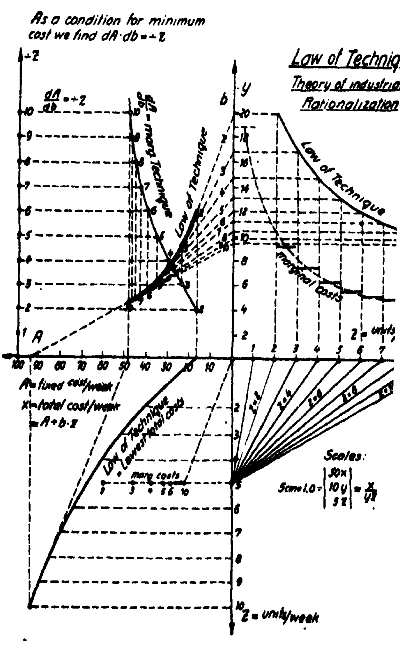

Figure 8

for the production of each quantity of units per week in the present technological situation. These lines are shaded and are seen to be envelopes of all the previously mentioned hyperbolas and straight lines for cost/unit and total cost.

Fig. 7 is another example of the same sort with a *continued graph* for the *Law of Technique*, $A = f_1(b)$, $y = f_2(z)$, $x = f_3(z)$, and further the demand function $p = \phi(z)$ for our product. We shall find the short-run optimum quanlity to produce and then the optimum plant for this production.

From $p = \phi(z)$ we find the corresponding points of the marginal revenue $= d(p.z){:}dz$. The intersection of this latter curve with the marginal cost $dx{:}dz$ gives the point of monopoly $z_m = M$. A tangent to $A = f(b)$ with direction z_m touches the curve at the point of technique, as expected, and in our example this point is $(A,b)_k = (40,\ 5.6)_{8.7}$. The planned price will be 12.6 \$/unit and the planned profit 20.9 \$/week.

Fig. 8 represents the *theory of rationalization*. Seeking the condition for the minimum of the total cost $x = A + bz$, where $A = f(b)$, we differentiate and take $dx{:}db = 0$, finding $dA{:}db = -z$ as a condition for minimum cost.

The figure shows the curve $dA{:}db$ drawn for the empirical law of technique (having reached the lowest point this curve will be discontinued by harmonization or repetition). The condition is identical with the results in the previous figures, *i.e.*, that a tangent to the law of technique forming an angle arctan $-z$ with the y-axis will touch the law of technique (in the Ab-system) in the best point of technique for producing z units/week.

Fig. 9 shows an example of the *accounting plan* for a business or sector as a whole with its own internal price system found for the sectors shown in the previous figures.

The purchased goods and services are adjusted (so as to have constant internal prices) in the *price-difference accounts*, and go to the account at the bottom (stocks, etc.), whence (1) as *direct contributions* to *unfinished product accounts*, (2) through the *producing sectors* as *indirect contributions* to the same account, whence the calculated products go to the *finished products accounts*, whence the turnover goes to the market. The *resulting profit* = market sales profit + sector profit + price-difference profit. (These accounts can, of course, be organized in many other ways according to the structure of the business.)

For a business as a whole we assume that we know the market price for the different quantities to be sold. The internal prices are constant corresponding to the previously mentioned estimates in the different sectors.

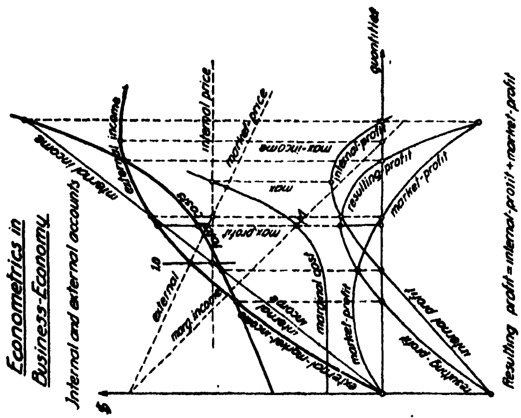

Figure 10

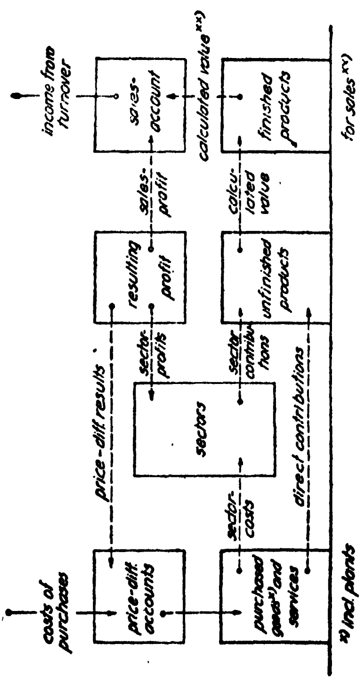

Figure 9

We choose a quantity 1.0, where price and revenue are equal on the graph, and with the known market prices we can now draw the *revenue* for each value of z. The horizontal internal price line for the product intersects the *internal revenue line* at $1.0 = z$.

The marginal cost and the marginal revenue intersect in the point of monopoly M giving maximum resulting profit. The resulting profits can now easily be drawn as shown at the bottom.

It is, as I see it, of great importance that economics does not neglect the cost problems which can bridge the gap between economics and the technical sciences, which in reality must always have an economic aim.

Technological research is at present too far ahead of technical-economic research.

Economists and technicians would benefit national production by finding a way of closer collaboration.

Résumé

La présente note se limite principalement à la théorie 1) du coût de production et 2) de la rationalisation des établissements industriels à coût marginal constant.

La première de ces théories illustre la nécessité d'utiliser, autant que possible, toute la capacité disponible (production maximum par unité de temps), au besoin par une expansion successive des secteurs dont la capacité insuffisante entrave l'utilisation d'autres secteurs. Le second problème consiste à déterminer pour chaque secteur la technique (quelle usine, méthode de travail, organisation, etc.) le mieux appropriée pour obtenir le coût minimum de la quantité à produire.

Le premier objet d'investigation technico-économique est le secteur (unité d'activité ayant sa propre comptabilité). Le secteur peut être un travailleur, un lieu de travail, une machine, un bureau, un atelier, une entreprise, une industrie, etc. Chacun de ces secteurs est, séparément, l'objet de la théorie de l'économie des entreprises.

Les coûts d'un secteur de production peuvent être subdivisés en deux parties: (1) *cout fixe,* qui est la raison d'être du secteur; et (2) *cout marginal,* supposé ici être proportionnel à la production. La première partie est subdivisée en differentes colonnes de "quantes" uniformes (coûts fixes indivisibles).

La figure 1 montre la subdivision en coûts fixe et marginal;

La figure 2 (*un quante*) illustre *la Loi de Capacité,* c'est-à-dire le coût par unité et le coût total comme fonctions des quantités produits;

La figure 3 montre la même loi, *pour une colonne* de quantes; et les figures 4-5 montrent la même loi, devenue *la Loi de l'Harmonie* pour *un secteur* intégral, montrant l'influence de chaque colonne et de chaque quante sur le coût.

Les figures 6-8 montrent *la Loi de Technique* illustrant les usines le plus appropriées pour les différentes quantités produites, et *la Théorie de la Rationalisation.*

Les dernières figures 9-10 illustrent un plan de comptes et une entreprise considérée dans son ensemble et montrent le profit comme une somme de profits internes et commerciaux.

Les figures 2-8 donnent des exemples numériques, permettant la lecture des quantités aux échelles.

Il est de la plus grande importance que la science économique ne néglige pas ces problèmes dont l'objet est d'établir un pont entre la science économique et les sciences techniques (qui, en réalité, doivent toujours avoir des objectifs économiques). La recherche technologique est à présent trop en avance sur la recherche technico-économique.

Mr. Jantzen's paper was discussed by Messrs. François Divisia and I. Jantzen.

BUSINESS PLANNING AND STATISTICAL ANALYSIS[1]

by Charles F. Roos
President, The Econometric Institute, Inc. (United States)

Many companies in the United States today use statistical analyses and forecasts derived from them to plan (1) sales and advertising, (2) inventory and price policy, (3) wage and labor policy, (4) salesmen's compensation and bonuses, (5) capital investment, and (6) financing operations. The degree of use, of course, varies company by company. In some instances the use of statistical analysis is limited to visual comparison of time series. In other cases analyses are very comprehensive and employ the latest methods of statistics. The purpose of the present paper is to sketch some of the more usual approaches.

Factors affecting sales. Sales are of primary importance to every management. The average merchant accepts without argument the fact that his sales are affected by changes in consumers' purchasing power. The demand for steel, on the other hand, is less closely related, but still depends fundamentally on present and past income. Other factors also importantly affect retail sales—for example, a change in the birth rate, increase in income received by women and other new workers and by certain racial groups, wartime decrease in the production of durable consumer goods, and increase in proportion of older people in the population. Even though management takes into account all regular factors, chance or unpredictable elements may partially affect the forecasts. Some of these, however, can be taken into account. For instance, failure to recognize the temporary nature of buying due to the Soldiers' Bonus in 1936 led to overexpansion in 1937 and the subsequent slump. In 1946, however, many merchants through the help of statistics avoided making undue commitments.

Increasing numbers of companies are comparing their sales performance with disposable income (income payments less personal taxes) or purchasing power, and determining functional relationships. These relationships serve as a basis for measuring performance and for forecasting, on the basis of reliable forecasts of income. Comparisons with industry sales and regional sales are also helpful.

Concept of normal or standard inventory. Return on investment, which is the ultimate measure of the success of a business enterprise, is

[1] This paper appears in detail in *Charting Your Business*, New York, Funk & Wagnalls, 1948.

probably influenced more by price and inventory policies than by any other phase of a business. A standard inventory may be defined as the amount that should be invested in inventory in order to provide adequately for production or sales based on forecasted volume of business for a given number of months in the future and at the same time lead to a maximum return on investment *if prices are unchanged.* Whether to hold more or less than the standard inventory, and so be subject to the possibility of gain or loss due to price changes, should be determined by the outlook for the industry and business in general. This will involve studies of the market for the product and of prices, as influenced by a variety of factors.

Average hourly earnings and average weekly pay. Comparison of the company's figures with industry and regional figures as reported by the Bureau of Labor Statistics has been extremely helpful to management, and has led in some cases to installation of labor-saving machinery to counteract absolute or relative increases in labor costs.

Hours of work. The standard work week of 72 hours in 1860 has gradually shrunk to 40 hours or less. The economic consequences of this have been important; mass leisure has made possible mass markets for products that once would have been luxuries for the few. The trends of leisure both within an individual industry and in the country as a whole are of importance to management.

Cost of financing. A growing business is frequently in need of new capital, and must determine whether to obtain it by borrowing or by stock issues. For intelligent decision on this point, various monetary statistics must be studied.[2]

Profits and dividends. Profits are the net result of management's ability to make sales at prices that cover all costs of production at less than capacity operating levels. In a well-managed company the ratio of cost of production or expenses to sales tends to decrease as sales volume increases. Investigations of substantial deviations of expenses from the line of average relationship to sales have usually been fruitful.

Summary. Business profits thus depend upon management's ability to do many things well: to budget operations correctly with respect to future trend of business; to maintain satisfactory inventory position; to schedule new plant construction wisely; to anticipate labor conditions and meet them with suitable policies; and to learn the implication of social trends and develop the policies necessary to meet them. Leading corporations in the United States are today carefully following all these trends.

[2] See C. F. Roos and Victor S. Von Szeliski, "The Determination of Interest Rates," *Journal of Political Economy,* 1942. See also the business service, *Economic Measures,* published by the Index Number Institute, Inc., New York.

Résumé

Les entreprises aux Etats-Unis ont recours à la technique moderne de la statistique pour la direction de leurs affaires. C'est suivant cette méthode que les entrepreneurs établissent aujourd'hui les directives relatives aux stocks et aux prix, au contrôle de la qualité des produits, aux ventes et à la publicité, à la rémunération et aux primes des vendeurs, aux salaires et aux heures de travail, aux investissements de capitaux et aux opérations de financement. Certains commerces de détail ont recours à la technique moderne du calcul de corrélation pour déterminer les effets probables sur les ventes, du pouvoir d'achat local, des variations du taux de natalité, du nombre des grossesses, des revenus des femmes, du rapport du nombre des ouvriers de moins de vingt ans et des ouvriers âgés à l'ensemble de la population, etc.

Alors que les revenus personnels de la clientèle sont probablement l'index statistique le plus profondément étudié par les entreprises, plusieurs autres index servent également à déterminer les prévisions de vents. Par exemple, dans l'industrie automobile, le nombre de voitures en circulation, leur âge et leur état; dans l'industrie de l'acier, l'état des voies ferrées, du matériel roulant et des machines, etc.

On peut, à bon droit, espérer réduire l'amplitude des fluctuations dans les affaires en recourant plus largement à la technique statistique moderne. Dans le passé, à diverses reprises, le commerçant moyen a vraiment mal préjugé de son potentiel de vente.

En 1936, par exemple, le paiement de primes aux soldats pendant une courte période a causé un accroissement des revenus personnels et, pendant quelques mois, une très forte augmentation des ventes au détail. Mais le commerçant moyen n'a pas reconnu le caractère temporaire de ce facteur et a passé des commandes en quantitè excessive pour l'automne et pour l'hiver. De leur côté, les fabricants ont été trompés par l'afflux soudain des commandes passées par les commerçants et se sont lances dans des programmes d'extension exagérée. Commerçants et fabricants se sont alors inquiétés de leurs stocks excessifs et ont modifié leurs programmes à l'extrême en sens contraire.

Par contre, en juin 1946, des commerçants de premier plan, reconnaissant, grâce aux statistiques, que leurs engagements étaient excessifs par rapport aux perspectives de vente et à l'augmentation de leurs stocks, ont réduit leurs ordres de façon draconienne et ont évité des surplus en quantité excessive après la guerre; ceci a permis de maintenir la production dans le voisinage de la capacité des ateliers d'occuper totalement la main-d'oeuvre disponible, et de libérer des marchandises pour l'exportation à un moment où le besoin s'en faisait sentir d'extrême urgence.

Les engagements des commerçants de détail par rapport aux ventes étant actuellement inférieurs à la normale, on peut compter sur une augmentation de leurs nouvelles commandes, situation qui limitera les disponibilités de marchandises pour l'exportation, l'automne et l'hiver prochains.

Mr. Roos's paper was discussed by Messrs. Michel J. J. Verhulst, Kalle T. Jutila, Jacques Dumontier, and the speaker.

ECONOMETRICS OF INTERNATIONAL ECONOMIC RELATIONS

Wednesday, September 10, at 4:00 p.m.

CHAIRMAN :

Jacob L. Mosak (United States),
Division of Economic Stability and Development of Economic Affairs, (United Nations)

SOME REMARKS ON THE PROBLEM OF DOLLAR SCARCITY*

by J. Tinbergen
Director, Central Planning Bureau (Netherlands)

1. THE BASIC PROBLEM OF DOLLAR SCARCITY

At the prevailing exchange rates and price levels it appears that, considering the current items of the balance of payments of the United States, demand for dollars surpasses their supply. Or in other terms: American exports to the rest of the world (taking goods and services together) surpass imports from the other countries. For the moment, one of the chief reasons is the heavy reconstruction demand in many countries, particularly European, together with the low level of productivity in these areas. Many experts expect, however, that the disequilibrium will remain to some extent after the reconstruction period. The possibility of a permanent disequilibrium will hamper, they fear, even the action needed to solve the temporary difficulties, Granting credits for reconstruction to war-hit countries is not attractive unless in the long run an equilibrium will develop. It seems worth while, therefore, to investigate in some more detail the possibilities of restoring the equilibrium in the current items of the American balance of payments.

The problem of dollar scarcity would be nonexistent if a permanent flow of capital from the United States would be accepted as a normal

* The author wants to express his thanks to Prof. A. Smithies, Dr. H. Staehle, Dr. J. J. Polak, Mr. Colin Clark, Mr. J. Hartog, and the other participants in the discussion, for various improvements in the analysis presented.

element in the world economy. Such a flow may be accepted as far as it is directed to underdeveloped countries; most European countries would not, however, after the reconstruction period, want to continue capital imports from the United States. Of course it may, nevertheless, upon closer examination, prove to be the best solution. To begin with, we shall investigate the possibilities of restoring equilibrium in the purely current items.

A priori, there are many aspects of the problem. American imports may be increased by new technical developments in the other countries, or by better sales promotion; exports may be curtailed by higher American home consumption or by deliberate cutting of imports by the receiving countries; developments in natural resources may exert an influence, etc. In this paper I want to focus upon the orthodox economic method available, *viz.*, a change in the terms of trade, or to say it in other words: a reduction in costs of production (by a change either in living conditions or in productivity) in non-American countries. The other aspects, important though they are, will be neglected.

Our analysis will consist of two parts: a theoretical analysis of the mechanism at hand (section 2) and an attempt to measure some of the most relevant coefficients and to draw some provisional conclusions (section 3). A few remarks on the nature of the disturbing forces may precede. In a general way it may be said that imports into the United States are expected to be, even in more normal times, "too low," exports to be "too high," particularly imports from and exports to the European countries. To some extent, American trade policy may be the reason; much will depend on the outcome of the new multilateral negotiations. Apart from that, there is the prospect of a changed relation between Western Europe and the Far East. Formerly, the Far East supplied some necessary raw materials to the United States, whereas Western Europe supplied capital goods to the Far East. There is a tendency now for the United States to make over that capital-goods supply and, at the same time, to demand less of Far East raw materials (*e.g.*, rubber). It is evident that all this points to a decline in the competitive position of Europe, either by economic or noneconomic developments. And the only purely economic response would seem to be in a reduction of European supply prices.

2. Theoretical Analysis of Equilibrium in Balance of Payments

We want to understand the essential features of the problem. For that purpose a model will be used satisfying the minimum requirements that must be posed.

First of all, there must be the question of two territories. We call them the United States and the Rest of the World. United States

variables will be indicated without a prime, Rest of the World variables with a prime.

Our primary interest concerns the balance of payments between these two territories. We assume it to consist of the current items only, since

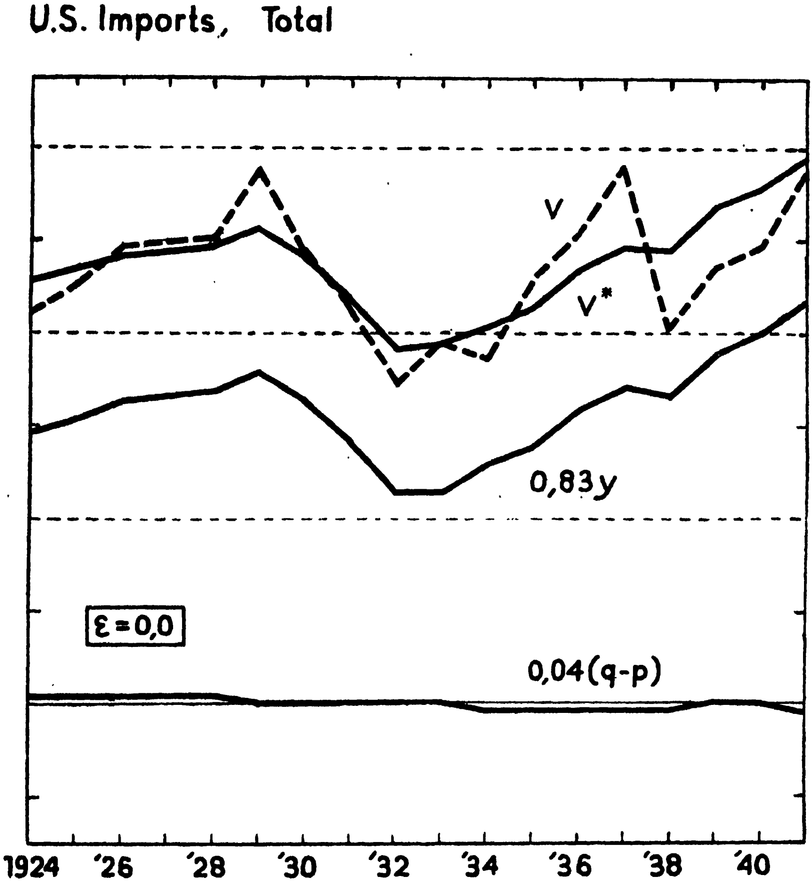

Figure 1

we are interested in the possibility of equilibrium in these items. Including goods and services when speaking of imports and exports, we may simply say that equilibrium in the balance of payments is identical to the equality of the value of imports and exports of the United States. The value of imports will be indicated by V, their volume by v and their price level by q. Exports of the United States are imports into the other territory; their value, volume, and price level (in foreign currency) are written as V', v', and q', respectively. All these symbols are assumed to refer to deviations from some initial equilibrium the figures for which

are indicated by barred values: $\bar{V}, \bar{v}, \bar{q}, \bar{V}', \bar{v}', \bar{q}'$,. The disturbance of equilibrium may be due to some consequences of the war to be specified and localized later. Writing k for the exchange rate of the dollar in terms of foreign currency, the equilibrium condition for the balance of payments runs:

$$\bar{V}'+V' = (\bar{V}+V)(\bar{k}+k). \tag{1}$$

As a first approximation, only linear terms in the deviations will be considered. Since $\bar{V}' = \bar{V}\bar{k}$, equation (1) reduces to:

$$V = \bar{V}k+\bar{k}V. \tag{2}$$

By definition,

$$\begin{aligned} \bar{V}'+V' &= (\bar{v}'+v')(\bar{q}'+q') \text{ or} \\ V' &= \bar{v}'q'+\bar{q}'v', \end{aligned} \tag{3}$$

and similarly:

$$V = \bar{v}q+\bar{q}v. \tag{4}$$

Choosing our units such as to make $\bar{q} = \bar{q}' = \bar{k} = 1$, we shall have $\bar{V}' = \bar{V} = \bar{v}' = \bar{v}$ and hence (2) turns into:

$$\bar{v}q'+v' = \bar{v}k+\bar{v}q+v. \tag{5}$$

Since our problem is essentially one of relative price levels, we shall have to introduce, apart from import price levels q and q' for each of the territories (measured in their own currency), home price levels p and p'. In order to simplify as much as possible, it will be assumed that export price levels are identical with home price levels. (It would not be difficult to substitute more complicated hypotheses for this simplest set-up.) It follows that p measures, in dollars, the same thing as q' in foreign currency; consequently,

$$\bar{q}'+q' = (\bar{k}+k)(\bar{p}+p)$$

or

$$q' = k+p. \tag{6}$$

Similarly,

$$p' = k+q. \tag{7}$$

So far, only purely *technical* relations (including definitions) have been introduced. We now turn to the economics of the problem. This

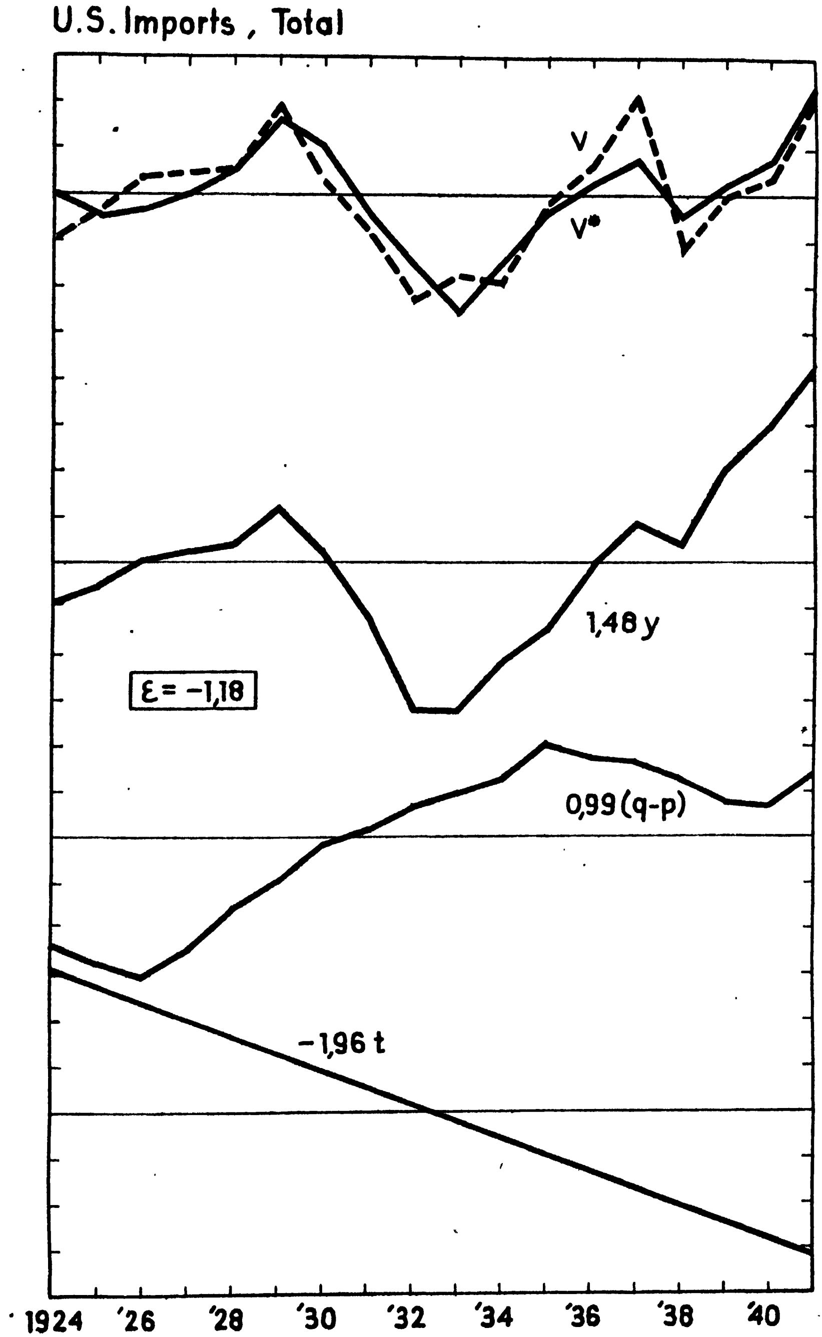
U.S. Imports, Total
V
V*
1,48 y
ε = −1,18
0,99 (q-p)
−1,96 t
1924
'26
'28
'30
'32
'34
'36
'38
'40

Figure 2

comes to considering the *demand* and *supply* functions for each of the flows v and v'.

As to demand, our hypothesis will be that demand for imports depends on:

(i) total volume of production u, which may be written as the sum of production for the home market x and production for exports v';

(ii) import price level $\bar{q} + q$; and

(iii) home price level $\bar{p} + p$.

We shall even specialize somewhat more by assuming, in addition, that the two latter variables act only through their ratio; in linear approximation $q-p$. Denoting the elasticity of imports by ε, and the marginal import quota by π, we have, for the United States:

$$v = \pi(v'+x)-\bar{v}\varepsilon(q-p). \tag{8}$$

For the Rest of the World, similar hypotheses will be made. In addition, however, it will be assumed that there is an extra demand for imports s, which is the chief cause of the disequilibrium with existing exchange rates and prices and which may be due, say, to a loss of competitive power by capital destruction. Hence we have:

$$v' = \pi'(v+x')-\bar{v}\varepsilon'(q'-p')+s. \tag{9}$$

A complete solution of our problem will only be possible, therefore, if we are informed on the determinants of x and x' also. There are two possible approaches here, which may be called for shortness the "Walrasian" and the "Keynesian." The Walrasian solution should be taken under conditions of stable and high employment; the Keynesian under conditions of depression. According to the first, the volume of total production is determined by productive capacity c; assuming $\bar{u}$ to be equal to c, we have $u = 0$, and the first term in (8) and (9), right-hand side, vanishes altogether. According to the Keynesian solution, there will be a relation between total real expenditures x and total real income y:

$$x = \xi y. \tag{10}$$

Since it may be shown by a somewhat extensive calculation that

$$y = x, \tag{11}$$

the consequence is that $x = y = 0$ and hence, in this case (8) and (9) reduce to:

$$v = \pi v'-\bar{v}\varepsilon(q-p), \tag{8'}$$

$$v' = \pi' v-v\varepsilon'(q'-p')+s. \tag{9'}$$

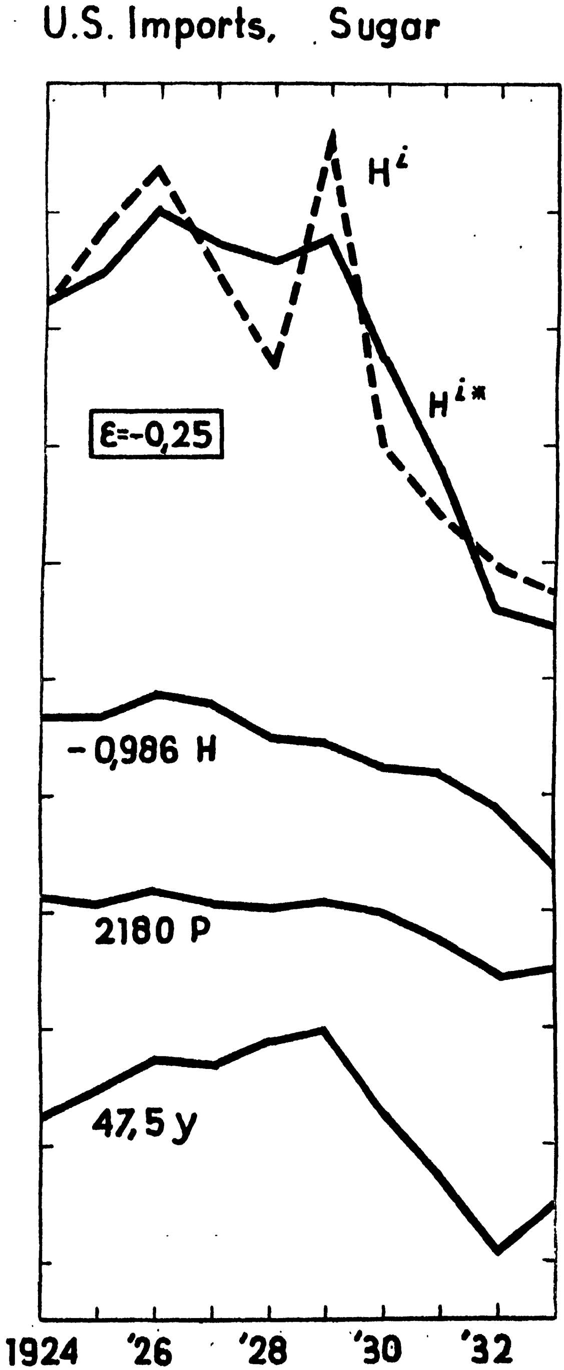

Figure 3

The *supply* equations will be given the form of price-fixation equations. Indicating by l the price of home productive effort (*e.g.*, labor) and by π_1 the marginal quota of such effort in a unit of home-produced goods, we have:

$$p = \pi q + \pi_1 l, \tag{12}$$

and correspondingly:

$$p' = \pi' q' + \pi_1' l'. \tag{13}$$

The price l may be linked up with p by:

$$l = \lambda p + l_0, \tag{14}$$

and similarly,

$$l' = \lambda' p' + l_0'. \tag{15}$$

Here the terms l_0 and l_0', represent some autonomous change in the price of productive effort (*e.g.*, wage rates) that may be used as a deliberate instrument of economic policy. We may call them the "indices of autonomous price policy."

The system of equations now established will be sufficient to solve our problem.

It seems instructive to do this in the following way. We may first express *all the items of the balance-of-payments equation* (except the disturbance term) *as functions of* $q-p$ and, as a next step, express $q-p$ as a function of k and the indices of autonomous price policy l' and l_0'. In both the "Walrasian" and the "Keynesian" approach it appears possible to transform the balance-of-payments equation (5) into an equation of the following type:

$$\gamma(q-p) = s/\bar{v}. \tag{16}$$

For the "Walrasian" approach:

$$\gamma_w = 1-\varepsilon-\varepsilon'; \tag{17$_w$}$$

whereas for the "Keynesian" approach:

$$\gamma_k = \frac{(1-\pi\pi')-(1-\pi')\varepsilon'-(1-\pi)\varepsilon}{1-\pi}. \tag{17$_k$}$$

From equations (6), (7), and (12)—(15) we deduce:

$$q-p = \alpha k + \beta \tag{18}$$

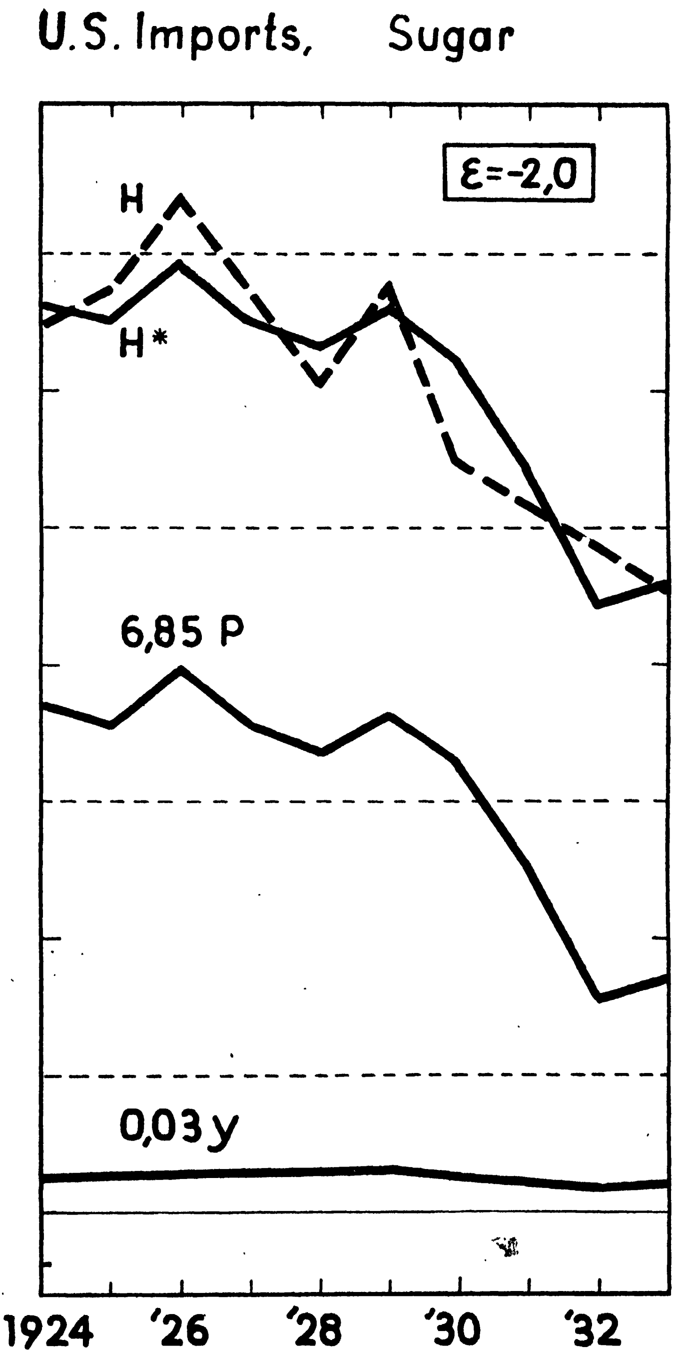

Figure 4

in which $\alpha = \dfrac{\pi\pi' + \mu\mu' - \pi'\mu - \pi\mu'}{\pi\pi' - \mu\mu'}$, $\mu = 1-\pi_1\lambda$, $\mu' = 1-\pi_1'\lambda'$;

(20) $$\beta = \frac{(\mu'-\pi')\pi_1 l_0 - (\mu-\pi)\pi_1' l_0'}{\pi\pi' - \mu\mu'}.$$

These rather complicated expressions reduce to simpler ones if the two territories are assumed to be identical in structure; an assumption that may have didactic value, when we are studying these problems.

We are now able to draw some interesting *conclusions* about the possibilities of restoring equilibrium in the balance of payments. From (16) we see that this may be done by an adaptation of the "terms of trade," $q-p$; the equilibrium being obtained for:

(21) $$q-p = \frac{s}{\gamma\bar{v}}.$$

Some very rough estimates of the figures involved will be considered in section 3.

Such a policy will, however, not succeed if γ happens to be equal to zero; and it will be difficult to perform if γ is a small figure. According to (17_w) and (17_k) it depends on the elasticities ε and ε' whether or not this special case will present itself. The "Walrasian" case has been discussed repeatedly in the literature.[1] The critical values for ε and ε' are rather low. The "Keynesian" case has so far not been put forward.[2] Here the critical values are somewhat higher. Taking, *e.g.*, $\pi = \pi' = 0.1$, we have:

(22) $$\varepsilon + \varepsilon' = 1.1.$$

How, now, can we obtain the desired adaptation in the terms of trade? This question is answered by equation (18).

There are two ways:

(i) a change in the nominal rate of exchange k; or

(ii) a change in autonomous price policy in either of the territories.

Equations (19) and (20) tell us to what extent k or l_0 and l_0' must be changed in order to obtain the desired change in the terms of trade. *Here again there is a condition to be fulfilled in order to let the mechanism work*: if $\alpha = 0$, no change in k, however large, will help; if $\beta = 0$,

[1] *Cf.* Alfred Marshall, *Money, Credit and Commerce*, Appendix J, 1923.

[2] I suggested a somewhat different approach in the *Revue de l'Institut International de Statistique*, Vol. 9 (1941), p. 36, where, however, the economy of the second territory was not considered completely.

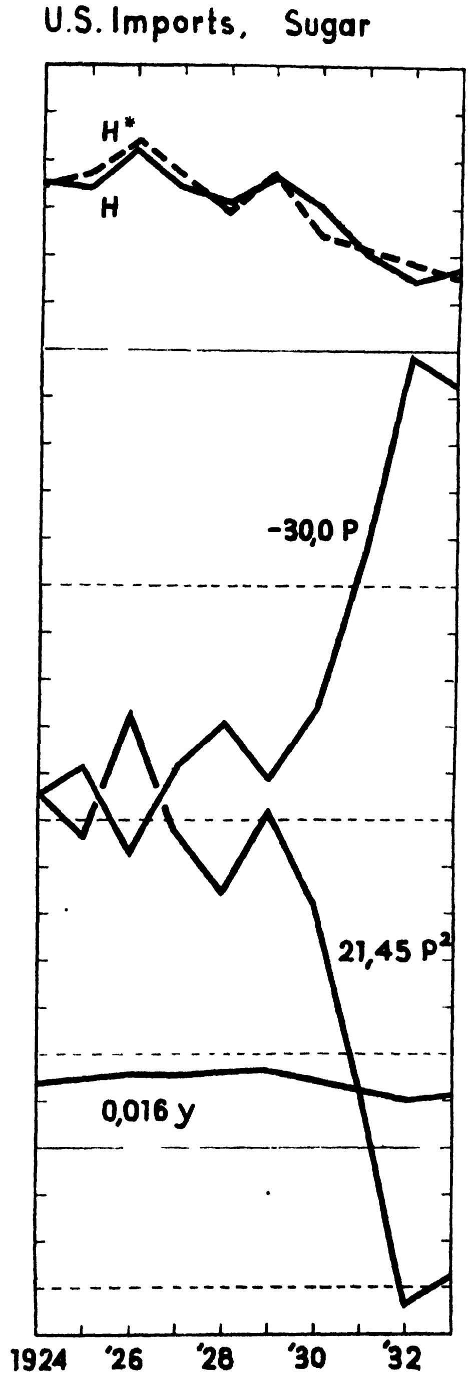

Figure 5

autonomous price policy will be impossible. A closer examination of α teaches us that $\alpha = 0$ either for:

$$(23) \qquad (a) \qquad \pi = \mu \quad \text{but} \quad \pi' \neq \mu',$$

$$(24) \qquad (b) \qquad \pi' = \mu' \quad \text{but} \quad \pi \neq \mu.$$

The case where both $\pi = \mu$ and $\pi' = \mu'$ presents complications which we will not now discuss. Case (a) comes to the assumption that

$$(25) \qquad \pi + \pi_1 \lambda = 1.$$

This may materialize in various ways. First, it may be that, in addition, $\lambda = 1$; an example is to be found in labor contracts prescribing proportionality between wage rates and cost of living. In this situation there will be proportionality between all prices: p, q, and l. This is commonly supposed to hold true for long-term reactions. For static systems this is the generally accepted feature, narrowly related to the statement that, in principle, only price ratios are relevant and not the absolute level of prices. In a sense, case (a) is therefore also typically "Walrasian." Under these conditions, changes in nominal exchange rates will not be helpful to restore equilibriun, since in the long run all prices will show proportional changes and the terms of trade will not change.

It may, however, even be that (25) is true, without $\lambda = 1$. Marginal quota for imports and internal factor cost may add up to more than one whereas the response of l to changes in p may be less than proportional, to quote the most probable example. In this situation too, manipulation of exchange rates would not lead to changes in the terms of trade.

From (20) it will be seen that for $\mu = \pi$, but $\mu' \neq \pi'$, autonomous price policy will be possible for the United States; if $\mu' = \pi$, but $\mu \neq \pi$, it will be possible for the Rest of the World.

Summarizing, we may state that there are *two possible reasons why a manipulation of nominal exchange rates may be unsuccessful* in restoring a disequilibrium in the balance of payments, *viz.*,

(i) if $\mu = \pi$ no change in terms of trade will result and even if this should be the case where $\mu \neq \pi$,

(ii) no change in the gap in the balance of payments may occur if the elasticities ε and ε' add up to about 1 or 1.1.

If situation (i) present itself, other types of price policy, exemplified by what we called *autonomous price policy*, must be tried. If case (ii) is present, even these policies will not help; the only way of restoring

equilibrium in the balance of payments will then be the *manipulation of capital items*, *i.e.*, cancelling of debts or granting of new credits, in whatever form.

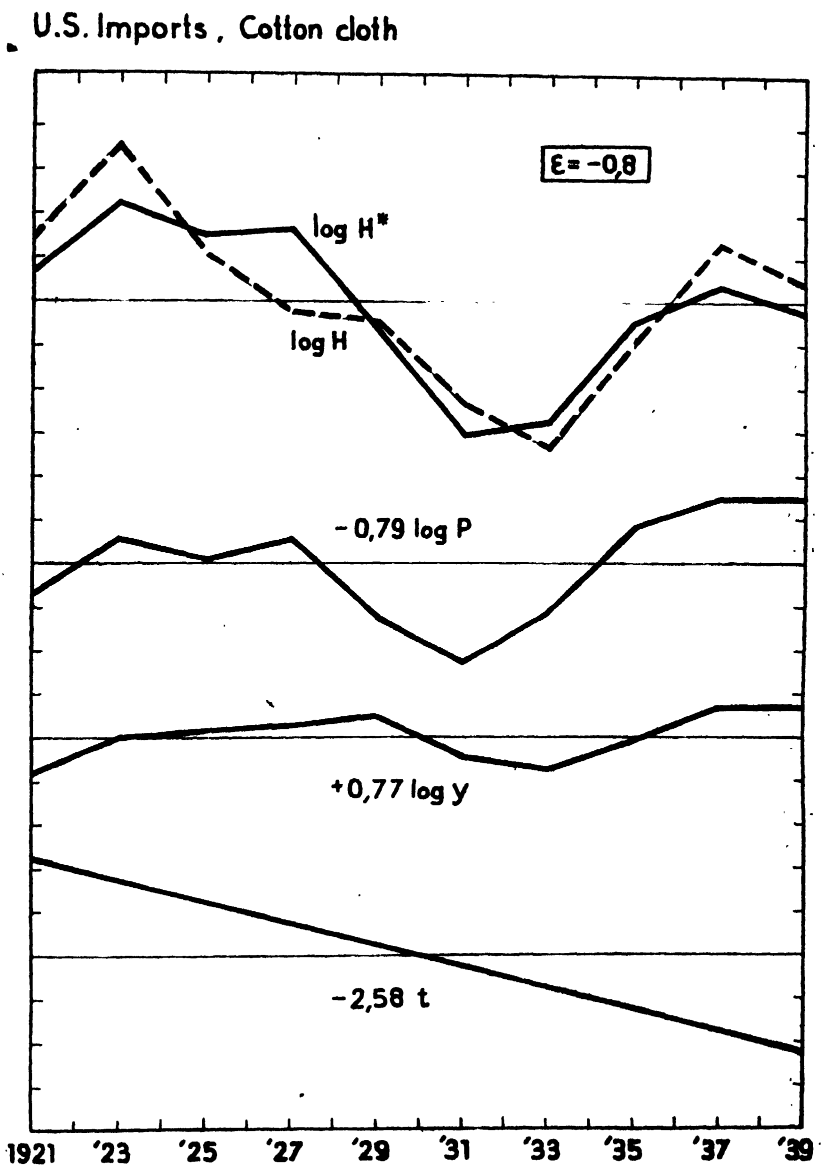

Figure 6

It goes without saying that our very simple model may be made more complicated in various ways. These complications will be imperative

as soon as a good approximation to reality is required for purposes of practical policy. It is to be expected, however, that these more complicated models show even more singular situations than this simple one and this makes the study of the simple model so useful. More complicated assumptions will be necessary anyhow if dynamic models are required. The introduction of certain lags may be the first step. For the above-stated singular values of π and ε, the corresponding dynamic systems will show explosive movements.

In the static models, the introduction of more complicated demand functions may be necessary, *e.g.*, with different coefficients for the competing price levels. Although this would be a departure from pure statics it might be useful as a first approximation to "middle-long-run" problems.

3. Statistical Determination of Elasticities for the United States

From the foregoing theoretical analysis it will be clear that it is particularly interesting to gain an insight into the probable values of ε and ε' *i.e.*, the elasticities of American imports and Rest of the World imports as defined in section 2. *A priori* views as recently expressed are contradictory. In a general way most economists would assume that the elasticities are not as low as in the "special case," in which the dollar problem cannot be solved by changes in the terms of trade. For the particular case of the United States (*i.e.*, ε) some would, however, admit the possibility.[3] Many business men and politicians are of the opinion ε is very low indeed. They point to the high degree of "autarky" of the United States as to raw materials and to the virtual impossibility for European producers to compete in the field of manufacturing. In addition, one could point to the large demand for home-produced services in the United States which is an immediate consequence of the high standard of life. There is probably some exaggeration in this pessimism. As to the competitive power of the United States it is interesting to quote the investigation made by the International Labor Office, at the request of Henry Ford, in 1930. To my knowledge it is the most exact attempt to compare price levels of strictly comparable goods in the United States and elsewhere. The differences were not large; in many cases, non-American prices were lower.

A first attempt to measure the elasticity of American imports was made by Hinshaw,[4] who found a figure of 0.48. As variables he

[3] *Cf.* O. H. Korican, "Aims of our Foreign Investment Policy," *Harvard Business Review*, Vol. 24.

[4] R. Hinshaw, "American Prosperity and the British Balance of Payments Problem," *The Review of Economic Statistics*, Vol. 27 (1945), p. 1.

used, however, only American national income and the price level of American imports; not, therefore, any competitive price index. I have tried out several methods to give an explanation, by correlation analysis, of the fluctuations in American imports, using as explanatory

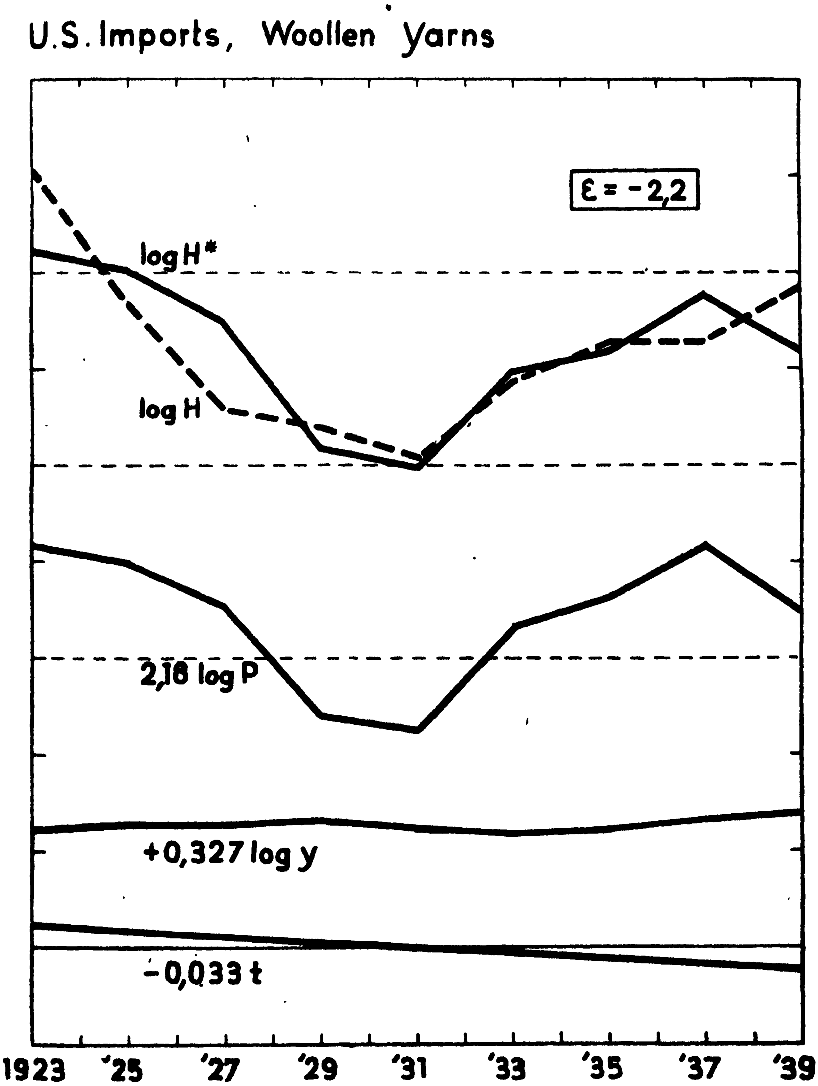

Figure 7

variables national income at 1929 prices, an index of import prices (taken from Hinshaw; duties included), and a wholesale price-index, reweighted according to the composition of imports (using broad subdivisions only).

From 1924 to 1941 the import quota shows only irregular fluctuations, whereas the ratio between import and home prices fell considerably. There is no correlation between these two ratios; which would point to a low elasticity, but does not permit of any numerical estimate. Fairly good correlations are obtained if a regression equation with free coefficients is tried:

$$v = \alpha u + \beta q + \gamma p,$$

but a bunch map shows large margins of uncertainty although always the signs are correct. The most probable results point to elasticities at least as low as those found by Hinshaw.

In order to get more accurate estimates separate commodities were considered. The general idea was to use a different procedure for goods immediately competing with American products (sugar, cheese, herring, woollen and cotton manufactures) and for goods not immediately doing so (rubber, tin, bulbs). For the first group it will be possible to take account, in the explanations, of competing prices in the strict sense. For the second group this is impossible; here only the general home price level may be assumed to present "competing prices" but in a wider sense. Similarly, for the first group the volume of production may either be total production of all goods (or real national income) as a general index of activity or the volume of home production of the special good considered, whereas for the second group only the general index can be used.

Some results obtained for commodities of the first group are given in Table 1. The corresponding graphs are represented in Figures 1–10.[5] There are a number of fairly high elasticities of substitution among them.

For the second group a few results are available from other authors, which have been assembled in Table 2.

[5] In all the graphs the dotted line represents the actual value of the variable to be explained, whereas the full line indicated with an asterisk indicates the explanation obtained by the inclusion of the explanatory variables whose symbols are indicated in the graph. The contribution of each separate explanatory variable is indicated by the full curves shown in addition.

The meaning of the symbols as far as these are identical with the variables used in the text may be taken from the text. The other symbols have the following meaning:

H = ratio between quantities imported or exported and corresponding production in importing country;

P = ratio between import price and home price;

ε = elasticity of substitution corresponding with formulae in text;

t = time;

t' = estimated influence of quota systems in non-American countries.

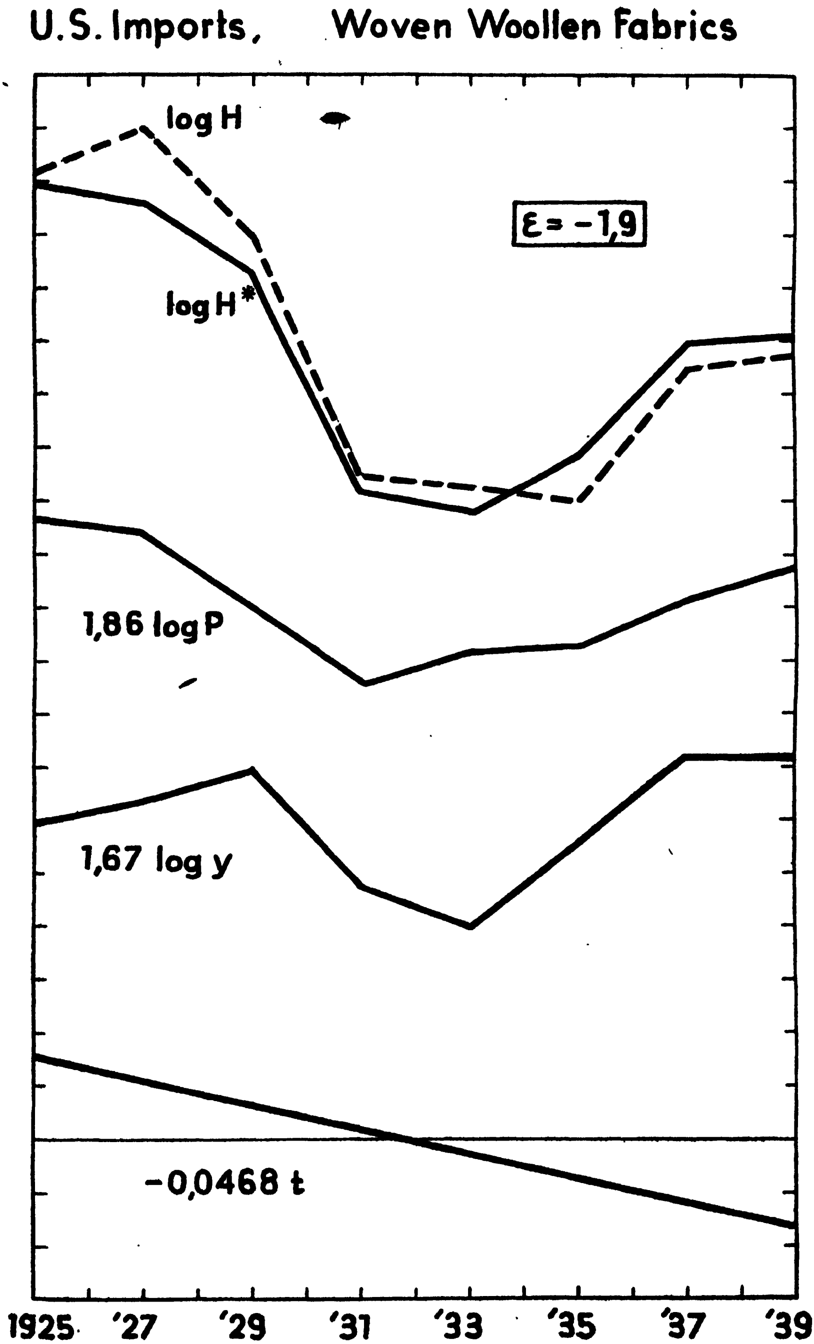
U.S. Imports, Woven Woollen Fabrics
log H
ε = −1,9
log H*
1,86 log P
1,67 log y
−0,0468 t
1925
'27
'29
'31
'33
'35
'37
'39

Figure 8

TABLE 1

Substitution Elasticities of Demand, American Imports

Commodity	Period	Explanatory variables: Index of production	Explanatory variables: Competing price	Elasticity
Cheese	1921–1933	1. real national income; home prod. cheese[2]	home price cheese[3]	0.0
		2. home prod. cheese[2]	do.[3]	1.0
Herring from Holland*	1929–1939	1. national income	price of all imp. herring	1.1
		2. all imp. herring[2]	do.[3]	0.8
Sugar	1924–1933[4]	real national income; home production[2]	home price sugar[3]	2.0
Cocoa*	1929–1939	national income[2]	price cocoa beans	2.3
Cotton cloth	1923–1939[5]	real national income; home production[2]	home price of cotton woven goods over 12 inches[3],[6]	0.8
Woollen yarns	1923–1939[5]	real national income; home production[2]	home price of woollen yarns[3],[6]	2.2
Woollen woven fabrics	1923–1939[5]	real national income; home production[2]	home price of woollen woven fabrics[3],[6]	1.9

[1] Apart from price of commodity considered.
[2] With *a priori* elasticity = 1.
[3] With *a priori* elasticity equal to that of price of commodity considered (but with opposite sign).
[4] Since 1934 market regulation.
[5] Odd years only, being the only years for which census data are available.
[6] In addition, a linear trend was introduced.
* Taken from unpublished calculations by G. P. Kamermans.

TABLE 2

Elasticity of Demand, American Imports

Commodity	Period	Explanatory variables[1]	Author and publication	Elasticity
Bulbs[2]	1923–1936	National income	J.B.D. Derksen and A.L.G. M. Rombouts, "The Influence of Prices on Exports, *De Ned. Conj.*, *Special Memoir. Nr.* 1, pp. 33–34	0.6
Rubber	1921–1934	Automobile production; Total number of motor cars	J.B.D. Derksen, "De Vraag naar Rubber," *De Nederl. Conjunctuur*, August, 1936, p. 19	0
Tin	1923–1936	Automobile production; Tin-plate production	M.J. Schut, *Tinrestrictie en Tinprijs*, Haarlem, 1940, pp. 26–27	0.3

[1] Apart from price of commodity considered.
[2] Average for hyacinths and other bulbs.

The elasticities found here are lower. They are not, however, substitution elasticities. Nevertheless it may be that they are the relevant

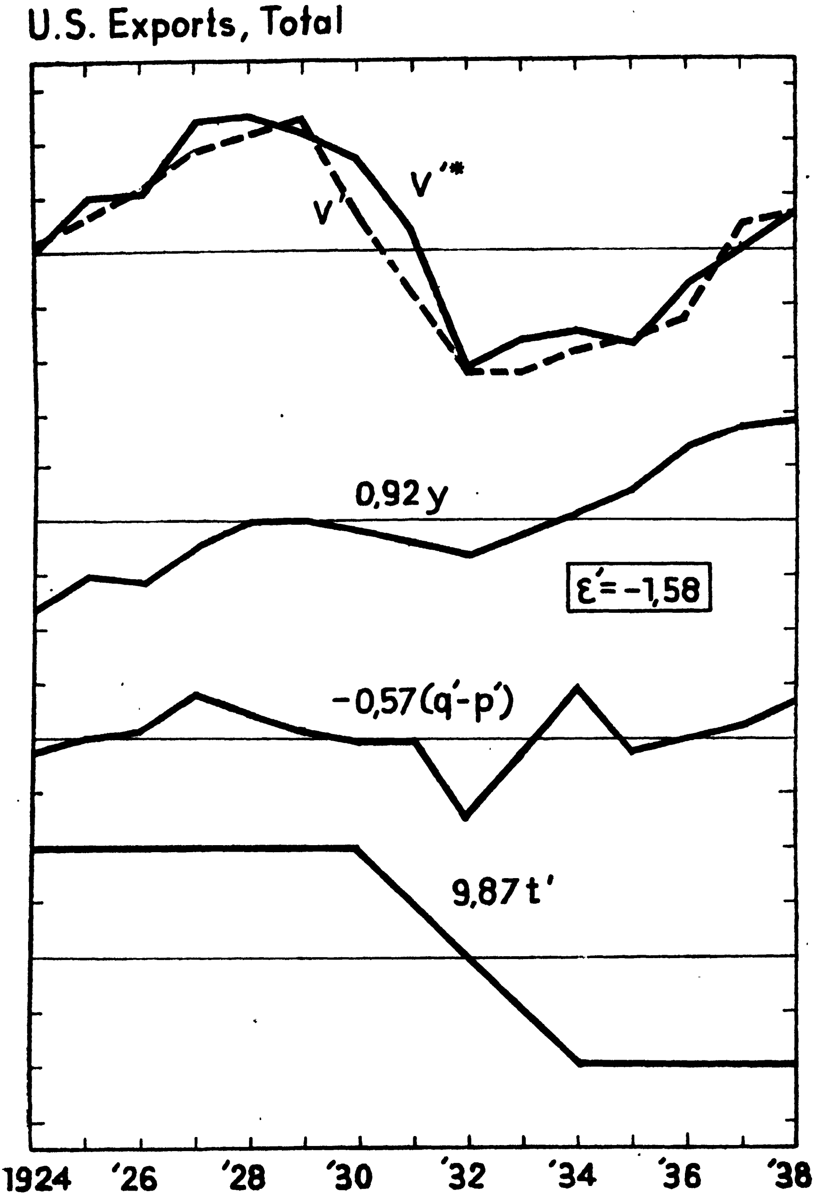

Figure 9

elasticities for our problem. This depends on whether the demand for these goods depends only on their own prices or whether the general

internal price level in the United States exerts an influence too. In the last case the elasticity with respect to their own price found in the quoted analyses may be the combined effect of the two influences and the net effect of their own price may be larger than suggested by the elasticities indicated. There seems to be some reason, howėver, to assume that in the cases now considered the difference is not so large.

In order to get an impression of the average elasticity corresponding with the figures shown in Tables 1 and 2, the individual elasticities have been weighted and averaged. As weights, the value of imports in 1929 have been used; the amplitude of price fluctuations has been assumed to be equal for all in the case in which we are particularly interested, *viz.*, a general change in the terms of trade. The average elasticity thus found amounts to 0.9.

The elasticity E' of "Rest of the World Imports" or of American exports was estimated in the following way: An attempt was made to explain the fluctuations in the volume of American exports v' by a combination of the variables:

y' = real income of the "Rest of the World,"

z = ratio of export price index of the United States (in gold) to export price level of "Rest of the World,"

t = a very rough index of "trade barriers," assumed to be zero from 1924-1930, increasing regularly from 1930 to 1934 and remaining constant after 1934.

Both y' and z were estimated in two different ways, which turned out to yield very similar results. The influence of t appeared to be large; in view of the rough character of this series this means that our results are very uncertain. The values found for ε' are —1.8 and —1.3 respectively. These values may be tested also by estimates for some individual commodities, *viz.*, wheat, cotton, and motor cars.

For wheat, some estimates bearing on the substitution of various kinds of wheat for each other were published recently.[6] Only one figure is given for exports of American wheat; here a substitution elasticity of —5.3 to —11.3 with respect to Canadian wheat is found. From the other figures it is clear, however, that the elasticity is highest for countries showing the same season; hence it is probable that the elasticity of substitution between American wheat and all other wheat is lower than the one between American and Canadian wheat. A fair average may be the median of all the figures presented, which is —5.6; still a very high one.

[6] J. Tinbergen, "Some Measurements of Elasticities of Substitutions," *The Review of Economic Statistics*, Vol. 28, (1946), p. 109, particularly p. 112.

For cotton, figures are to be found in the same publication; a figure of -2.9 is the most representative.

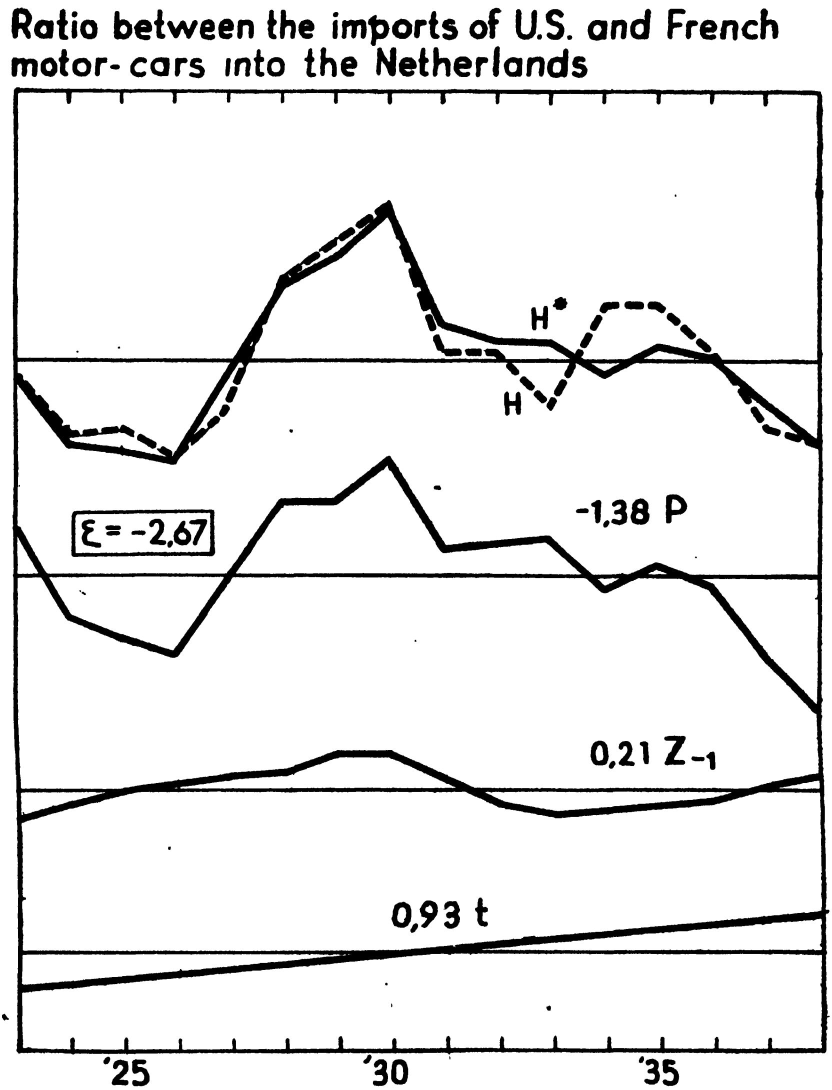

Figure 10

Finally, some figures are available for the substitution elasticity between American and other motor cars; they are between -2 and -2.5.

Clearly it will be necessary to refine and amplify the analyses just reviewed. As a provisional impression we may state that ε' cannot be far from -2.

It follows that $\gamma = 1-\varepsilon-\varepsilon'$ will be in the neighborhood of -2.

4. Interpretation and Estimation of the Shifts

For $s = 0$, all variables (measured as deviations from the "equilibrium values") are equal to zero, *i.e.*, the system shows the equilibrium that would have occurred if the "shift" s in the demand for American products had not existed. Apart from this situation I, two other situations are considered. It is assumed that the present state of affairs, if reconstruction demand is disregarded, is characterized by (i) the existence of the shift s whereas (ii) the price levels p, q, l, p', q' l', and k are not yet "adapted," *i.e.*, have their previous values, *i.e.*, zero. This implies that in equations (14) and (15) l_0 and l_0' are equal to zero too. The corresponding set of values may be called "situation II". Finally, a third situation III is considered, and, one might say, aimed at, *viz.*, the situation in which the values of all prices are such as to offset the consequences of s on the balance of payments. As already indicated, this may be obtained either by introducing values of l_0 and l_0' different from zero, or by changing the value of k (previously assumed to be zero) or by both methods.

The question of finding the desired values for the prices may be answered in two stages, the first being to indicate $q-p$ (and $q'-p'$) and the second being to indicate by what values of l_0, l_0', or k these values may be obtained. In the present paper this second stage was already discussed from the theoretical viewpoint in section 2, but will not be considered from the statistical side. Only the first stage will be considered.

In order to do so, it is necessary to estimate the value of s, or rather of $s/\bar{v}$, *i.e.*, the ratio between the shift in demand for American products and the equilibrium value of imports and exports. This again requires the interpretation of s. At closer examination this interpretation is a different one for the Walrasian and the Keynesian model. In the first case, where total volumes of production $x+v'$ and $x'+v$ cannot be changed, we have, in situation II, since also $p = q = p' = q' = 0$, $v = 0$ and $v' = s$. This means that s equals the *export surplus*.

In the Keynesian model things are not as simple as this. It appears that s is a multiple of the export surplus, or, to say it the other way round, that the export surplus is a fraction of s only. The Keynesian model comes down to the following representation of what happens: the original shift s leads to an export surplus of the United States, which in its turn increases the volume of production inside and decreases

production outside the States. In response, imports into U.S. are increased and imports into the Rest of the World decreased, which again affects production in both areas.

The final result is an export surplus generally smaller than s together with an upward shift in American production and a downward shift in non-American production, both considerably larger than the export surplus. Under conditions of full employment this cannot work; as soon as this state of affairs is approached, the Walrasian model will have to be substituted for the Keynesian.

The magnitudes of the shifts in production, exports and imports depends, in the Keynesian model, on the values of ξ and ξ' which are not exactly known. They may be quite near the values $+1$. The more these values are approached, the larger the shifts in production become and the less probable it becomes that the Keynesian model can portray the situation. Our calculations will therefore be based on the Walrasian model. This choice may also be justified in quite another way, *viz.*, by introducing the assumption that the governments and international agencies concerned will be able to maintain full employment.

On the hypothesis that s represents the export surplus after correction for the present temporary extra demand for reconstruction purposes it is still difficult to indicate its magnitude. We assume it is half that of v, or that $s/v = 0.5$. Closer examination would, of course, be very useful.

For any given value of s/v formula (21) teaches us what value of $q-p$ will be necessary in order to re-establish the equilibrium in the balance of payments. The result is, with our esitimate:

$$q-p = -0.25,$$

meaning that a lowering of the terms of trade of some 20 to 30 percent would be necessary.

The corresponding reduction in real incomes in the non-American countries will depend on the other coefficients, but as a first approximation will be $\pi\,(q-p)/(1-\pi)$, *i.e.*, about 1/9 of 0.25 or 0.03.

If the Keynesian model should be applicable, however, larger figures would be found.

It is too early yet to discuss the practical implications involved. The first thing to be done is to improve measurements of the coefficients ε and ε' and the ξ's. The provisional impression is, however, that a solution of the problem of dollar scarcity along the lines of price adaptation is not hopeless beforehand.

Résumé*

1. *Problème fondamental de la pénurie de dollars*: la demande de dollars dépasse l'offre:

(i) au cours de la période de reconstruction européenne;

(ii) au cours des années ultérieures. Ne constitue peut-être pas un problème si l'exportation permanente de capital est acceptée par le public; situation modifiée en Europe occidentale; rapport avec l'Extrême Orient.

2. *Analyse théorique de l'équilibre de la balance des paiements*. La demande et l'offre dépendent des transactions courantes et des transports de capitaux.

Transactions courantes: principalement importations et exportations. Dépendent du revenu réel, des niveaux "réels" de prix et des entraves (artificielles) au commerce (droits de douane et contingentements).

Selon la théorie traditionnelle, l'équilibre des transactions courantes peut-être restauré par une adaptation appropriée des termes d'échange, c'est-à-dire, des niveaux de prix et (ou) des cours de change.

Un examen plus poussé révèle, cependant, que cette méthode ne réussit pas toujours. L'équilibre peut-être presque indifférent (indéterminé). Il est utile de distinguer entre les changements des cours de change nominaux et réels, et entre quatre cas pouvant être combinés comme suit: a) plein emploi ou capacité inutilisée (en un certain sens: cas Walrasien et Keynesien);

b) élasticités "normales" des importations et exportations ou élasticités "spéciales."

3. *Détermination statistique des élasticités pour les Etats-Unis*. Les opinions a-priori sont contradictoires. La plupart des économistes pensent que les élasticités ne sont si basses comme l'on suppose dans le cas spécial; beaucoup de politiciens et d'hommes d'affaires pensent qu'elles le sont. L'élasticité faible des importations aux Etats-Unis pourrait être expliquée par les vastes ressources naturelles des Etats-Unis, la production en séries et le niveau de vie élevé (impliquant demande considérable de services d'origine intérieure).

Données sur les prix relatifs aux Etats-Unis et à l'étranger.

* A full French translation of this paper appeared in *Revue d'Economie Politique*, 1948, pp. 36 ff., under the title, "Observations sur le problème de la rarété du dollar."

Tentatives d'estimation de l'élasticité de la demande. L'essai de Hinshaw. Niveaux de prix à utiliser.

Multicollinearité. Essais sur marchandises isolées: sucre, automobiles, articles de laine, fromage.

Mr. Tinbergen's paper was discussed by Messrs. Jacques Rueff, J J. Polak, Kantilal L. Dalal, and the speaker.

UNE PRESENTATION GEOMETRIQUE DU DOUBLE EQUILIBRE DU CHANGE ET DU COMMERCE INTERNATIONAL

par F. Divisia

Professeur au Conservatoire des Arts et Métiers, a l'Ecole Polytechnique, et a l'Ecole des Ponts et Chaussées (France)

La présentation proposée ci-dessous comporte des imperfections assez graves, et c'est l'objet de cette communication, de mettre en discussion ses défauts et, tout de même, son utilité possible.

Exposé

Considérons deux pays seulement, par exemple Angleterre et France, et tout d'abord, une seule marchandise.

Supposons d'abord les deux pays économiquement isolés l'un de l'autre; dans chacun d'eux, les courbes d'offre et de demande déterminent des prix p_a et p_f, que nous appellerons: "prix de structure"; si, ensuite, les deux pays sont mis en communication, il se fait un transfert q_1 de la marchandise, lequel porte les prix à des niveaux p_a' et p_f' que nous appellerons "prix actuels".[1] A l'équilibre, ces prix sont tels que:

$$p_f' - £p_a' = t \tag{1}$$

£ étant le cours de la livre à Paris, et t, le coût du transfert.[2] Cette condition permet de calculer p_f', p_a', q et donc la demande de devise (positive ou négative) qp_a':

[1] On peut montrer que les résultats ci-dessous, établis ici pour une marchandise produite à cout croissant dans les deux pays, sont, en général, valables pour toutes autres, notamment pour les marchandises à cout constant et pour les marchandises non produites dans le pays importateur. Le cas exceptionnel ou l'exposé tombe en défaut correspond vraisemblablement à celui qui a été signalé séparément par M. J. Tinbergen dans la *Revue de l'Institut International de Statistique*, "Unstable and Indifferent Equilibria in Economic Systems," 1941, et dans son livre: *International Economic Cooperation* (1945).

[2] En effet, par frais de transfert nous entendons ici, non seulement les frais de transport et frais commerciaux, publicitaires et autres, non déjà compris dans le cout de production, non seulement les droits de douane, mais encore, la différence de prix, mesurant le défaut de fluidité de la clientele indigène vis-à-vis de la marchandise étrangère—que ce défaut de fluidité soit du à son ignorance, à ses habitudes, au sentiment nationaliste, aux entraves au commerce autres que les droits de douane,—voire à des prohibitions d'importation, auquel cas il suffit de prendre $t_1 = p_f - £p_a$.

La figure 1 montre qu'on peut écrire:

$$
(2) \qquad \begin{cases} \dfrac{p_f - p_f'}{p_f} = F\left(\dfrac{q_i}{Q_f}\right), \\[2ex] \dfrac{p_a' - p_a}{p_a} = G\left(\dfrac{q_i}{Q_a}\right), \end{cases}
$$

F et G étant des fonctions croissantes indépendantes du choix des unités et en rapport avec la forme des courbes de coût marginal et de demande, de la marchandise.

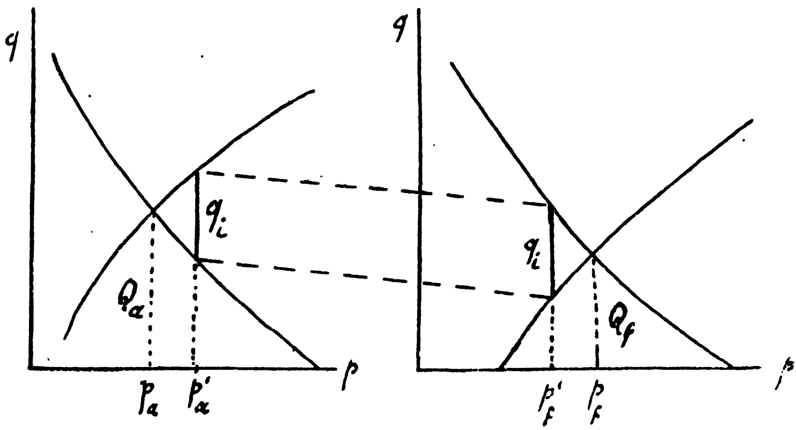

Figure 1

Des équations (1) et (2) éliminons p_a' et p_f' et posons:

$$
(3) \qquad \frac{p_f}{p_a} = \pi_f \quad \text{et} \quad \frac{t_i}{p_a} = \tau_i\ ;
$$

il vient:

$$
(4) \qquad \pi_f\, F\left(\frac{q_i}{Q_f}\right) + \pounds\, G\left(\frac{q_i}{Q_a}\right) = y_i
$$

avec

$$
(5) \qquad y_i = \pi_f - \pounds - \tau_i\,.
$$

Pour une marchandise donnée, π_f, τ_i, Q_f, Q_a sont donnés.

L'élimination de £ entre (4) et (5) fait apparaître q_i comme une fonction de y_i, et c'est une fonction croissante.[3]

La demande correspondante de livres à Paris est: $d_i = q_i\, p_a'$;[4] c'est une fonction croissante de y_i, car la figure montre que p_a' est une fonction croissante de q_i, donc de y_i.

Or y_i est susceptible d'une représentation graphique commode (fig. 2): Soit un axe vertical des prix en *francs*. Traçons la droite horizontale correspondant au cours £ de la livre à Paris. Faisons correspondre à la marchandise considérée une chandelle C_i ayant sa base sur la droite £ et son sommet au niveau de π_f, et portons à partir de la base la hauteur τ_i; la hauteur de la partie hachurée de la chandelle sera: $y_i = \pi_f - £ - \tau_i$. Quant à la demande de la devise correspondante, d_i, fonction croissante de y_i, si nous admettons grossièrement

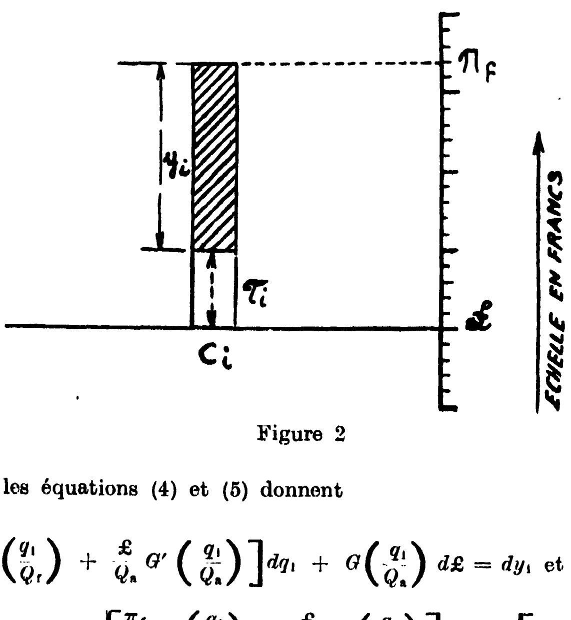

Figure 2

[3] En effet les équations (4) et (5) donnent

$$\left[\frac{\pi_f}{Q_f} F'\left(\frac{q_i}{Q_f}\right) + \frac{£}{Q_a} G'\left(\frac{q_i}{Q_a}\right)\right] dq_i + G\left(\frac{q_i}{Q_a}\right) d£ = dy_i \text{ et } d£ = -dy_i,$$

d'où:
$$dq_i \left[\frac{\pi_f}{Q_f} F'\left(\frac{q_i}{Q_f}\right) + \frac{£}{Q_a} G'\left(\frac{q_i}{Q_a}\right)\right] = dy_i \left[1 + G\left(\frac{q_i}{Q_a}\right)\right].$$

Or F et G étant des fonctions croissantes (voir figure) les facteurs de d_q et de d_y sont positifs.

[4] Nous supposons ici que le transfert est payé en francs par l'acheteur à un commerçant importateur français; ce n'est pas là une hypothèse restrictive. Si par exemple, le transport est assuré par un armateur anglais payé en £, il suffira de considérer qu'il constitue une autre importation q_i donnant lieu de son côté à une demande de livres $d_i' = d_i'\,(\pi_f - £ - \tau_i)$, τ_i désignant les frais de commission nécessités par le transport; la partie de ces frais payable en £ pouvant, à son tour, être considérée, de même, comme une troisième importation.

que cette fonction soit linéaire, elle sera de la forme Ky_i,[5] puisqu'elle s'annule avec y_i, et elle sera représentée par l'aire hachurée ayant une base de largeur K. (Si la forme linéaire paraît inadmissible, nous pouvons supposer la chandelle matérialisée par une plaque dont l'épaisseur varie avec y_i, de telle sorte que le poids de la partie hachurée, nul avec y_i et croissant avec lui soit proportionnel à d_i.)

Reste à représenter l'équilibre, conditionné par l'égalité de l'offre et de la demande globales de devise anglaise sur le marché de Paris

Équilibre de la balance commerciale

Classons toutes les marchandises par ordre de π_i croissant et représentons chacune d'elles par un segment de droite horizontal dont l'ordonnée soit π_i et dont la longueur soit égale au coefficient K. Nous

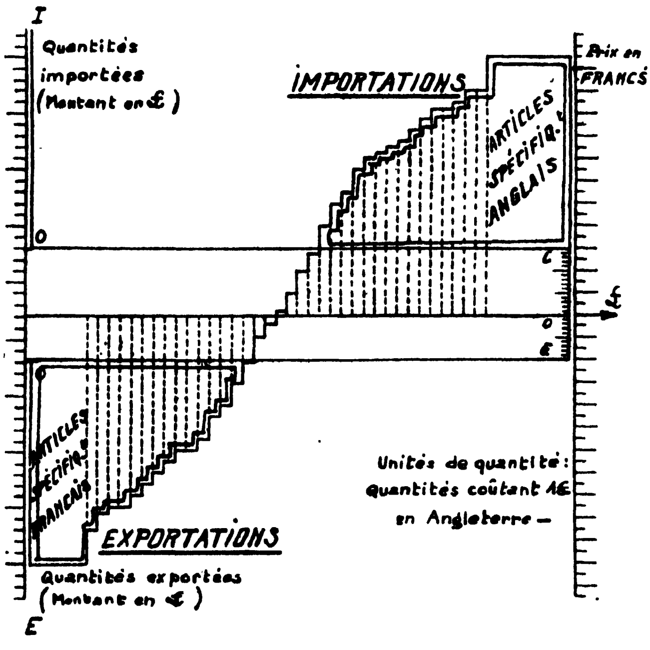

Figure 3

avons vu que ce coefficient dépend uniquement des courbes d'offre et de demande intérieures avant l'établissement du courant d'échanges; nous obtenons donc ainsi une courbe en escalier (fig. 3) qui *dépend uniquement de la structure intérieure initiale*—économique et monétaire —des deux pays, et constitue, par suite, une *donnée* ferme du problème;

[5] K étant un coefficient différant d'une marchandise à l'autre.

nous l'appellerons "courbe de structure." Nous pouvons d'ailleurs (et peut-être même, devons) y inclure, non seulement les marchandises du commerce extérieur, mais aussi la totalité des autres, en portant toutefois notre attention sur ce que nous considérerons alors comme frais de transfert.[6]

Ceci étant, traçons l'horizontale £ correspondant au cours (inconnu) de la livre à Paris et, de part et d'autre, les parallèles aux distances τ_i et τ_e représentant, en francs et à l'échelle, les frais de transfert, (supposés, pour simplifier, être les mêmes pour toutes les marchandises); nous obtenons ainsi une sorte de curseur, qu'on peut matérialiser par une réglette de largeur $\tau_i + \tau_e$, portant en £ l'index du cours de la livre et dont il suffit de rechercher la position d'équilibre.

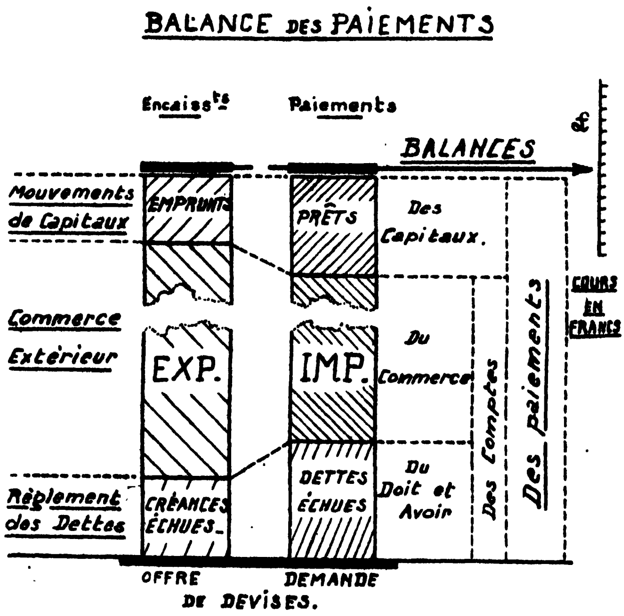

Figure 4

Cette position doit réaliser l'égalité de l'offre et de la demande globales de devises, c'est-à-dire, comme nous l'avons vu, l'égalité des aires

[6] A première vue, on pourrait croire que les immeubles comportent des frais de transfert infinis et ne peuvent intervenir dans le commerce international; en réalité, si un Anglais résidant à Londres achète une maison en France, le prix devra en être réglé par le marché des changes; s'il s'agit d'une maison de rapport, on peut considérer qu'il s'agit là seulement d'un placement anglais en France; mais il se peut aussi qu'il s'agisse d'une maison à usage personnel, que son propriétaire viendra habiter de temps à autre. Dans tous les cas, les frais de transfert sont ici représentés par ceux du titre de propriété.

(ou des poids) des deux surfaces limitées par la courbe de structure et séparées par le curseur. (Surfaces encadrées d'un double trait sur la figure.)

Introduction de la balance financiere

Dans la réalité, les offres et demandes de devises ne portent pas que sur les ventes de marchandises mais aussi sur les paiements autres que commerciaux, répondant au règlement de dettes et à des prêts.

En définitive, l'équilibre du change est représenté par la fig. 4, exprimant l'égalité des offres et demandes de devise de toute nature: car si la balance des comptes (ou balance des paiements exigibles) n'est pas en équilibre, la différence constitue, *ipso facto*, un prêt ou un emprunt.

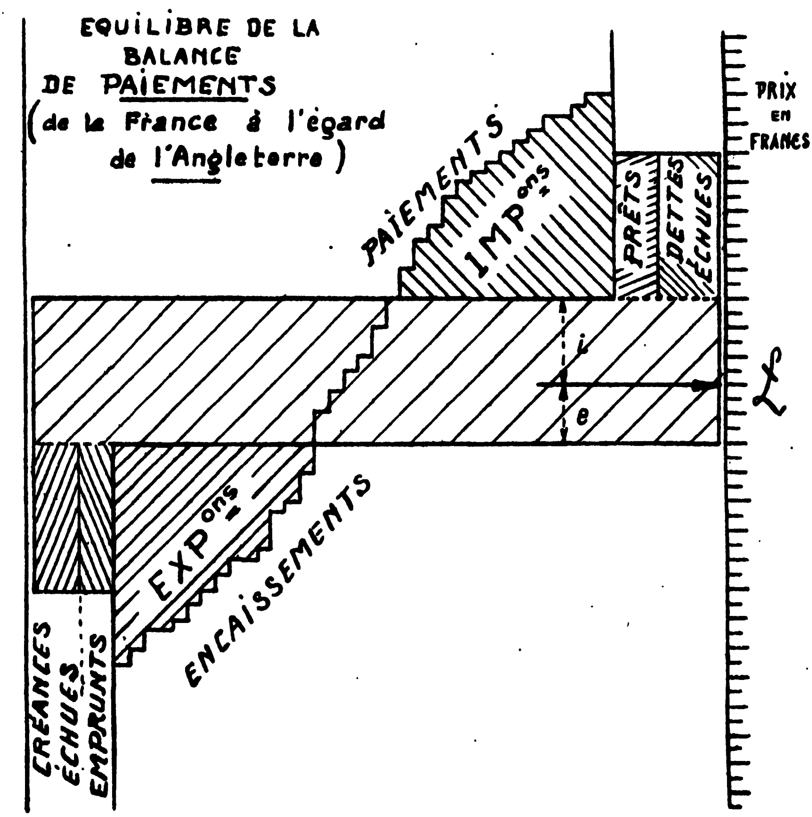

Figure 5

L'épure donnée plus haut constituera une représentation de l'équilibre international, si, aux surfaces inconnues figurant les paiements d'exportations et d'importations, on ajoute des surfaces figurant les

placements et les dettes; et si on fait abstraction du mouvement des capitaux flottants (qui peut dépendre du niveau des changes, mais n'a en général, qu'une faible importance et une action toute momentanée), ces éléments, en gros indépendants du niveau du change, peuvent être considérés comme *des données* et représentés par des surfaces constantes attachées au curseur (fig. 5). La figuration obtenue donne ainsi une vue à la fois complète et simple des problèmes commerciaux, financiers, monétaires—assez complexes—mettant en jeu l'équilibre international.

Dans cette figuration, l'équilibre du curseur détermine tout à la fois le niveau du change, la nature et les montants des marchandises importées ou exportées.

Applications

La représentation géométrique ci-dessus ne paraît guère permettre des déterminations numériques, en raison de la difficulté de relever la valeur des éléments de sa construction; notamment, la courbe de structure qui lui sert de base, est relative à un état économique hypothétique antérieur à l'établissement des échanges.

Toutefois on peut observer que le problème pratique n'est pas de déterminer *ab ovo* la situation d'équilibre, mais d'étudier les modifications qu'elle peut subir sous l'influence de tel ou tel facteur; ce qu'il importerait donc de connaître, ce sont seulement, d'une part, les variations des aires (ou des poids) plutôt que leurs valeurs elles-mêmes, d'autre part, la forme de la courbe de structure au voisinage du curseur.

Même non construite numériquement, la figure peut guider des relevés d'observation tendant à la détermination de certaines élasticités, qui paraissent devoir être d'un grand secours déjà à simple titre d'ordres de grandeur, dans des problèmes pratiques importants.

Enfin, et d'ores et déjà, la représentation paraît utile comme simple image pour clarifier les questions de change et de commerce extérieur, et tous les problèmes qui s'y greffent.

Le problème des transferts

On sait qu'après la guerre de 1914, à propos du paiement par l'Allemagne de sa dette de réparations, on s'est demandé si un pays peut assurer le paiement d'une dette massive par les moyens ordinaires du marché libre des changes, sans désorganiser sa monnaie. A l'époque, M. Rueff montra théoriquement qu'il existe toujours un niveau de change assurant un excès suffisant des exportations sur les importations pour procurer les devises nécessaires au paiement, et que l'achat de ces devises sur le marché des changes par l'Etat debiteur fait précisément

monter le change à ce niveau, qu'ainsi, en définitive, il n'y a pas de problème de transfert.

Effectivement, si on se reporte à la fig. 5, on voit que, quelle que soit la grandeur de la surface rectangulaire "Dettes échues," il existe toujours une position du curseur telle, que les aires délimitées assurent l'équilibre de la balance des paiements: le niveau du change s'élève en conséquence.

Toutefois, dans le cas de monnaie convertible en or, un courant de transfert important risque de porter le change audessus du gold-point de sortie et d'assécher l'encaisse de la banque d'émission. Dans ce cas, il convient de passer d'abord au cours forcé, pour que le change puisse s'élever sans encombre au niveau requis; ce niveau atteint et les courants suffisamment stabilisés, on peut revenir à la convertibilité, moyennant une dévaluation convenable de la monnaie correspondant à la nouvelle position d'équilibre du curseur.

Ce n'est qu'au cas où on se refuse à accepter toute dévaluation de la monnaie qu'il existe un problème de transfert, le transfert possible étant alors limité par la condition que le niveau du change ne dépasse pas le gold-point de sortie. Peut-on, alors, calculer *a priori* l'importance de ce maximum du transfert possible? C'est une question d'élasticité de la balance commerciale relativement au niveau du change. Précisément, dans les travaux qu'il publia à l'époque, M. Rueff a donné des évaluations numériques de disparités des changes, et des dénivellations de balance commerciale corrélatives; il est curieux qu'il n'ait pas songé à en dégager des mesures d'élasticité; mais toutes ces notions n'étaient pas alors connues comme aujourd'hui.

Il est facile de voir comment est représentée cette élasticité sur la fig. 5. Si on appelle e et i les longueurs des bases de la courbe de structure sur les deux bords du curseur, on voit qu'à une élévation $d£$ du curseur correspondra une variation des exportations $dE = ed£$ et une variation des importations $dI = -id£$, en sorte que la balance $E - I$ variera de $dE - dI = (e + i)\, d£$. Inversement, toute connaissance des élasticités de E et de I par rapport à £ renseignera numériquement sur la fig. 5.

La parité des pouvoirs d'achat

On présente assez souvent le change comme déterminé par la condition de réaliser la parité des pouvoirs d'achat, sans d'ailleurs que cette théorie ait jamais fait, à notre connaissance, l'objet d'un exposé vraiment précis. La présentation géométrique peut nous montrer ce qu'on peut en penser au juste.

Si on admet que la balance financière est négligeable vis-à-vis de la balance commerciale, la position d'équilibre du curseur, réalisant à peu près

l'égalité des aires de la courbe de structure, se situera dans le milieu de cette dernière; d'autre part, comme l'horizontale qui marque le niveau du change est elle-même dans le millieu du curseur, la valeur d'équilibre de la livre sera une valeur médiane dans l'ensemble des rapports de prix $\pi_f = p_f \; p_a$ représentés par la courbe de structure. Nous avons ainsi une vue très générale, et très simple, de la parité des pouvoirs d'achat.

Mais cette parité est extrèmement grossière: d'une part, la balance financière n'est souvent pas négligeable, et d'autre part, l'horizontale qui représente le niveau du change ne passe pas au milieu du curseur, dans tous les pays (et ils sont nombreux) où les frais de transfert à l'importation sont systématiquement et notablement supérieurs aux frais de transfert à l'exportation, par suite de l'existence de droits de douane.

Au demeurant, quand on parle de parité des pouvoirs d'achat, ce n'est pas relativement aux prix de structure, hypothétiques, mais par rapport aux prix actuels.

Il est possible de voir comment ces derniers se situent sur la figure:

L'équation (1) donne:

$$\frac{p_f'}{p'_a} - £ = \frac{t}{p'_a} = \tau \frac{p_a}{p'_a},$$

d'où:

$$\pi_f' = \frac{p_f'}{p_a'} = £ + \tau \frac{p_a}{p_a'},$$

et il est facile de voir sur la fig. 1 que le coefficient de τ est une fonction décroissante de q, égale à 1 pour $q = 0$ et atteignant une valeur comprise entre 0 et 1 lorsque q atteint sa valeur maxima Q_a. On voit ainsi que la courbe des rapports des prix actuels se situe tout entière sur le curseur, au-dessus de l'horizontale de niveau £ pour les importations, et au-dessous pour les exportations; par suite, ce niveau £ a une valeur médiane dans l'ensemble de ces rapports, et il vérifie donc la parité des pouvoirs d'achat relativement aux prix actuels (de façon d'autant plus serrée que le curseur est plus étroit).

Mais cette propriété du change à l'équilibre n'est pas déterminante de l'équilibre, puisqu'elle existe indépendamment de la position du curseur.

Ce qui donne à penser que toute théorie de la détermination du change fondée sur la parité des pouvoirs d'achat est inconsistante.

Conclusion

Si la représentation géométrique proposée ci-dessus conduit peut-être à certains résultats, elle possède, assurément, de graves défauts, dont le plus gros est, sans doute, qu'elle néglige complètement l'interdépendance générale des biens et les modifications de structure qui en résultent.

Pourtant, comme c'est là une lacune classique de l'oeuvre des Cournot, Dupuit, Marshall, Colson, ... elle ne paraît pas devoir être nécessairement dirimante, puisque cette oeuvre s'est montrée fort utile, déjà au point de vue de la connaissance qualitative, mais aussi au point de vue économétrique: C'est que, si elle ne considère que l'effet primaire, il arrive, dans certains problèmes, que l'effet secondaire, dû aux réactions d'interdépendance, soit négligeable, ou même complètement nul.

Tel est le cas, par exemple, du calcul de la perte sèche d'un impôt indirect fait par Dupuit; dans un article publié dans le *N°* de juillet 1938 *Econometrica*, M. Hotelling a repris le calcul en tenant compte des interdépendances, et il a abouti à la même formule.[7]

Sans doute l'utilisation de la méthode des équilibres partiels à des déterminations économétriques comporte, de ce chef, un risque sérieux; mais le danger n'a pas l'importance qu'on pourrait être tenté de lui attribuer, si l'économètre connaît ce risque, et il le connaît. Assurément, une prise directe en synthèse, dans la ligne de l'oeuvre de Walras, se présente *a priori* comme beaucoup plus satisfaisante et plus sûre; mais elle est infiniment moins maniable, et ce n'est pas un mince inconvénient pour les travaux économétriques requis par les problèmes pratiques. Quant à la méthode macroscopique, elle n'est pas non plus, de son côté, exempt de dangers, lesquels, pour être moins apparents, n'en sont sans doute pas moins pernicieux.

Résumé

Let us consider only two countries and, at first, one good only. If two countries are economically isolated from each other, the supply and demand curves determine, in every one, the prices p_a and p_f as "structural prices," (*a* for Angleterre, *f* for France). Suppose now that communication is established, resulting in the flow of q units of the good; the new prices p_a' and p_f' will be called "actual prices." At the equilibrium, these prices are such that $p_f' - £p_a' = t$, £ being the current price of the pound sterling in Paris, and t being the cost of transporting the good. By means of this equation, we can calculate p_a', p_f', q and, therefore, the demand (positive or negative) for foreign exchange, qp_a'. This is found to be an increasing function of the term $y = P - £ - T$, P being the ratio of the structural prices P_f and p_a, that is to say the French price of the quantity, the English cost of which is one pound, and T being the transportation cost of this quantity.

[7] Résultat contesté, mais à tort à notre avis, par M. Allais.

In a graph where amounts in French currency are plotted vertically, the demand for foreign exchange, supposed to be proportional to y, is shown by the area of a rectangle whose bases have ordinates P and $£ + T$; and, generally speaking by the weight of a plate of metal of a variable density limited by this rectangle.

Now, putting side by side such rectangles in order of increasing P, we can show the relative position of all structural prices of both countries; the bases of these rectangles form a step-curve which depends only on the initial structures of both countries and is therefore a datum of the problem. If we draw the horizontal straight line the ordinate of which is L, and on both sides of it the parallel lines D_i and D_e, (we call T_i and T_e the import and export unit transportation costs and suppose, for simplicity, that they do not differ for the several goods) the exchange equilibrium implies the equality of the supply and demand of foreign exchange and consequently, the equality of the areas above D_i and below D_e.

Now let the rate of exchange be unknown, and let the straight lines D_i and D_e be the sides of a moving ruler with a width $T_i + T_e$. When the ruler is moved into a position that makes the areas I and E equal, then the equilibrium level of £ and the equilibrium amounts and values of imports and exports are determined.

This presentation is rather imperfect and inaccurate. It aims at stimulating a critical examination of its shortcomings as well as of its possible usefulness.

The most important defect is the complete neglect of the general interdependence of goods and of the consequent modifications of structure. This does not seem, however, to be fatal: the same gap appears in the work of Cournot, Dupuit, Marshall, Colson, ... work which has proved very useful for the qualitative knowledge as well as for econometrics. To be sure, the method considers only first-order effects. But in certain problems secondary effects are negligible or even vanish entirely (for instance, in Dupuit's calculation of the net loss of a tax; see Hotelling, *Econometrica*, July 1938).

With all its faults, the graph gives at least a picture of the international economic equilibrium, whilst the usual explanations of this equilibrium, based upon the purchasing-power-parity theory, are, in this respect, inconsistent.

Mr. Divisia's paper was discussed by Messrs. J. Tinbergen and Jacques Rueff.

EXCHANGE-RATE STABILITY CONSIDERED

by Lloyd A. Metzler[1]

Associate Professor of Economics, The University of Chicago (United States)

Résumé

The violence of exchange-rate fluctuations in countries which in the past, have had unregulated markets for foreign exchange has frequently led to the conjecture that a system of market exchange rates may be inherently unstable. The condition of stability, of course, is that a rise in the price of foreign currency shall reduce the excess demand for such foreign exchange. The relation of this condition to elasticities of demand for imports and elasticities of supply of exports has been investigated in considerable detail by Alfred Marshall, Mrs. Joan Robinson, C. W. Bickerdike, A. J. Brown, and others. All of the investigators have considered a world economy consisting of only two countries, and the conclusions they reached are as follows: (1) If exports are produced under constant price both at home and abroad, stability requires that the sum of the important demand elasticities at home and abroad must exceed unity. (2) If the elasticity of supply of exports is zero at home and abroad, the exchange market is always stable regardless of the demand elasticities. (3) In the intermediate case, stability requires that $\eta_1\eta_2(e_1+e_2+1) + e_1e_2(\eta_1+\eta_2-1)$ shall be positive, where η_1 and η_2 are import demand elasticities at home and abroad and where e_1 and e_2 are export supply elasticities.

Brown considers an additional complication in which each country's exports consist partly of imported materials, and concludes that the presence of such imported materials increases the stability of the market. It is easily shown, however, that this conclusion is incorrect. If exports are produced at constant supply price, the condition of stability, after allowing for imported raw materials, is: $r_1\eta_1+r_2\eta_2 > 1$ where η_1 and η_2 are elasticities of demand for imports of finished goods, and where r_1 and r_2 are the ratios of the value of exports retained by domestic producers to the total value of exports. Since these ratios are less than unity, and will be considerably less than unity if imported materials comprise a large part of the cost of exports, it follows that the exchange market may be unstable even when $\eta_1 + \eta_2$ exceeds unity.

If exchange markets are stable as of given supply and demand schedules, the possibility still remains that they may be made unstable

[1] The speaker was not able to be present and the paper was read by title. Only abstracts are available to publish.

by secondary movements of income. In the depreciating country, income rises as a result of the export surplus induced by depreciation, and in the rest of the world income falls. The income adjustments thus increase the demand for imports in the depreciating country and reduce the demand for that country's exports. Mrs. Robinson argues that the secondary changes in the balance of trade cannot completely offset the primary changes since income in the depreciating country can remain higher than before depreciation only if the balance of trade remains more favorable. The Robinson argument assumes, however, that savings depend only on money income. If we consider the reaction of savings to an adverse movement in the terms of trade, it becomes possible for the secondary income movements to offset completely the initial change in the balance of trade.

Résumé

L'ampleur des fluctuations des cours de change dans les pays où, dans le passé, le marché des changes n'était pas contrôlé, a souvent fait penser qu'un système où les cours de change sont formés par le marché libre serait essentiellement instable. La condition de stabilité est, évidemment, qu'une hausse du prix d'une monnaie étrangère réduira l'excédent de la demande de cette monnaie. La relation de cette condition aux élasticités de la demande de produits d'importation et aux élasticités de l'offre des produits d'exportation a été etudiée en détail par Alfred Marshall, Mrs. Robinson, C. W. Bickerdike, A. J. Brown, et d'autres. Toutes ces études sont basées sur l'hypothèse d'une économie mondiale constituée de deux pays seulement. Les conclusions auxquelles on est arrivé sont les suivantes: 1) Si les marchandises d'exportation sont produits à prix d'offre constant à la fois dans le pays considéré et à l'étranger, la stabilité requiert que la somme des élasticités de demande des produits d'importation dans le pays et à l'étranger soit supérieure à l'unité; 2) Si l'élasticité d'offre des produits d'exportation est nulle dans le pays et à l'étranger, le marché des changes est toujours stable, quelles que soient les élasticités de demande; 3) dans le cas intermédiaire, la stabilité requiert que $\eta_1\eta_2(e_1 + e_2 + 1) + e_1e_2 (\eta_1 + \eta_2 - 1)$ soit positif, η_1 et η_2 étant les élasticités de demande à l'importation dans le pays et à l'étranger, et e_1 et e_2 étant les élasticités d'offre à l'exportation.

Brown considère un cas plus complexe où les exportations de chaque pays sont constituées partiellement de matériaux importés; il aboutit

à la conclusion que cette hypothèse fait augmenter la stabilité du marché. Il est cependant facile de démontrer que cette conclusion et fausse. Si les exportations sont produites à prix d'offre constant, et s'il est tenu compte des matières premières importées, la condition de stabilité est: $r_1\eta_1 + r_2\eta_2 > 1$ où η_1 et η_2 sont les élasticités de demande de produits finis importés, et r_1 et r_2 sont les rapports de la valeur des exportations retenues par les producteurs du pays, à la valeur totale des exportatinso. Du fait que ces rapports sont inférieurs à l'unité, et seront considérablement moindres que l'unité si les matières importées constituent une grande part du coût des exportations, il s'ensuit que le marché des changes peut être instable même quand $\eta_1 + \eta_2$ excède l'unité.

Si le marché des changes est stable pour des lois données d'offre et de demande, il est encore possible qu'il devienne instable par suite des variations secondaires dans les revenus. Dans le pays dont la monnaie se déprécie, le revenu augmente à la suite de l'excédent d'exportation causé par la dépréciation, tandis que dans le reste du monde le revenu diminue. L'accroissement de revenu fait augmenter la demande d'importations dans le pays à monnaie dépréciée et fait diminuer la demande des produits d'exportation de ce pays. Mrs. Robinson prétend que les changements secondaires de la balance commerciale ne peuvent compenser complètement les changements primaires puisque le revenu dans le pays à monnaie dépréciée ne peut demeurer à un niveau supérieur à celui d'avant la dépréciation que si la balance commerciale continue à être plus favorable. L'argument de Mrs. Robinson suppose, cependant, que l'épargne dépend uniquement du revenu monétaire. Si la réaction de l'épargne aux fluctuations adverses des termes d'échange est prise en considération, il est possible que les variations secondaires de revenu annullent complètement la modification initiale de la balance commerciale.

L'analyse de la stabilité d'un système de marchés multiples n'est pas aussi aisée. En général, les conditions de stabilité formulées par Hicks ne s'appliquent pas aux marchés de ce type. La thèse qu'un pays, dont l'importance dans l'économie mondiale est restreinte, a des chances de bénéficier d'une dépréciation de sa monnaie, néglige le fait que la dépréciation de la monnaie d'un pays peut entraîner des difficultés dans la balance des payements et, en outre, des dépréciations monétaires dans d'autres pays.

ECONOMIC GROWTH AND FLUCTUATION

Thursday, September 11, at 4:00 p.m.

CHAIRMAN:

J. J. Polak (*United States*)
International Monetary Fund

THEORY OF ECONOMIC GROWTH

by Colin Clark
Director, Bureau of Industry, Financial Adviser to the State Government, and Under Secretary for State, Department of Labour and Industry, Queensland (Australia)

The primary measure of economic growth must be real income obtained per hour worked. Comparison of real incomes necessitates fixing some base for defining the relative value of different goods and services. The base chosen is the U.S.A. during the period 1925-34, and a quantity of goods and services exchangeable for $1 over that period is defined as an International Unit (I.U.)

In comparing the real income of a peasant and of an urban community we must bear in mind that considerable expense incurred in the transport and distribution of foodstuffs for the urban community are not necessary for a peasant family supplying their own requirements. In other words, food produced for their own use by a peasant population should be valued at the *retail* price payable in an urban community. In gauging the real income of primitive communities we must, therefore, *impute* to them an income representing the difference between the wholesale and retail value of their food consumption. This imputed income is a large proportion of the total real income in the poorest communities. The following table shows six countries selected as typical of the different phases of economic growth to be found in the world at the present day.

TABLE 1

	Income Level I.U. Per Person at Work		As Percentage of National Income			
	Per Hour	Per Year	Imputation of Income from Valuation of Food Consumption by Farm Families at Retail Prices	Consumption of Farm Products at Wholesale Value	Consumption* of Products of Large-Scale Manufacture	Consumption of Services and Small-Scale Manufacture
China	0·03	138	59	19	1	21
India	0·08	223	46	24	3	27
Italy	0·18	395	9	21	21	49
Netherlands	0·44	1,054	2	15	20	63
Australia	0·64	1,421	—	13	16	71
U.S.A.	1·00	2,222	1	10	22	67

* Including goods representing new investment. Excluding depreciation and replacement of *manufacturing* equipment, and excluding value of raw materials and of tertiary services incorporated in manufactured goods.

The real product of China, it is seen, consists mainly of foodstuffs, worth, per hour of work done, about 3 cents valued at American retail prices and nearly two-thirds of this is imputed income. At American wholesale prices it would be worth only a little over 1 cent.

The decline with increasing income in the proportion of income spent on farm products is a well-known phenomenon. With further advances in real income it may even fall below 10 percent. The demand for manufacturing services is more interesting. The poorest communities are supplied by handicraft men and small-scale manufacturers. In the early stages of economic growth the demand for manufactures rises very rapidly (*e.g.*, in South Africa and Finland it rose to about 27 percent of income in 1936-38). With the lapse of time and as the country becomes wealthier, manufacture finds that it has to serve a *replacement* demand rather than a new demand (for capital goods and for durable consumption goods) and its relative importance in the national income begins to fall. In U.S.A. this stage was reached as early as 1920.

In the economically advanced communities an increasing proportion of demand must be for those objects of consumption which have a high income elasticity of demand, which are largely services rather than goods. There is also an increasing demand for hand-made and specialty goods produced in small workshops rather than for the mass-produced goods that are acceptable in poorer and less cultured communities.

The manner in which a country's working population is distributed between agriculture, manufacture, and tertiary industries is determined by:

1. The relative demand for the three types of products.
2. The extent to which farm produce or manufactures are imported or exported. (Most services cannot be internationally traded.)
3. The relative productivity of labour in the different industries.

Agriculture occupies about 70 percent of the working population in China and 64 per cent in India, although food consumption (at whole-sale prices) represents much lower proportions of those countries' total incomes. This is because the productivity of labour in agriculture in those countries is very low owing to excessive density. Similarly in U.S.A. the proportion of the working population engaged in agriculture is much higher than the proportion of the national income devoted to the consumption of farm products, because of low relative efficiency of labour in agriculture. It would be higher still if it were not for the amount of primary produce imported.

The first stage of economic growth, therefore, is to transfer labour away from diminishing-returns agriculture to manufacture, where increasing returns are possible. As these increasing returns cannot be obtained until manufacture is already on a substantial scale, the process will not occur automatically and must be assisted by tariffs or subsidies. At this stage of development rapid accumulation of capital is necessary.

The second phase is reached when the relative demand for manufactures levels out or falls off. At the same time capital requirements, relative to increments of income, tend to become smaller while the capacity to save increases. Countries that have reached this phase must either make large external investments, or find some other new use for their savings. Otherwise extensive unemployment will ensue.

Assuming this problem has been solved, the main economic question for the economically advanced countries is to secure efficiency in the service industries (building, retail distribution, etc.). Though these industries can never yield increasing returns on so striking a scale as manufacture, there are indications that very satisfactory progress has been made in certain directions though much still remains to be done.

Résumé

Le revenu réel par heure de travail constitue le critérium principal du développement économique. La comparaison des revenus réels nécessite

la définition d'une unité de base, afin de pouvoir exprimer la valeur relative des différents biens et services. L'"Unité Internationale" choisie est la quantité de biens et services que pouvait acheter en moyenne $1 aux Etats-Unis au cours de la période 1925-34 (*cf.* tableau 1).

L'on voit que le produit réel de la Chine est constitué en ordre principal de produits alimentaires valant, par heure de travail, environ 3 cents en termes de prix de détail américains; en termes de prix de gros 1 cent seulement.

Avec accroissement du revenu réel la diminution de la fraction du revenu dépensé pour les produits agricoles est un phénomène bien connu. De nouveaux progrès dans le revenu réel pourraient réduire cette fraction à moins de 10 pour cent.

La demande de services manufacturiers est plus intéressante. Les communautés les plus pauvres sont servies par les artisans et la petite manufacture. Aux premiers stades du développement économique, la demande de produits manufacturés croit très rapidement. Après un certain temps, et à mesure que le pays devient plus riche, l'industrie manufacturière doit faire face à une demande de *remplacement* plutôt qu'à une demande de produits nouveaux (biens de production et biens de consommation durables) et son importance relative dans le revenu national commence à décliner. Aux Etats-Unis ce stade a été atteint dès 1920.

Dans les communautés économiquement avancées, une fraction croissante de la demande s'adresse aux objets de consommation dont l'élasticité de la demande est élevée par rapport au revenu; ces objets sont plutôt des services que des biens.

La distribution de la population active d'un pays entre l'agriculture, l'industrie manufacturière et les autres industries est déterminée par:

1. La demande relative des trois types de produits.
2. La mesure dans laquelle les produits agricoles et les articles manufacturés sont importés ou exportés. (La plupart des services n'ont pas de marché international.)
3. La productivité relative du travail dans les différentes industries.

C'est pourquoi le premier stade du développement économique consiste en un transfert de main-d'oeuvre de l'agriculture, où les rendements sont décroissants, vers le secteur manufacturier, où les rendements croissants sont possibles. A ce stade de développement, une accumulation rapide de capital est nécessaire.

Le second stade commence lorsque la demande relative de produits manufacturés cesse de croître ou tend à décroître. A ce moment, les besoins de capitaux par rapport à l'accroissement des revenus tendent à se réduire tandis que la capacité d'épargne augmente. Les pays qui

ont atteint ce stade doivent soit procéder à des investissements extérieurs considérables, soit trouver quelque autre emploi de leur épargne. Sinon, un chômage étendu s'en suivra.

Si ce problème est supposé résolu, le principal problème économique des pays avancés est d'assurer l'efficacité du secteur des services (bâtiment, commerce de détail, etc.). Quoique ces branches d'industrie ne pourront jamais produire des rendements croissants dans la mesure atteinte par l'industrie manufacturière, il y a des indices qui permettent de croire que des progrès très satisfaisants ont été réalisés dans certaines directions, bien qu'il reste encore beaucoup à faire à cet égard.

Mr. Clark's paper was discussed by Messrs. J. Tinbergen, Hans Staehle, J. J. Polak, Benjamin H. Higgins, Abba P. Lerner, Jacob Marschak, W. S. Woytinsky, François Divisia, Evsey D. Domar, R. L. Anderson, Michal Kalecki, Julius Wyler, and Jacques Dumontier.

LA LOI DE DIVERGENCE

par B. Chait

INDUSTRIEL

[Dans l'absence de M. C h a i t, sa communication fut presentée par François D i v i s i a, Professeur au Conservatoire des Arts et Métiers, à l'Ecole Polytechnique, et à l'Ecole des Ponts et Chaussées (France)]

Ayant, en l'absence de M. Chait, à vous présenter sa communication, je vous demanderai de l'indulgence pour l'exposé d'une oeuvre qui est, à mon sens, particulièrement importante et complexe. Je vais tâcher très brièvement de vous en donner l'essentiel, tel que je le considère. Il s'agit en somme d'une théorie synthétique dynamique, décrivant le fonctionnement et l'interdépendance des marchés. M. Chait a employé ici ce mot "*marché*" dans une acception très précise. Il s'agit, en somme, d'un lieu d'élaboration des biens, d'une unité productive, cette unité pouvant d'ailleurs être une entreprise, une industrie, ou même un groupe d'industries.

Si l'on considère l'ensemble de la vie économique, on peut la regarder comme un flux de biens et de services, à partir des matières premières et du travail, jusqu'aux produits de consommation finale. Depuis leur origine, ces biens se transforment en traversant une série successive de marchés. Tous ces marchés comportent, soit une élaboration industrielle, soit, tout aussi bien, une élaboration commerciale, en sorte que leur ensemble représente l'intégralité de la vie économique.

M. Chait considère cet ensemble, cet univers économique, pour l'instant du moins, sous l'aspect purement matériel. Il considère des flux de biens et ces flux auront une expression mathématique dans des quantités. Je passe sur l'objection de la non comparabilité des quantités de biens différents. Placez-vous, je vous prie, sur un plan beaucoup plus abstrait et beaucoup plus général.

Ces unités économiques que M. Chait appelle "marchés" sont liées, en somme, les unes aux autres, par *le lien de fournisseur à client.* Si vous considérez une matière première et toutes ses transformations jusqu'au produit de consommation finale, il y a un chapelet d'élaborations successives et ce chapelet, M. Chait lui a donné le nom de "*filière*". On peut représenter schématiquement cette filière par une série de marchés, que la marchandise traverse successivement au cours de son élaboration industrielle ou commerciale.

Une telle filière est une filière théorique. Nous savons que, dans la réalité, un marché est fournisseur non pas d'un seul marché mais de plusieurs. Lui-même, n'a pas nécessairement un seul marché fournisseur, il peut en avoir plusieurs, recevoir par exemple, du fer, du charbon, etc. M. Chait nous amène ainsi à considérer l'ensemble de l'activité économique comme portant sur un ensemble de flux de richesses débités dans *un réseau de filières polyvalentes*, chaque marché étant le noeud de sortes de conduits qui le relient d'une part à tous les marchés fournisseurs et d'autre part à tous les marchés clients. Les biens s'écoulent dans ce réseau jusqu'à devenir des produits de consommation finale, déversés sur le marché du consommateur, où en somme (si nous faisons abstraction de la monnaie) ils s'échangent les uns contre les autres.

Vous voyez que nous avons là une vue extrêmement générale, et au fond très réaliste, de la vie économique. M. Chait a réussi à lui donner une représentation analytique complète. Et cela l'a amené à étudier le fonctionnement d'un marché, puis le lien du marché avec tous les autres.

Si vous considérez un marché quelconque, ce marché est caractérisé dans la filière, par un flux d'entrée E, par un flux de sortie S et par un stock existant à l'intérieur σ. La situation du marché en question est parfaitement définie par la valeur du flux d'entrée, par la valeur du flux de sortie, et par la valeur du stock. Déjà, il y a un lien entre ces quantités: si nous appliquons le vieil aphorisme que rien ne se perd, rien ne se crée, vous voyez que la différence entre le flux d'entrée et le flux de sortie est égal à l'accroissement du stock.

Pour un marché M quelconque, nous avons à tout instant la relation $E - S = d\sigma/dt$.

Déjà cela nous amène à considérer *l'interdépendance de marchés successifs sur la filière.* Dans le cas d'une filière monovalente, cas le plus simple, le lien entre deux marchés successifs est extrêmement facile; le flux de sortie du marché M_i est égal au flux d'entrée du marché M_{i+1}. Si bien que nous allons trouver des équations de récurrence qui nous permettront d'écrire les liaisons entre le flux de départ de la filière, le flux d'arrivée, tous les flux intermédiaires et les variations de tous les stocks. Soit, si n est le nombre de marchés de la filière, $n-1$ équations entre les $2n$ flux et les n stocks.

Si, maintenant, nous passons au réseau réel complexe polyvalent, eh bien, le calcul reste le même, sauf que le flux d'entrée dans le marché M_i est égal à la somme de ceux des flux de sortie des marchés M_{i-1} qui y aboutissent et que le flux de sortie de ce marché est égal à la somme des flux d'entrée dans les suivants. Vous voyez que cela peut donner lieu à l'écriture d'un système d'équations assez volumineux, assez complexe aussi: c'est un système d'équations différentielles puisque nous y trouvons les dérivées $d\sigma/dt$.

M. Chait a réussi à donner un traitement mathématique de ce système d'équations différentielles récurrentes, et il apporte là des équations qui sont fondamentales pour le fonctionnement de toute économie. En effet, je vous ferai observer que ce qu'il écrit ici, ce sont simplement des équations de conservation, c'est-à-dire presque des équations de pure logique ayant, par conséquent, une portée universelle.

On pourrait presque dire que, jusque là, la présentation n'est pas économique, en ce sens qu'elle s'appliquerait à bien d'autres choses. Qu'on ait par exemple, un réseau de distribution d'eau avec des conduits et des réservoirs, le fonctionnement technique de ce réseau serait décrit de la même manière par les mêmes équations. Là où, vraiment, je pense que l'économique intervient, c'est, autrement que par ces équations purement logiques ou mécaniques, dans les relations qui maintenant existent, ou peuvent exister, pour un marché donné, entre les flux et le stock, ou leurs variations. Pour apercevoir ces liaisons (et ces liaisons manquaient pour la solution du système des équations où il y avait plus d'inconnues que d'équations), M. Chait a dû, tout naturellement, étudier les stocks, *le rôle des stocks.* Et il a donné là une présentation qui, pour être brève, n'en constitue pas moins, d'ores et déjà, beaucoup plus qu'un simple embryon de la théorie des stocks économiques. Cette théorie est extrêmement importante, qu'il s'agisse de l'étude théorique de l'économique, qu'il s'agisse des investigations statistiques. Vous savez que jusqu'à une date assez récente, on peut dire que les stocks ont été la terreur des théoriciens aussi bien que des statisticiens.

M. Chait aperçoit, dans les stocks, premièrement, un rôle technique qui est d'assurer la continuité des flux. C'est qu'en effet, pour des raisons de nécessité commerciale, de nécessité économique, les flux d'entrée et de sortie ne sont pas nécessairement liés; le flux d'entrée est en rapport avec les possibilités de fournitures, le flux de sortie est en rapport avec les possibilités commerciales de vente. Il n'y a donc aucune raison qu'un lien systématique s'établisse entre les deux. Si le flux d'entrée était rigoureusement et constamment égal au flux de sortie, le stock serait constant; et au fond c'est là la présentation de l'économie classique de Walras qui lui a permis d'éliminer justement les stocks de sa construction théorique. En réalité, il n'en est pas ainsi et le stock est constitué pour, justement, réaliser le raccordement entre l'entrée et la sortie. Par là, vous apercevez la nécessité d'un stock technique destiné à parer aux irrégularités d'approvisionnement. Ce stock technique a un aspect probabiliste, dans la mesure où on émet des hypothèses sur les possibilités d'approvisionnement ou les possibilités d'interruption de l'approvisionnement, d'interruption accidentelle. De même, il y a un stock commercial destiné à raccorder la production au rythme des demandes, avec un élément de calcul précis si on supposait les

débouchés donnés, et, en fait, un élément probabiliste qui provient de ce que le fournisseur ne sait pas bien quel sera le rythme des demandes.

M. Chait ajoute également une autre fonction du stock: les stocks dont nous avons parlé (techniques ou commerciaux), ont encore un rôle que l'on peut appeler technique au sens général. Le producteur doit savoir les calculer et si ces stocks sont déficients, il y a des incidents dans la gestion. Mais, en outre, le producteur peut vouloir constituer des stocks supplémentaires pour se donner de la liberté d'action, et c'est là un stock économique. Dans ce stock économique, existe notamment le stock spéculatif qui fait que l'entrepreneur désire acheter ou vendre ses biens plutôt à certaines époques qu'à d'autres.

Pour connaître le comportement du producteur à l'égard de son stock, eh bien, c'est évidemment à l'observation statistique que l'on devra faire appel. Les idées théoriques sur les stocks seront donc complétées par des observations sur le lien des stocks avec le rythme des flux de production et de vente; et si on admet que l'observation statistique puisse nous fournir sur ce point des normes de fait assez générales pour pouvoir être regardées comme caractéristiques d'une économie donnée, cela permettra d'introduire *des équations supplémentaires de liaison entre flux et stock*, qui compléteront le système; on obtient alors une description dynamique de son fonctionnement. Se basant sur certaines observations statistiques (d'ailleurs encore rudimentaires, car son effort a surtout porté sur la construction théorique) M. Chait a émis certaines hypothèses, vérifiées en certains cas, sur le comportement des stocks, et il en a tiré des calculs complets, dont la solution lui donne l'évolution du système.

Il arrive ainsi à poser des liens entre l'évolution des flux d'entrée et l'évolution des flux de sortie, entre le rythme de production des biens primaires (matières premières et biens semi-finis) et le rythme de production des biens finis, ou même des biens finaux, notamment sur le marché du détail. Car le commerce du détail, naturellement, est inclus dans la filière, c'est un marché placé après les autres, à l'extrémité de la filière.

M. Chait est arrivé de cette manière à trouver *des lois générales de liaison entre l'amplitude des fluctuations de certains marchés et l'amplitude des fluctuations de certains autres*. Cela tient, à ce que, étant donné le comportement d'un marché, il n'y a aucune raison pour que le rythme du flux d'entrée soit égal au rythme du flux de sortie, mais les deux sont liés à travers le stock, lui-même lié aux flux par le comportement des producteurs.

C'est en quoi consiste, de la façon la plus générale, je ne dis pas sa loi de divergence, mais au moins l'idée de cette loi. A partir de là, M. Chait a retrouvé, mais sur un plan bien plus général, le principe

d'accélération. La théorie du multiplicateur se trouve également incluse dans son étude.

Vous voyez qu'il y a là, par conséquent un moyen de traitement théorique d'investigations statistiques qui me parait, personnellement, d'une puissance extraordinaire. Et je crois que cette puissance tient notamment à ce que cette construction, fondée sur l'observation statistique, a une base de départ entièrement déductive, et qu'elle a un caractère absolument général. C'est en somme un systême économétrique dans toute la splendeur du mot.

Outre ces lois retrouvées à partir du principe d'accélération, je vous signalerai un autre résultat qui me paraît de grande importance. En raison de la division des filières, M. Chait a trouvé que lorsque, pour une cause quelconque, peut être accidentelle, on produit une perturbation sur un marché, cette perturbation se propage le long des filières par le jeu des équations de récurrence et il a trouvé que dans certains cas, la perturbation atteint un marché, dit "marché zénithal,"qui subit la perturbation maxima et que, dans le cas général, par suite de la ramification des filières, la perturbation se partage et donc s'amortit, jusqu'à une certaine limite au delà de laquelle ce qu'il en reste peut être considéré comme négligeable.

Vous voyez que, tout en bénéficiant d'une présentation absolument générale qui nous donne la possibilité de traiter l'univers économique dans son entier, nous pouvons tout de même faire des études particulières, parce que nous savons les limites de notre domaine. Si nous nous penchons sur une perturbation donnée, nous pouvons calculer à partir d'où nous pouvons négliger le reste de l'économie. Il me semble qu'il y a là une justification possible de cette méthode des équilibres partiels dont je vous ai parlé dans ma propre communication. L'équilibre partiel sera ici, tout simplement, l'équilibre général limité à la petite partie de l'univers économique où il y a intérêt à le considérer, ce qui se passe dans le reste de cet univers étant tenu pour négligeable.

Etant donné que nous sommes très à court de temps, je ne veux pas abuser plus longtemps de votre attention. J'ai certainement défiguré l'exposé de M. Chait; me trouvant obligé, pour le rapporter, de l'interpréter, je puis n'en avoir pas pénétré tous les aspects.

Et maintenant, abandonnant ma fonction de rapporteur officiel pour prendre une position plus personnelle, je voudrais vous dire qu'il y a, à mon avis, dans cette construction théorique, une des très grandes conquêtes de l'économique sythétique. Et je la considère comme d'autant plus grande que, tout en étant une présentation de synthèse, une présentation très générale, elle reste dans le domaine de l'économie microscopique; en effet, l'étude proprement économique de M. Chait porte sur le marché individuel, sur la fonction du stock dans l'entreprise. Nous

savons qu'il existe, à l'heure actuelle, un gros, un très gros problème, le problème de l'agrégation, qui consiste à savoir si l'on peut légitimement passer de l'étude microscopique à l'étude macroscopique. Personnellement j'estime que, jusqu'à nouvel ordre, l'étude économique est l'étude microscopique, parce que nous pouvons y toucher du doigt l'élément individuel, le comportement d'un homme ou d'une entreprise. Dans l'oeuvre de M. Chait, nous trouvons un moyen d'aller jusqu'à la synthèse collective, à partir de ce que je considère comme un inévitable point de départ, l'étude microscopique.

J'ajoute, enfin, que la construction de M. Chait est tellement abstraite, que ce qu'il considère sous le nom de marché peut être, comme je vous disais, tout aussi bien, une entreprise ou un groupe d'entreprises agrégé d'une manière quelconque, pourvu que ces entreprises soient directement liées sur le réseau de filières. Mais cet élément de réseau peut encore, tout aussi bien, représenter une usine, voire un atelier, à l'intérieur d'une entreprise. Car nous pouvons considérer l'entreprise unitaire comme étant déjà elle-même un agrégat, un réseau traversé par des flux, dont la description très précise nous est donnée, d'aille,sur par la comptabilité en partie double, où chaque poste comporte un flux d'entrée, le débit, un flux de sortie, le crédit, et un stock, le solde. Il y a là toute une conception de l'entreprise particulièrement précieuse pour l'étude de son fonctionnement, par exemple en ce qui concerne la rémunération des travailleurs, et je crois, pour ma part, qu'il y a là également tout un domaine extrêmement important. En somme, dans cette théorie d'unités successives de plus en plus grandes pouvant être prises comme noeuds du réseau de filières, je crois vraiment que M. Chait, comme Pascal, embrasse l'infiniment grand et l'infiniment petit.

Résumé

The law of divergence is a synthetic dynamic theory which describes the operation and the interdependence of markets, a market being defined as the organization of goods, industrial, commercial, or otherwise.

The totality of economic life is composed of a multitude of flows of goods and services, starting with raw materials and labor and ending with consumer goods.

Markets are connected with each other through the relationship of seller and buyer. The totality of transactions through a succession of

markets forms a stream. General economic activity thus forms a collection of flows of wealth in a network of *polyvalent streams*, where each market is a junction of channels connecting it on the one hand to all its suppliers and on the other to all its buyers. Goods flow through this network until they become consumer goods in the last market where they are in effect exchanged for others.

Each market is the sum of an inflow, an outflow, and a stock, connected by the equation $E - S = d\sigma/dt$, where $E =$ inflow, $S =$ outflow, and $\sigma =$ stock.

Setting the outflow of one market equal to the inflow of the next market and repeating the operation for the n markets of a stream, we get $(n-1)$ equations among the $2n$ flows and the n stocks.

The connection between stocks and flows, found from statistical observations, leads to another series of equations.

From the system of differential equations thus obtained, combined with the special equations of the model chosen for each market, and with the help of operational calculus, it is possible to deduce:

1. From the amplitude of the movements of a certain number of markets, the amplitude of the movements of certain other markets.

2. Their respective lags.

On a broader plane we can deduce from them a law of divergence that contains as a special case the principle of acceleration as well as the theory of the multiplier.

Finally it is easy, by this method, to limit the zone of action in a network of streams by a perturbation starting from one of its markets.

Discussion

*Professor Tinbergen:**

I have very little to add to the valiant summary given by Professor Divisia of Dr. Chait's very important work. But permit me in a few minutes just to add some remarks of my own which might further clarify to you the true nature of Dr. Chait's work.

I think the particularly interesting feature is that he brings stocks and equipment into the same group of phenomena, and next that many of his results are obtained by applying one experimental relation, namely the relation of proportionality between stocks and sales, a relation that in many cases has been established by statistical observation. That very simple relation, if applied to a chain of markets, in fact introduces differential equations of a higher order than the first order only; and it is by the combination of these two sorts, namely the linear relationship between stocks and sales on one hand, and the fact that he considers a number of markets that are placed behind each other, that he gets the possibility of explaining economic fluctuations even without introducing lags: something that is interesting from the purely theoretical point of view.

On the other hand, as Professor Divisia has very clearly pointed out, Dr. Chait doesn't dwell upon this special case. He also gives us more general possibilities. And I might perhaps end by telling you that just at this moment, at the National Bureau of Economic Research, interesting statistical investigations are going on, relating to such a system of markets. I hope very much that that will give us an opportunity of bringing into the center of attention some of Dr. Chait's ideas.

Professor Koopmans:

I would like to ask some clarification on the following point: What types of equations do occur in Mr. Chait's system? As I understand it, Mr. Chait aims at an explanation of economic fluctuations by the use of an equation system; and, from the explanations that have been given by Professor Divisia and Professor Tinbergen, I understand that these equations are of at least two types. One type of equations could be described as an identity (the type of equation that occurs in hydrodynamics) which says that the total amount of goods going into a

* Since Mr. Chait was not able to be present, his paper was presented by Professor Divisia, and the discussion by Professors Tinbergen and Divisia and others is included for completeness.

certain domain less the amount of goods going out of it must be balanced by the change of the stocks in that domain. This can be declared to be an identity. Now in addition to that Mr. Tinbergen has also mentioned something that, if I understand him correctly, is a behavior equation which says that stocks are held proportional to sales; in any case that was so in one of the alternatives considered by Mr. Chait.

My question is the following: Does the putting together of these two types of equations, flow identities and proportionality between stocks and sales, suffice to create a complete system from which economic fluctuations can be consistently and fully explained?

Professor Tinbergen:

I think the situation is this, that Dr. Chait paid particular attention to the two types of equations that have now been discussed. As to the other equations that of course are necessary in order to complete his system, he doesn't make, as far as I am aware, assumptions that are different from the assumptions that we are accustomed to or, I may say, we were accustomed to in 1936 when he wrote his book. So, although he certainly pays some attention to these questions, it is not that part of his book which is the most original part, I think in some of his examples, for instance, he assumes that total demand is determined by income in the way it is done by the Keynesian equations. I am not sure whether he introduces in any of his examples some laws about price formation or whether he—as Keynes did—makes the system into some sort of a real system only. But the chief thing to be said is that the new elements in his systems are the elements indicated by Professor Divisia.

Professor Marschak:

Perhaps the system will have a periodic solution if, in addition to time-derivatives, Mr. Chait has time-lags, corresponding to the production periods of various firms?

Professor Tinbergen:

Dr. Chait, I think, considered two cases. First of all, he considered the case where the lags do not exist or are negligible. In that case he nevertheless succeeds in getting a theory of fluctuations by the very fact that more than two succeeding markets are considered, or even for one market, if industrial or commercial equipment is introduced as a variable. And next, he contains derivatives of Z_{i-1} (maybe of different orders) meant to represent the enterpreneurs' reactions. The final result may be a variable between Z_{i-1}. The lag is not introduced *a priori*, but explained by the various derivatives and the consequences is that it need not be constant.

Professor Divisia:

Je voudrais simplement fournir un complément de renseignement à Mr. Koopmans sur la question qu'il a posée. Le travail de M. Chait, autant que j'aie compris, n'est pas du tout une tentative d'explication des cycles: c'est beaucoup plus. M. Chait m'a dit qu'il a eu l'idée de son travail, alors qu'il se promenait dans une usine de fabrication à la chaîne. L'idée lui est aussitôt venue à l'esprit que la vie économique, c'est tout simplement une chaîne. Et cette idée a pris figure dans une peinture. Sachant les mathématiques, il a fait la peinture en équations. Il a trouvé ainsi, la possibilité de tirer de là toutes sortes d'idées sur des observations à faire,—ce qui est fort important—et d'ores et déjà, toutes sortes de conclusions. Tout au moins dans sa première étude, mais, je crois, également dans son livre, M. Chait a, avant tout, considéré *la solidarité la plus générale qui s'exerce entre tous les marchés, par récurrence sur le réseau* de filières. Ce n'est qu'ensuite qu'il a envisagé toute une série de cas concrets, en vue de trouver, par l'investigation statistique, les équations qui lui manquaient. C'est ainsi qu'il est tombé sur des solutions touchant au problème des oscillations, mais on pourrait presque dire que ce ne sont là que des cas d'application ce sa présentation théorique générale. Je crois vraiment qu'il a été beaucoup plus loin que la seule étude des cycles, par un très bel effort d'abstraction. Cet effort est tellement général et a abouti à un mécanisme tellement abstrait, qu'il garde sa valeur, pour quelqu'explication particulière que ce soit.

M. Rueff:

J'ai été extrêmement intéressé par le résumé qu'a présenté M. Divisia. Je ne connais pas l'ouvrage de M. Chait, mais j'avais moimême rencontré sur ma route la notion de "flux."

J'ai indiqué à la page 35 de la première édition de mon *Ordre Social* l'importance qu'elle me paraissait présenter. Je crois, en particulier, que les phénomènes économiques n'entreront commodément dans le temps que lorsqu'ils seront décrits par le langage des flux.

Je voudrais vous indiquer rapidement comment je vois l'utilisation de la notion de flux en théorie monétaire.

Je considère, non comme M. Chait, l'entreprise, mais le marché.

En chaque période de temps le volume des droits qui viennent se vider, sur le marché, c'est le flux d'entrée; le volume des droits qui viennent se remplir, c'est le flux de sortie.

Cette présentation conduit immédiatement à la théorie de variations du niveau général des prix. En effet, tant que le premier des flux est égal au second, le niveau général des prix reste inchangé.

Or, il ne peut y avoir différence entre les deux flux que s'il y a différence entre le montant global des encaisses effectives et celui des encaisses désirées.

Je n'insiste pas. Je veux seulement marquer que je suis entièrement d'accord avec M. Divisia—donc avec M. Chait—pour penser que la théorie des flux est un puissant instrument d'analyse économique.

Professor Divisia:

Je voudrais dire à M. R u e f f, que, effectivement, l'observation qu'il vient de faire, porte sur un aspect du travail de M. C h a i t (ou un prolongement de ce travail) que pour ma part, je considère comme essentiel. Je ne crois pas que M. Chait ait déjà publié des développements sur ce point, mais cela, à mon avis, n'a aucune importance parce que c'est *inévitablement inclus* dans sa présentation.

M. Chait s'est d'abord borné à nous décrire un réseau de filières matérielles, traitant de quantités de biens. Mais au bout de la filière, nous arrivons aux produits de consommation finale, qui s'échangent les uns contre les autres. Or ils s'échangent sur le pied de leurs valeurs, si bien qu'on est inévitablement conduit à considérer, après le réseau de flux des biens matériels *en quantité*, le même réseau de flux mais *en valeurs*. Là encore, il y a pour chaque entreprise, un flux d'entrée, un flux de sortie et un stock, qui n'est autre que le montant de l'actif diminué du profit nominal cumulé. Et alors, si on considère, ces deux réseaux de filières se correspondant noeud à noeud et flux à flux, l'un où coulent des quantités de richesses, l'autre où coulent des valeurs; le prix y apparaît comme le quotient des flux en deux points correspondants de ces résaux. Car un flux de valeur divisé par un flux de quantité, c'est un prix. Vous voyez qu'on arrive à une définition extrêmement générale du prix, qui en particulier, est complètement indépendante du régime politique.

D'autre part, si nous regardons avec M. Chait, l'ensemble de l'économie comme un vaste travail à la chaine, nous sommes obligés de nous dire que, si les biens matériels circulent, il y a autre chose aussi qui circule, qui est la monnaie. Et alors M. Chait nous a donné du même coup, une description des *filières monétaires* avec, encore, pour chaque élément un flux d'entrée, un flux de sortie, et un stock. Les flux de monnaie sont égaux, et de signes contraires, aux flux de valeurs ci-dessus; et le stock de monnaie dans l'entreprise c'est sa trésorerie. Il y a là, par conséquent, ce que, pour ma part, je considère comme la fondation de la théorie monétaire, dont l'aspect qu'elle présente ici n'est pas autre chose que la théorie des stocks appliquée aux signes monétaires.

J'ai l'impression que la présentation très abstraite de M. Chait aura d'énormes retentissements, en donnant lieu à toutes sortes de

cristallisations concrètes fort importantes. Justement dans cette direction il y en a une, que je veux signaler, en raison de son importance, c'est la théorie de la banque. Elle est incluse dans la correspondance entre le réseau des filières de marchandises et le réseau des filières de monnaie, au sens le plus large de ce mot; car au fond, si tous les crédits bancaires ont l'air d'être des crédits en monnaie, il sont, en réalité, des crédits en marchandises, puisque tout ce que l'homme fait, il le fait avec des marchandises et non avec de la monnaie. Et donc, pour connaître le volume des crédits disponibles, il faut connaître le comportement des stocks de marchandises disponibles dans les filières. Lorsque nous aurons franchi ce pas, nous aurons, je crois, vraiment une théorie efficiente en ce qui concerne la politique bancaire. Vous voyez comment, en définitive, nous retrouvons, dans la présentation abstraite générale de M. Chait, l'industrie, le commerce, la monnaie, la banque. Et peut-être son utilisation viendra-t-elle y ajouter encore quelques autres châpitres.

M. Rueff:

Je voudrais dire à M. Divisia que je pense comme lui: l'étude des variations de prix peut être très efficacement approchée par comparaison entre le flux de marchandises et le flux de pouvoir d'achat.

Mais il est alors indispensable de mettre en lumière le lien qui existe entre les deux flux. Ils ne sont pas indépendants et, pour ma part, je n'ai réussi à dégager ce lien qu'en montrant qu'en chaque période le premier était identique au second, majoré des éventuelles divergences entre encaisses effectives et encaisses désirées.

Mr. Chait's paper was further discussed by Messrs. Everett E. Hagen, J. J. Polak, and Sidney Alexander.

THEORY OF CHOICE AND UTILIZATION OF RESOURCES: I

Friday, September 12, at 9:30 a. m.

CHAIRMAN :

Abraham Bergson
Columbia University (United States)

RENDEMENT SOCIAL ET PRODUCTIVITE SOCIALE

par M. Allais
Professeur d'Economie Théorique a l'Institut de Statistique de l'Université de Paris (France)

La présente communication a pour but de présenter certains des résultats auxquels nous ont mené nos travaux de 1940 à 1946. Les démonstrations nécessitant de très nombreux développements et figurant dans nos différents ouvrages, nous nous bornerons ici à en énoncer brièvement les grandes lignes.

1. Théorie restreinte du rendement social

En représentant les psychologies individuelles à un instant donné par des fonctions de satisfactions

$$(1) \quad S_0^i = S_0^i\,(A_0^i,\ B_0^i, \ldots, C_0^i, \ldots,\ A_q^i,\ B_q^i, \ldots, C_q^i, \ldots, A_p^i,\ B_p^i, \ldots, C_p^i),$$

où $A_q^i, B_q^i, \ldots, C_q^i$ représentent les quantités de services (A), $(B), \ldots, (C)$ consommés aux différentes époques par un individu, et les techniques de production à chaque époque par des fonctions du type

$$(2) \qquad H_q = H_q\,(X_q,\ Y_q, \ldots, Z_q),$$

où H_q représente une production donnée et X_q, Y_q, ..., Z_q les quantités nécessaires des facteurs de production (X), (Y), ..., (Z), il est possible

d'écrire, pour des conditions aux limites données et compte tenu des conditions de liaison du type (2), les conditions de maximation d'une quelconque des satisfactions S_0, les autres restant constantes.

Si on définit alors une situation de rendement social restreint maximum comme une situation où il n'est pas possible d'augmenter la satisfaction d'au moins un individu *a l'instant* t_0 sans diminuer celle de quelque autre au même instant et si on interprète les relations trouvées, on arrive alors au théorème suivant:

"Dans une économie de type quelconque, la condition nécessaire et suffisante pour que pour des psychologies et des techniques de production, des caractéristiques foncières, un équipement matériel initial et final, et une structure démographique donnés, le rendement social restreint y soit maximum, est quil existe explicitement ou implicitement un système de prix, équivalent à celui qui existerait dans une économie d'échanges caractérisée,

"a) par le libre choix des individus;

"b) par la concurrence parfaite des entreprises dans le secteur où la concurrence est *physiquement possible*, c'est-à-dire dans le secteur où la meilleure technique de production (*au sens physique*) est constituée par la juxtaposition d'entreprises analogues (production de la fonte par exemple) (secteur différencié);

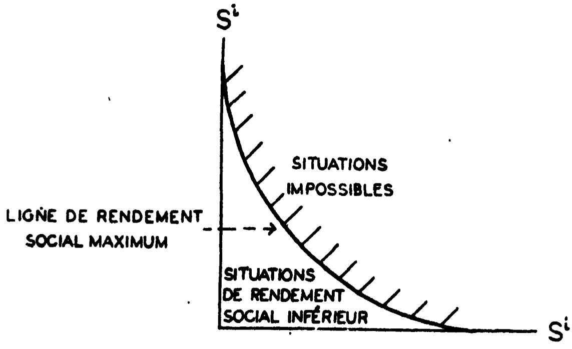

Figure 1. Redement Social et Répartition

"c) par la minimation du coût moyen et la vente au coût marginal, c'est-à-dire à un prix égal au coût de l'unité en plus, dans le secteur où la concurrence est *physiquement* impossible, c'est-à-dire dans le secteur où la meilleure technique de production (*au sens physique*) est constituée par une seule entreprise (production du gaz dans une ville par exemple) (secteur non différencié);

"d) par une répartition individuelle de la propriété des revenus matériels des facteurs de production (travail, sol, et capitaux matériels)

correspondant à la répartition qui reste ainsi arbitraire, des différents services consommables dans l'économie considérée;

"e) par une absence d'intervention de l'Etat sur le marché du capital."

Ces conditions impliquent en particulier qu'il y ait entre chaque couple de dates un taux d'intérêt *unique* dont la valeur équilibre l'offre et la demande correspondantes de capital et qu'il n'y ait aucune intervention de l'Etat pour modifier l'équilibre spontané du marché de l'épargne.[1]

Pour une structure donnée, il existe une infinité de situations de rendement social maximum, chacune d'elles correspondant à une certaine répartition des services consommables.

2. Théorie généralisée du rendement social

Si on fait intervenir, non pas seulement les satisfactions *présentes* S^i_0, mais également les satisfactions *futures* S^i_1, S^i_2, ..., S^i_p et si on définit comme situation de rendement social généralisée maximum, ou plus simplement de rendement social maximum, une situation où il n'est pas possible d'augmenter la satisfaction S^i_q d'un individu à un instant t_q sans diminuer soit sa satisfaction S^i_r à une autre époque, soit la satisfaction d'un autre indvidu à une époque quelconque, on obtient un théorème analogue où les conditions a, b, c, et d restent inchangées mais où la condition e) devient

"e′) par une certaine intervention de l'Etat sur le marché du capital modifiant la répartition dans le temps des revenus consommables des individus que ces derniers auraient faite en l'absence de cette intervention."

Ici les conditions trouvées impliquent encore que, pour une structure quelconque de l'économie, l'organisation la meilleure correspond à l'emploi d'un *système unique de prix* et d'un *taux d'intérêt technique*[2] *unique* à chaque instant dans le secteur production, et du même système de prix

[1] Nous avons dévéloppé la théorie restreinte du rendement social dans les ouvrages suivants:

A la recherche d'une Discipline Economique, Tome I, Paris 1943, pages 604 à 682; *Economie pure et Rendement Social*, Sirey, 1945, 72 pages in 8; *Economie et Intéret* (Dépositaire Librairie des Publications Officielles, 40, rue de Verneuil, Paris, 1947) No. 54 à 59.

[2] Le taux d'intérêt technique correspondant à un procès donné de production est le taux qui égalise la valeur actuelle des recettes à obtenir et la valeur actuelle des dépenses nécessaires.

dans le secteur consommation, mais ici les *taux d'intérêt psychologiques*[3] correspondant à ce secteur restent *arbitraires* et peuvent être quelconques.

On voit ainsi que si la maximation du rendement social généralisé exige l'emploi à chaque instant d'un taux *unique* d'intérêt dans le secteur production, elle laisse par contre *indéterminés* les taux d'intérêt psychologiques correspondant aux différents individus considérés aux différents instants.

L'unicité des taux d'intérêt qui caractérise les équilibres concurrentiels classiques n'a pas d'autre effet que de réaliser pour chaque individu une des multiples répartitions possibles de ses revenus consommables dans le temps et cette répartition ne peut en aucun cas être regardée comme préférable à toute autre.

Il en résulte que l'Etat est parfaitement fondé à intervenir sur le marché de l'épargne, par exemple pour faire en sorte que l'investissement réalisé soit plus grand que celui qui tendrait à se réaliser spontanément en dehors de son action.[4]

3. Théorie de la productivité sociale

Nous dirons que la productivité sociale d'un régime permanent de rendement social maximum est maximum lorsque les satisfactions présentes et futures sont maxima au sens de Pareto *pour un volume variable* de l'équipement en capitaux matériels existant à chaque instant, c'est-à-dire lorsqu'il n'existe aucune modification vituelle de cet équipement susceptible d'augmenter au moins une satisfaction, les autres restant constants.

Si on définit la productivité physique d'un procès indirect de production comme l'accroissement de production obtenu relativement au procès direct pour une même dépense de facteurs primaires de production (travail et sol), on peut montrer que dans un régime permanent la condition nécessaire et suffisante de la maximation de la productivité sociale est que la productivité physique des différents procès de production utilisés soit maximum.

En traitant la question mathématiquement on peut alors montrer que la condition nécessaire et suffisante pour que la productivité sociale d'une économie en régime permanent de rendement social maximum soit

[3] Le taux d'intérêt psychologique I entre les instants t_0 et t_1 d'un individu de satisfaction $S = S\ (A_0, A_1)$ est défini par la relation

$$\frac{\dfrac{\partial S}{\partial A_0}}{a_0} = \frac{\dfrac{\partial S}{\partial A_1}}{\dfrac{a_1}{1+I}}.$$

[4] Nous avons développé la théorie généralisée du rendement social dans notre ouvrage *Economie et Intérêt*, No. 60 à 66.

maximum, c'est-à-dire pour que la productivité physique des différents procès de production y soit maximum, est que le taux d'intérêt caractérisant l'équilibre concurrentiel dans le secteur production soit nul.

Dans ces conditions la liaison entre la productivité valeur et la productivité physique des procès indirects de production peut se representer par Figure 2.

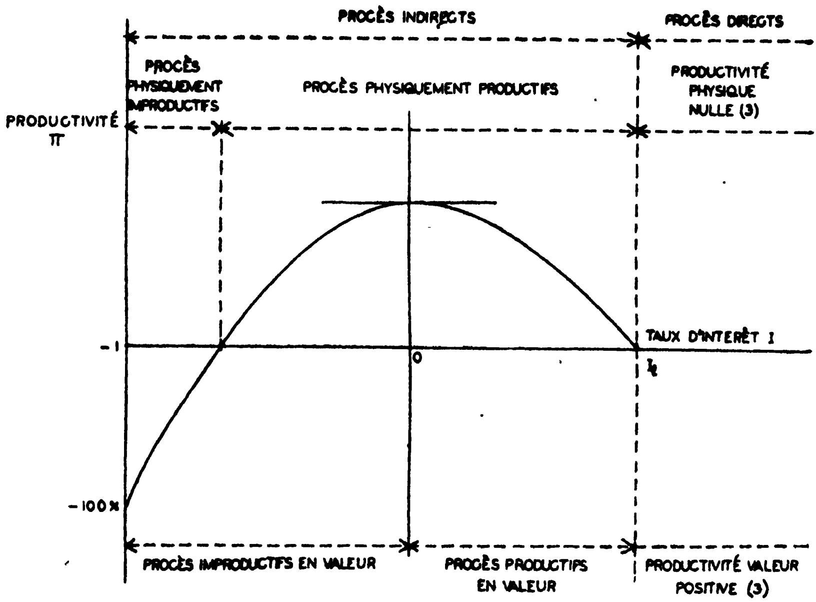

Figure 2. Productivité Physique et Productivité Valeur.

On peut montrer à partir des données statistiques de 1914, dernière année où un équilibre concurrentiel peut être considéré comme existant approximativement dans les différents pays du monde occidentel, l'augmentation de revenu national réel qu'il eût été possible d'obtenir, pour un pays dont le taux d'intérêt pur était de 5% en abaissant ce taux d'intérêt de 5% à 0, aurait été d'environ 20%.[5]

4. Possibilité d'annulation du taux de l'intéret

Le taux de progrès technique n'étant que de quelque pour cent par an, nos économies peuvent être considérées en première approximation comme en régime permanent de sorte que la théorie de la productivité sociale peut leur être appliquée. La question se pose ainsi de savoir si on peut annuler le taux d'intérêt pur.

[5] Nous avons développé la théorie de la productivité sociale dans notre ouvrage *Economie et Intéret* No. 67 à 77.

Or on peut montrer que le taux d'intérêt ne peut actuellement descendre en dessous d'un certain minimum on raison

1°) de l'appropriation privée du sol,

2°) de la prime de liquidité de la monnaie,

et que l'annulation du taux de l'intérêt ne peut être obtenue dans une économie concurrentielle à l'équilibre que si

1°) le sol est approprié collectivement,

2°) l'unité de monnaie circulante est dissociée de l'unité de compte et se devalorise avec le temps par rapport à cette unité de compte.[6]

Résumé

1. Limited Theory of the Social Yield

If a situation of maximum restricted social efficiency is defined as one where there is no possibility of increasing the satisfaction of at least one individual at a moment without reducing, at the same moment, that of another, it is found that the necessary and sufficient condition for the social yield to be at a maximum is that there exists in the economy, explicitly or implicitly, a system of prices equivalent to the system which would exist in an economy of exchanges characterized:

a) by the free choice of the individuals;

b) by the perfect competition of the enterprises when this competition is physically possible;

c) by the minimization of the average cost and the sale at marginal cost when competition is physically impossible;

d) by a given distribution of the material incomes of the production factors (labor, land, and material capital) between the individuals;

e) by the nonintervention of the State in the capital market.

2. Generalized Theory of the Social Yield

If, in the definition of social efficiency, one takes into account not only present satisfactions but also future satisfactions, the preceding conditions remain unchanged except condition (e) which becomes:

[6] Nous avons développé ces indications dans notre ouvrage *Economie et Intéret*, No. 114 à 139.

e') by a given intervention of the State in the capital market modifying the distribution over time of the consumable incomes (of individuals), compared with the distribution that would take place in case of nonintervention.

3. Theory of the Social Productivity

We shall say that the social productivity of a stationary economy where the social yield is at a maximum is maximum when the present and future satisfactions of the individuals are at a maximum in Pareto's sense for a variable volume of equipment existing at each moment, *i.e.*, when at each time there exists no virtual modification of this equipment to which would correspond an increase of one satisfaction, the other satisfactions remaining constant.

This definition given, it is possible to show that the necessary and sufficient condition for the social productivity of a stationary economy, characterized by a social yield at a maximum, to be at a maximum is that the rate of interest characterising the production competitive equilibrium be zero.

4. Conditions for the Rate of Interest to be Zero

In a competitive economic equilibrium one may obtain the reduction to zero of the rate of interest only if:

1. the land is collectively owned,

2. the unit of circulating money (*monnaie circulante*) is dissociated from the unit of account (*unité de compte*) and is devalorized in relation to the unit of account as time goes on.

Mr. Allais' paper was discussed by Messrs. Jacob Marschak, Abba P. Lerner, Leonid Hurwicz, Jacques Rueff, Jacques Dumontier, and the speaker.

OPTIMUM UTILIZATION OF THE TRANSPORTATION SYSTEM*

by Tjalling C. Koopmans

Professor of Economics, The University of Chicago, and Research Associate, Cowles Commission for Research in Economics

The purpose of this paper is to give an application of the theory of optimum allocation of resources to one particular industry. I shall, therefore, not speak on that theory in general. I shall use one of its basic propositions, which was very admirably put forth in the paper presented by M. Allais. This proposition says that a system of prices corresponding to marginal costs is necessary to guide the optimum allocation of resources in a productive system. If cost is minimized in each branch of production on the basis of such a system of prices, each unit of any (divisible) factor of production will be used in such a manner that its contribution to the satisfaction of ultimate consumers is highest.

For this proposition to be valid, it is not necessary that such prices are established in a market where exchange of goods takes place; they may also be accounting prices determined only for the purpose of guiding allocative decisions. I shall give examples of both kinds of prices.

It may be useful, indeed, to consider applications of this proposition to particular industries. The meaning of the marginal cost concept is not always obvious to the engineer, manager, or business economist. It is true that, where perfect competition exists, the mechanism of the market will bring about prices reflecting marginal cost. In a sphere like transportation, however, where perfect competition does not prevail throughout the industry, specific analysis is needed to bring out in quantitative terms what the marginal cost is in any particular case, and how it can be determined.

In order to simplify our problem, I shall consider a homogeneous transportation system, that is, a system in which there is only one type of moveable equipment. For instance, there is only one type of ships all of the same carrying capacity, speed, and other characteristics. Or, there is only one type of railroad cars, or highway trucks.

Let us first consider the case of a railroad connecting only two terminals, A and B, a case which has also been discussed by Pigou in his

* The text of this paper follows closely the stenographic transcript of the original verbal presentation. It will be reprinted in Cowles Commission Papers, New Series, No. 34. A monograph giving a more systematic exposition of the subject is in an early stage of preparation.

book, *The Economics of Welfare.*[1] Let us assume that there is a given demand for five trains each day to go loaded with goods from A to B; that there is a demand for only three trainloads daily to go from B to A. Let us express cost simply in terms of equipment tied up, *i.e.*, in train-days incurred daily. Then, if we wish to transport an additional trainload from A to B, that increase in demand will require an additional train to be run daily from A to B loaded with goods. The cost incurred directly by that movement is the sum of the times spent loading in A, moving to B, and discharging in B, by one train. But it will also be necessary to move the train back empty from B to A, because we assume no change in the requirement of three loaded trains daily in that direction. The marginal cost in this case, expressed in equipment time committed each day, corresponds therefore to the whole turn-around time of one train, loading, moving, discharging, moving back. On the other hand, the marginal cost of adding one trainload daily from B to A is given only by the time spent loading in B and discharging in A, because the time spent moving would have to be spent in any case, to approximately the same amount, as a result of the fact that otherwise that train would have to be moved empty. We thus find a sizable difference in marginal cost according to the direction of transportation. This was clearly recognized by Pigou. It is difficult to understand why he regarded this difference as of comparatively small importance.

Since most transportation systems connect many terminals, we shall now consider how the determination of marginal cost works out in a general network of routes. Let us assume, however, that the *program* of transportation is constant in time. Constant daily or monthly requirements for transportation from each terminal in the network to each other terminal are assumed to be given. Let us assume further that the *performance times* involved in the various tasks of loading, moving, discharging, are constants in time and in the sense that on each route they are independent of the number of trains or ships that carry out these tasks. This implies an assumption of absence of congestion.

We shall again assume that the cost of a program can be expressed in amount of equipment required, or, synonymously, in equipment time committed in each unit of time. This is not as unrealistic as it may seem. There have been situations where equipment time was the decisive element of cost. For instance, in the shipping problems of the two World Wars, the controlling bottleneck was the number of ships available. All other costs, like wages and fuel, even though important by themselves, were negligible compared with the opportunity cost of

[1] 1920 Ch. XV, 85, p. 266.

using a ship for one highly urgent purpose rather than for some other highly urgent purpose.

I shall distinguish, for any program, the *direct cost* and the *indirect cost.* The direct cost of the program (or of any increment thereto) is the equipment tied up at any time in loading, loaded movements and discharging (or its increment). The indirect cost arises whenever there is a departure from perfect balance in the program. In general, certain terminals will receive more goods than they dispatch, and other terminals will be in the reverse situation. Generally, a continual movement of empty equipment is required from points of equipment surplus to points where there is a deficit. The amount of equipment inevitably tied up in empty movements is called the indirect cost of the program.

In a transportation system that is not too unbalanced, the direct cost is by far the more important element in total cost. But in the marginal cost of given increments to the program, the indirect cost is always important and deserves a good deal of study. It has a more complicated structure than the direct cost, and it enters into marginal cost in a more subtle way.

As an example for the discussion of this problem I have chosen the flows of dry cargo on the ocean shipping routes of the world in the year 1925. For the study of indirect cost, we need only consider the net shipping surplus of each port or area of limited size. We can roughly assume that the net dry-cargo shipping surplus of an area is proportional to the net excess of the weight of all goods (other than mineral oils) arriving in sea-borne trade over the weight of all such goods departing. In Table 1, such net receipts figures are computed for areas designated by "representative ports" and indicated by dotted lines on Figure 1. Let us simplify our problem by assuming that the figures of 1925 are constant flows applying through time for an indefinite period, without seasonal movement or other fluctuation or trend. Furthermore, let us calculate as if all traffic going to or from a particular area were going to or from its representative port. The representative ports which, by our assumption, have a net surplus of shipping are Lisbon, Athens, Yokohama. All other representative ports are shipping deficit ports.

Let us now for the purpose of argument (since no figures on war experience are available) assume that one particular organization is charged with carrying out a world dry-cargo transportation program corresponding to the actual cargo flows of 1925. How would that organization solve the problem of moving the empty[2] ships most economically

[2] Technically, ships here referred to as empty take in a certain amount of ballast for stability reasons. "Empty movements" are accordingly described in shipping parlance as "ballast traffic."

from where they become available to where they are needed? It seems appropriate to apply a procedure of trial and error whereby one draws

TABLE 1

Net receipts of dry cargo in overseas trade, 1925

Unit: Millions of metric tons per annum

(1)	(2)	(3)	(4)
Area represented by[1]	All cargoes other than mineral oils		
	Received	Dispatched	Net receipts
New York	23 ·5	32 ·7	–9 ·2
San Francisco	7 ·2	9 ·7	–2 ·5
St. Thomas	10 ·3	11 ·5	–1 ·2
Buenos Aires	7 ·0	9 ·6	–2 ·6
Antofagasta	1 ·4	4 ·6	–3 ·2
Rotterdam*	126 ·4	130 ·5	–4 ·1
Lisbon*	37 ·5	17 ·0	20 ·5
Athens*	28 ·3	14 ·4	13 ·9
Odessa	0 ·5	4 ·7	–4 ·2
Lagos	2 ·0	2 ·4	–0 ·4
Durban*	2 ·1	4 ·3	–2 ·2
Bombay	5 ·0	8 ·9	–3 ·9
Singapore	3 ·6	6 ·8	–3 ·2
Yokohama	9 ·2	3 ·0	6 ·2
Sydney	2 ·8	6 ·7	–3 ·9
Total	266 ·8	266 ·8	0 ·0

Source: *Der Guterverkehr der Weltschiffahrt*, Statistisches Reichsamt, Berlin, 1928.
[1] See Figure 1.
*The figures in columns (2) and (3) for this area contain an equal amount of traffic within the area, between smaller areas from which this area was composed.

tentative lines on the map that link up the surplus areas with the deficit areas, trying to lay out flows of empty ships along these lines in such a way that a minimum of shipping is at any time tied up in empty movements.

The lines on Figure 1 correspond to an optimal solution of that kind, if we can assume that time spent is proportional to navigational distance. The procedure of trial and error can be illustrated as follows: Each surplus area serves a number of deficit areas, and the type of experimental variation that one would explore is to shift a certain "marginal" deficit area from one surplus area to another, with compensation elsewhere. For instance, one might think of cutting the link from Lisbon to West Africa, substituting a compensating link from Lisbon to San Francisco; one might explore several other limited adjustments of that kind, calculating in each instance the (positive or negative) net saving of shipping so achieved, on the basis of the performance times involved in the

alternative movements. In that way one would arrive at what may be called a "local" optimum, that is, a routing plan of empty ships that cannot be improved upon by adjustments of the type described. The question arises whether one cannot mislead oneself in that way. Is it

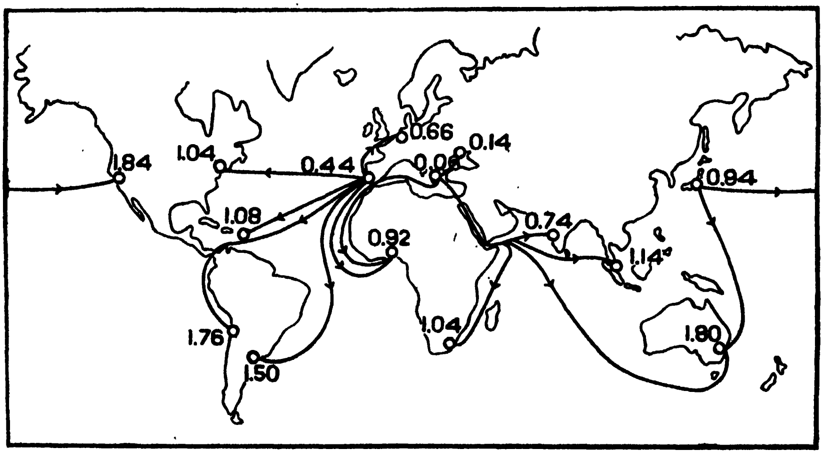

Figure 1. Optimal Routes of Empty Shipping Corresponding to World Dry-Cargo Flows in 1925.

The Figure shown with the representative port of each area represents the net shipping surplus of that area in millions of metric tons of dry-cargo capacity.

not possible that, by a very drastic rearrangement in the linking of surplus and deficit ports, another perhaps better optimum could be found which cannot be detected by any "small" rearrangement?

The question is answered by the *first theorem*: If, under the assumptions that have been stated, no improvement in the use of shipping is possible by small variations such as have been illustrated, then there is no—however thoroughgoing—rearrangement in the routing of empty ships that can achieve a greater economy of tonnage.

The reason for this statement is a mathematical one which can be only briefly suggested: The function we are minimizing, the total amount of shipping tied up in the various flows of empty shipping, is the sum of the monthly flows on all routes, each multiplied by the constant performance time involved in that movement. We are thus minimizing a linear function of the flows of empty ships under two types of restrictions. In a continuing program, the number of ships going into any area per unit of time, with or without cargo, must equal the number of ships going out. Therefore, there is a first set of restrictions in the form of linear equalities saying that the sum of all flows of empty ships out

of any area less the sum of all such flows into that area is equal to the shipping surplus of that area, as prescribed by the program. This surplus may of course be negative. There is a second set of restrictions which says that a flow of empty ships cannot be negative. This is a linear inequality. We are thus minimizing a linear function subject to linear equalities and linear inequalities in the variables involved.

If we take the flows of empty shipping on all possible routes as the Cartesian coordinates of a point in an n-dimensional space, then the set of all points satisfying these two types of restrictions has the following property: If we select arbitrarily two points of this set, then all points located between those two points on the straight line connecting them will also belong to the set, *i.e.*, satisfy the restrictions stated. A point set with this property is called a convex set, and further analysis shows that the minimum value of a linear function on a convex point set is unique: Any local minimum is the absolute minimum.[3]

We now come to the second problem to be discussed: how to find estimates of marginal cost. The constant program for which an optimal routing plan of empty ships has been found is now subjected to variation, not in time, but as a matter of comparative statics. Besides the

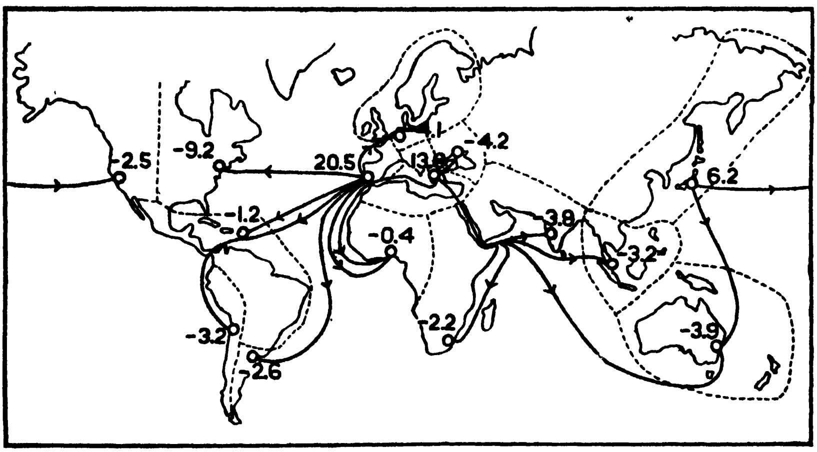

Figure 2. The Potential Function of Dry-Cargo Ships Corresponding to the Composition of Demand in 1925. Unit: Ship-Months.

constant program already considered, we consider another constant program which differs from the previous one only with respect to the amount of cargo to be shipped on just one route. This amount is

[3] It is possible that the minimum value is reached at different points simultaneously; instead of the one lowest point in a valley there is then a horizontal line constituting the lowest part of a valley, or even a low plain at the bottom of a valley, or its analogue extended into more dimensions.

increased by, say, one shipload a month. The calculation of marginal cost with respect to that change in the program can be performed with the help of a certain calculus illustrated by Figure 2.

For each port in which empty movements originate or terminate (or both) we define the value of a *potential function*, which is a valuation placed on the location of a ship in that port. This definition proceeds as follows: We assign an arbitrary value to the potential function in one arbitrary port, in our example the value zero in the port of Athens. From there we follow routes travelled by empty ships according to an optimal routing plan for the original (unchanged) program. In order to derive the potential in Bombay from the potential in Athens, we add the time involved in an empty movement from Athens to Bombay. We *add* because the movement from Athens to Bombay is in the direction of empty traffic. In the same way, this procedure defines the potentials in Odessa, Singapore, Sydney, Durban, and Lagos as certain positive figures. From any of these ports, we cannot go on along routes of empty shipping except by moving counter to the flow of such ships, as for instance along the route from Sydney to Yokohama. Therefore, in that case, we *subtract* the amount of time spent in the empty movement Yokohama-Sydney from the potential in Sydney in order to obtain the potential in Yokohama. In this way the potential is defined in any port, linked with Athens by the *graph* of optimal routes of empty shipping for the original program.[4]

I shall now formulate a rule for determining the marginal cost of a given change in the program. Let us take as an example the addition of one ship to the monthly loaded movement from San Francisco to Antofagasta. The marginal direct cost is simple — it is given by the time involved in loading, moving, and discharging, on that route. The marginal indirect cost, according to the *second theorem*, is equal to the loss in potential sustained by a ship while going from the port of departure to the port of destination. In our example, that loss is positive, because the potential at destination (1.76) is lower than at the port of departure (1.84). Therefore, the marginal indirect cost involved in this particular change in the program is $1.84 - 1.76 = 0.08$ ship-months, incurred monthly, or 0.08 of the continuous active availability of one ship.

[4] It can be shown that a closed circuit can be contained in the graph of optimal routes only if the performance times involved are such that the definition of the potential applied around the circuit does not lead to a contradiction. It is, however, possible for the optimal graph of empty traffic to break up into disconnected parts. In such special cases, differences in potential between ports on the same connected part are defined, but differences in potential between ports that are not connected by the graph are not defined.

Why is this theorem valid? It can be briefly indicated. If such an addition to the program is made, the net monthly surplus of ships in San Francisco is reduced by one; likewise, the net surplus in Antofagasta is increased by one. The flow of empty ships from Yokohama across to San Francisco can therefore be reduced by one ship a month. But that upsets the balance in Yokohama, and it will be necessary to move one additional ship monthly from Yokohama to Sydney, and so on. This dispenses with the necessity of sending one ship monthly from Athens to Sydney, and so on. The sequence of adjustments is closed when it is found ultimately that the monthly arrivals in Antofagasta of empty ships from Lisbon are reduced by one. Now, the algebraic sum of the time-expenditures and the time-savings involved in such a sequence of adjustments is precisely equal to the difference in potential between the end (Antofagasta) and the beginning (San Francisco) of a chain of routes of empty shipping, determined by application of the definition of potential along the chain.

In a war economy in which shipping is the essential bottleneck, the usefulness of marginal cost estimates as described is obvious. Such estimates are needed to guide decisions of programming authorities, for instance, in balancing competing claims for shipping services, or in determining the best source of a raw material on shipping grounds. It may be added without proof that the estimates described are applicable to finite (as distinct from infinitesimal) changes in the program, which are not so large as to require a change in the optimal routes of empty traffic.

What relevance does the foregoing analysis have to peacetime transportation problems where there is a market instead of an allocating authority, and where equipment time is not the only relevant measure of cost? I believe that the main part of marginal cost will still be arrived at along the lines described. In the first place, the equipment time committed by a change in demand is again to be accounted for, in the present case on the basis of the market valuation of equipment time (the opportunity cost of the use of equipment). In the shipping market, this valuation is expressed by the time-charter rate of a ship; in rail transportation no market quotation is available, but proper accounting procedures will reveal the net rental value to a railroad of the use of a car or train. In addition, the cost of fuel consumed and of labor to go with the equipment will also be roughly proportional to the time spent moving. Hence the same analysis is still largely valid for a considerable part, I would say the main part, of marginal cost.

How has the shipping market done its job without resorting to anything like the analysis described? To answer this question, we can make use of a theorem which M. Allais has already pronounced:

A perfectly competitive market automatically brings about pricing according to marginal cost. Therefore, to the extent that the tramp shipping market has been competitive—and that is to a very large extent through a long period in its history—the individual comparisons of alternative voyages made by many shipowners acting independently have broadly given effect to the process of minimizing the amount of shipping involved in empty movements; or rather of maximizing the amount of transportation that is performed by a given amount of shipping, which is an equivalent formulation. The totality of these individual decisions has furthermore produced a set of interconnected freight rates on various routes, reflecting marginal cost.

There is a definite need for an explicit analysis of marginal cost in rail transportation, where there is nothing like a competitive comparison of alternative courses of action by individual train owners. In the United States, movements of trains are laid out and rates are set by a number of railroad managements acting under the supervision of a regulatory agency of the government. As a result, I would surmise, the railroad rates have no connection whatever with marginal costs. The cases are rare in which rates in different directions are different, and I do not know of cases where a railroad's rate system has been made dependent on the composition of traffic. We must realize the social cost involved in this disregard of marginal principles—cost in terms of the decrease of social benefit that we derive from our transportation system. If rates do not reflect marginal cost, they provide no inducement or guidance toward private or public decisions regarding industrial location that will improve the balance in the use of the transportation system. For instance, in the United States, processing industries are more concentrated in the Northeast quarter of its area. Therefore, there is a net flow of raw materials from South and West to East, which is a larger movement in terms of weight or bulk than the reverse net movement of manufactured goods from the Northeast to the South and West. We are, of course, all made to pay for the extensive movement of empty cars thus necessitated, but we are not made to pay in such a way as to set up an incentive to change the situation. A system of railroad rates corresponding to marginal costs would quote higher rates per carload of goods carried toward the Northeast, where the predominant movement goes, than it would quote for the reverse direction. Such rates would contain just the optimal inducement to move processing activities away from the Northeast.

I must make one other qualification here. For a rate system according to marginal cost as regards different routes to be beneficial, it would likewise have to be in accordance with marginal cost as between different commodities. The present rate system also does not satisfy this criterion.

Commodities for the transportation of which the demand is inelastic are charged higher. It is uncertain whether the introduction of directional rates of the type that I have discussed, without at the same time abandoning discrimination between commodities, would lead to a better allocation of resources than the present rate system. It would certainly not lead to the optimum allocation.

It is, of course, well known that a system of pricing at marginal cost will imply operation at a deficit whenever and wherever the density of traffic is distinctly less than the capacity of the road. Other provocative features of marginal cost pricing are rates depending on the composition of demand by routes, possibly seasonal rates, possibly also contracts based on future rates, announced by the management of the railroad system and at any time subject to revision for contracts still to be concluded. It will be necessary to strike a balance between the cost to enterprise of uncertainty regarding future rate levels, the cost to railroads of announcing and applying changes in the rate structure, and the desirability of closely reflecting in rates the ever present fluctuations in the composition of demand. Further development of the foregoing analysis in a dynamic direction as well as factual study of fluctuations in demand are required before an approximately optimal railroad rate system can be formulated.

In conclusion, I wish to emphasize that a theory of optimal transportation rates, of which the present analysis is a small beginning, would provide an indispensable groundwork for any theory of the optimum geographical distribution of industry.

Résumé

Dans cette communication les principes de "l'économie du bien-être" sont appliqués à tout système de transport où les marchandises sont transportées à l'aide de matériel mobile (par vaisseau, wagon, camion, avion, etc.)

Considérons par exemple l'allocation d'une masse de transports maritimes, soit par une grande entreprise, soit par une autorité telle qu'il en a existé pendant les deux guerres mondiales. Dans un cas statique simplifié le programme consiste en une matrice A dans laquelle l'élément a_{ij} indique le nombre constant de vaisseaux uniformes requis chaque mois pour chargement au port "i" à destination du port "j". Le

coût total, c.à.d., le nombre de vaisseaux actifs qui est nécessaire pour l'exécution du programme se subdivise en *coût direct*—le nombre moyen en train d'être chargés, de naviguer avec cargaison, et d'être déchargés—et *coût indirect*—le nombre naviguant à vide vers un port de chargement. Le coût direct est une fonction linéaire des éléments de la matrice du programme, et des durées (supposées constantes) des mouvements ou opérations.

Le premier problème est la réduction au minimum du coût indirect, par un arrangement adéquat de trafic à vide. Une méthode tentative de solution est justifiée par le THEOREME 1. *Un arrangement de trafic à vide qui ne permet pas d'économies par un changement menu quelconque dans l'assignation de routes, ne permet pas d'économies par un changement intégral quelconque des routes assignées.*

Le coût marginal de chaque constituant a_{ij} du programme est la dérivée du coût total par rapport à a_{ij}. Cette dérivée est établie au moyen du: THEOREME 2. *Il est possible de définer un potentiel, c.à.d., une fonction* p_i *dans chaque port "i" touché par le trafic a vide, telle que le coût indirect marginal du constituant* a_{ij} *soit égal à la perte de potentiel* $p_i - p_j$ *subie par un vaisseu en effectuant un voyage pour constituant du programme. Le potentiel s'accroit le long des routes de trafic a vide, dans le sens de ce trafic, d'une quantité égale à la durée du voyage.*

Dans une économie de guerre, les autorites chargés du programme peuvent se servir du calcul du coût marginal décrit ci-dessus, p.e., pour déterminer la source la plus appropriée, du point de vue du transport, d'une matière première quelconque. Dans une économie qui s'assigne l'utilisation optimale des ressources, des taux de fret égaux aux coûts marginaux doivent guider les décisions de transport et de location industrielle. Le marché du *tramp shipping,* où la concurrence a été presque parfaite pendant une longue période, a connu un système de taux de fret approximativement égal à tout instant aux coûts marginaux anticipés. La manque de concurrence des chemins de fer a facilité le développement de systèmes de taux dégagées du coût marginal, et aussi entraîne des pratiques non-économiques dans le transport et la localisation industrielle.

Mr. Koopmans' paper was discussed by Messrs. Maurice Fréchet, M. Allais, and the speaker.

LES CHOIX DE L'ETAT PRODUCTEUR ET L'APPLICATION DE LA THEORIE DES SURPLUS DU PRODUCTEUR ET DU CONSOMMATEUR

par François Perroux

Professeur a l'Université de Paris

Directeur de l'Institut Science Economique Appliquée (France)

I. Limits de la note

Est-il possible de mesurer[1] et de quantifier les éléments qui, théoriquement, permettent de déterminer le *prix de maximum d'avantage collectif* ou *prix économiquement optimum* dans un groupe ou secteur (groupe de groupes) nationalisé?

Négligeant, en première approximation, les divers modèles possibles,[2] la "nationalisation" est considerée ici comme caractérisée par deux traits: 1°) l'action de l'Etat sur un *groupe* de firmes ou *groupe de groupes* (secteur);[3] 2°) la possibilité pour l'Etat de fixer le prix et l'output des firmes contenues dans le groupe ou d'imposer à ces firmes une règle uniforme de fixation de leur prix et de leur output.

La nationalisation est supposée orientée à une distribution optima des ressources par la voie des *rationalisations spécifiques*[4] de nationalisations.

II. La regle du cout marginale et la mesure de la perte

A—Mesure du Coût Marginal

Le principe du coût marginal[5] n'est pris ici ni comme un outil destiné à décrire l'équilibre aux conditions d'optimum, ni comme une recommandation normative de portée générale; mais comme une règle d'action pratique imposée à des firmes à l'intérieur d'un groupe.

La controverse contemporaine sur le coût marginal confirme que la difficulté réside dans l'indétermination de l'objet à calculer (limites entre frais fixes et frais variables, choix de la période). Elle montre, d'autre part, (Meade et Fleming) que pour tout accroissement de l'output

[1] Mesure réelle et non pas seulement mesure théorique.

[2] a) Trust d'Etat—b) Firmes d'Etat relativement autonomes sous un Centre de Coordination et d'arbitrage.

[3] Elle ne se réduit *pas* a la gestion d'*une* State enterprise.

[4] Rationalisations qui ne *peuvent* etre réalisées que par la voie de la nationalisation.

[5] Cout marginal des facteurs variables pour un équipement fixe donné et constant;

qui engendre une baisse appréciable du prix du produit et un accroissement appréciable du prix des facteurs, l'application pure et simple de la règle du coût marginal conduit à l'erreur. Un output calculé compte tenu des surplus du producteur et du consommateur est préférable à celui qui est déduit mécaniquement de cette règle.

L'Etat producteur ne peut se borner à prescrire, dans le groupe, l'application de la règle du coût marginal.

1°) La politique de nationalisation promet en fait et implique logiquement, si elle concerne des firmes capitalistes qui pratiquaient une politique restrictionniste, une extension substantielle du débit par comparaison au débit antérieur à la nationalisation. Cette extension met en jeu les éléments suivants (isolés ou combinés) 1) emplois additionnels de montants indivisibles de facteurs,—2) création de nouvelles unités de production,—3) création de groupes entiers d'unités de production. C'est la liste même des cas où les prix et l'output d'avantage collectif maximum ne peuvent pas être déduits mécaniquement de l'application de la règle du coût marginal.

2°) L'application de la règle du coût marginal par chaque firme est évidemment sans action sur l'obtention systématique des "external economies of scale" qui est l'un des maîtres objectifs des *rationalisations spécifiques de nationalisation* dans le groupe. Si ces rationalisations obéissent à un plan, elles éliminent des erreurs grossières et très visibles. Il peut être procédé à l'élimination des unités qui selon l'expérience courante ont des coûts marginaux anormalement élevés. Mais il est difficile d'aller plus loin car la comparabilité des coûts marginaux d'une unité aux coûts des autres est loin d'être parfaite. Deux effets distincts sont ainsi produits, de l'un à l'autre desquels on ne passe pas continuement: un effect of management qui est la conséquence du changement de gestion et de la préférence donnée au coût marginal;—un *effect of rationalisation* d'un groupe ou groupe de groupes qui forme le cadre dans lequel l'effect of management se développe.

3°) La seule hypothèse réaliste est celle où les choix de l'Etat englobent la totalité des décisions concrètes du producteur y compris le développement de l'équipement. Le développement rationnel de l'équipement pour le groupe forme un autre cadre indépendant de l'application de la règle du coût marginal et à l'intérieur duquel elle aura éventuellement effet.

Le groupe qui doit décider un investissement additionnel en capital fixe à réaliser au cours d'un certain nombre d'années, procédera à un calcul à la marge. Il tentera de prévoir la demande collective au bout de cette période, d'évaluer la dimension optima du groupe pour cette demande collective, d'évaluer le coût additionnel total, à la marge de l'unité de produit pour l'output correspondant à cette demande

collective (coût en facteurs fixes à ce moment, coût en facteurs variables au même moment.) L'important n'est plus le calcul à la marge[6] mais la correction de l'extrapolation des courbes collectives.

Les incertitudes concernent: a) la prévision de la demande collective: l'extrapolation le long d'une courbe de demande collective ne suffira pas en cas de changement dans la *structure* de la demande; c'est une nouvelle courbe de demande collective déplacée vers la droite qu'il faudra construire ou imaginer,—b) La prévision du niveau probable des coûts des facteurs variables au moment considéré,—c) la prévision du poids de l'amortissement au moment considéré, s'il est décidé d'exécuter l'amortissement selon un type et un rythme variables avec les opportunités économiques. Quelle que soit la proportion des coûts des facteurs variables et des coûts des facteurs fixes dans la somme que forme l'addition des deux éléments, le calcul visé effectué pour un groupe ou un groupe de groupes, est sans rapport avec l'application de la règle du coût marginal des facteurs variables et l'auscultation du marché par une firme isolée. Dans ce dernier cas, le chef de l'unité de production compare le prix du produit additionnel et le prix de la quantité additionnelle de facteurs variables à engager. Sa tâche s'épuise en une comparaison de deux prix, prévus sur une courte période à partir de prix connus. La simplicité toute apparente de cette tâche procède de ce que la définition du coût marginal supprime la difficulté de la classification concrète des frais en fixes et variables et les difficultés inhérentes au traitement des coûts correspondant à des facteurs durables ou très durables.

Le développement de la doctrine du coût marginal est un exemple remarquable d'"entrôînement" dans la théorie pure. Lorsqu'elle se dispense des difficultés de la quantification et de la mesure économétrique, elle substantialise un concept (coût marginal) et ne s'interroge plus sur son contenu ni sur les procédures de son obtention pratique.

Deux types de développement de l'équipement sont possibles: 1°) l'unité de production n'investit un montant additionnel de capital fixe qu'aussi longtemps que le prix du produit net additionnel attendu dépasse l'intérêt et l'amortissement du capital. La limite de l'investissement additionnel est alors l'égalisation du produit attendu et du coût (intérêt plus amortissement),—2°) l'unité de production investit à perte, la perte étant couverte par des ressources qui sont moins productives ou considérées comme telles en un autre point de l'économie. Cette subvention, qui pourrait, théoriquement, aussi bien couvrir des pertes sur frais variables, dissocie les coûts de l'unité de production de *son* produit et substitue l'équilibre d'un ensemble (économie nationale) à

[6] Cout "à la marge" en facteurs variables *et* fixes.

l'équilibre des composants de cet ensemble (firme nationalisée ou groupe).

L'Etat producteur, comme les grandes firmes capitalistes, établit ces plans *globaux* d'investissement[7] sur extrapolations particulièrement hasardées de courbes collectives d'offre et de demande. Ces calculs sont bien différents des calculs marginaux *limités* et *localisés* sur lesquels raisonne la théorie traditionnelle. Leur réussite ou leur échec nourrit des conséquences entièrement autonomes par rapport à l'application de la règle du coût marginal.

Lorsque l'on considère la nationalisation des groupes ou groupes de groupes, *pour des raisons spécifiques à ce modèle*, l'application de la règle du coût marginal ne suffit pas. Elle suppose un triple plan (rationalisation, production, taxation pour couvrir les pertes) qui forme le cadre dans lequel elle joue, mais qui ne peut se déduire d'elle.

B—Mesure de la Perte et de la Somme Compensatoire

Si elle tend à une redistribution des ressources en vue d'un optimum, la politique de nationalisation ne doit pas logiquement se préoccuper des pertes des firmes et groupes. Elle ne se comprend au contraire que par ces pertes, dans la mesure où celles-ci sont compensées par un transport de ressources économiques d'une zone où elles sont moins productives à une zone où elles le sont davantage. Ce même principe est à la base de l'acceptation de la perte "locale" résultant (a) de l'application de la règle du coût marginal quand le coût marginal est inférieur au coût moyen,—(b) de la mise en oeuvre du type 2 de politique d'investissement. La question est de savoir s'il est possible de mesurer: 1°) les pertes, (a) et (b), 2°) les ressources compensatoires.

La mesure des pertes (a) peut se faire ex post ou ex ante. Ex post, elle suppose la discrimination entre les pertes résultant de la vente au coût marginal et les pertes résultant éventuellement des insuffisances de la gestion dûes au fait de la nationalisation. Ex ante, elle suppose que l'on peut dans un groupe,—distinguer les firmes qui travaillent dans la partie ascendante et celles qui travaillent dans la partie descendante de leurs courbes de coûts,—connaître les courbes *objectives* de coût,—tenir ces courbes de coûts pour valables pendant la période considérée, et en déduire avec une précision suffisante le solde net (équilibre, gain ou perte de l'ensemble).

La mesure des pertes (b) est soumise aux incertitudes des extrapolations sur les courbes collectives d'offre et de demande dans le long run.

[7] Certains de ces plans d'investissement ne supposent pas seulement des *extension a la marge*, mais des "paris" concernant des *structures nouvelles*.

Ce total incertain devra être comparé en outre à un total de ressources compensatoires employées de façon moins productive. Pour les déterminer, on recourra soit à des présomptions générales fixées une fois pour toutes,—qui ne sont qu'un recours par voie détournée à la dangereuse "commodité" des social priorities,—soit à des mesures d'aires de surplus combinés de producteur ou de consommateur.

III. La mesure de la perte et les aires de surplus combinés

Soit dans les cas où (§II) la règle du coût marginal doit être complétée en prenant en considération les surplus, soit dans les cas où (investissement à perte) les pertes et les ressources compensatoires doivent être mesurées, s'impose le recours aux surfaces (triangles) de Dupuit, Hicks, Hotelling, considérées comme outils des "economics of welfare" modernes.

Cette méthode mesure théoriquement la perte sèche en surplus combinés du producteur et du consommateur, en cas de décroissance de l'output (ou de gain net dans le cas opposé). Quand il y a dans le groupe *A* un accroissement plus que marginal de l'output par déplacement des facteurs en provenance de l'industrie *B*, elle compare les gains faits par les consommateurs des produits de l'industrie *A*, à la suite de la baisse des prix, les pertes éprouvées par les consommateurs du produit de l'industrie *B* résultant de la diminution de l'offre, auxquels il faut ajouter les gains des propriétaires des facteurs déplacés de *B* en *A* et résultant de la hausse des prix des facteurs. Ce ne sont plus des diagrammes qualitatifs mais des calculs ou au minimum des évaluations ex ante de caractère concret et quantitatif qui devraient être obtenus, pour guider la politique de nationalisation. Ces évaluations concrètes devraient être l'oeuvre de l'Etat qui, seul, peut embrasser l'ensemble du groupe ou du secteur nationalisé. Les aires de surplus devraient fournir la mesure des virements de ressources productives que l'Etat provoque en faisant pratiquer par les firmes soumises à son contrôle, un prix et un output réputés propres à réaliser le maximum d'avantage collectif. A la recherche par "trial and error", serait substitué un calcul global de l'effet probable attaché à une politique déterminée du prix et de l'output.

Même si nous ne considérons qu'un groupe, nous rencontrons des difficultés présentement insurmontables, qui peuvent se répartir en deux familles:

a) *Des difficultés opérationnelles.* A un moment donné, nous ne connaissons pas les courbes expérimentales d'offre et de demande d'un bien mais le point de leur intersection, c'est-à-dire les quantités qui s'ajustent pour le prix pratiqué sur le marché. Restera à extrapoler en nous

prix des biens et services en provenance du secteur nationalisé dans le coût total des firmes du secteur libre.[9] Cette dernière mesure soulève des difficultés relativement mineures, sauf à observer que la proportion d'une catégorie quelconque de prix dans le total d'un coût étant *variable*, même en période normale, la proportion des prix des biens complémentaires en provenance du secteur nationalisé dans le coût des firmes privées étagées verticalement, l'est aussi. Quant à la mesure du degré de concurrence pour l'ensemble du secteur privé, ou de parties déterminées de ce secteur, aucune technique ne nous paraît pratiquement en situation de la fournir. Des supputations, purement qualitatives, devront donc être faites aux regards des effets verticaux de l'abaissement éventuel des coûts (prix) dans le secteur nationalisé.

V

Si la politique des nationalisations dans les pays occidentaux (Angleterre ou France par exemple) s'était préoccupée d'une justification économique, elle aurait dû prouver qu'elle repose sur des choix cohérents concernant le prix et l'output. Le choix d'un type de prix et d'output ne prend finalement son sens dans le concret que par la *mesure* des effets du changement de prix et d'output. Ni l'un ni l'autre de ces choix n'a été clairement et distinctement étudié. S'ils l'avaient été, le danger auquel est exposé l'Etat nationalisatéur *d'échanger les risques du monopole contre les risques de l'imprécision*, n'aurait pu être sous-estimé.

Le fonctionnement rationnel de cet hybride de l'économie de marché et de l'économie de plan qu'est l'économie à double secteur exigerait un calcul économique global d'un type nouveau. L'outil des aires de surplus combinés du producteur et du consommateur est approprié. Mais sa maniabilité reste pratiquement sensiblement nulle pour résoudre les problèmes concrets.

L'essai d'apprécier les difficultés de la traduction économétrique montrè la marge qui subsiste entre la description théorique de conditions d'optimum et l'énoncé de *regles praticables* propres à en rapprocher.

[9] La mesure de l'effet dépend plus précisément 1°) du pourcentage des prix des biens du secteur nationalisé dans le cout de production des entreprises visées, 2°) de la diminution de leur cout qu'elles obtiennent en étendant leur propre débit par l'effet d'un approvisionnement plus étendu en biens de production.

Résumé

I. Limits of Analysis. The theoretical model chosen is defined by: (1) the ability of the State to fix price and output of firms under control or to impose on them some uniform rule of price and output; (2) the putting of this policy into effect within the *group* of firms or a *group of groups* (sector) considered as clearly different from the management of a *single* State Enterprise. The question is one of determining the practical possibility of quantifying and measuring the elements which allow for fixing such prices and outputs.

II. Measuring Marginal Cost, Losses, and Means of Compensation. The indeterminacy of the object to be measured (distinction between fixed and variable costs, choice of the period) eclipses the difficulties of measurement. Furthermore, as regards the theoretical model concerned it can be shown that the mere application of the marginal-cost rule does not suffice; it is useful only within the framework of a threefold plan of rationalization, manufacturing, and taxation. An analysis of the concrete tasks that a State performs, to the extent (a) that it brings about the specialization of firms and (b) that it creates a substitute for free entry into the group, shows that it would only be playing with words to speak of the pure and simple application of the rule of marginal cost. The *aggregate* calculus, which constitutes the basis of the threefold plan, is lacking in definiteness regarding the (anticipated) losses and the elements of compensation.

III. Measuring the Areas of Combined Producers' and Consumers' Surpluses. The areas should be calculated *ex ante* in order to determine the prices and the outputs affording maximum collective advantage.

The conditions of the model chosen amplify not only the operational difficulties but also the difficulties related to the object to be measured (determination of group *B* which will provide the factors of production for the expansion of output in group *A*; evaluation of the diminution of output in *B*).

IV. Vertical Transmission of an Eventual Lowering of the Cost (Price) in the Nationalized Sector. The transmission depends on (1) the degree of competition in the free sector, (2) the degree to which the State is able to absorb or prevent the surpluses of nationalization. The relative effects of the prices of complementary goods originating in the public sector on the cost of production of private firms vertically related to this sector are variable. The best measure of the degree of competition in the whole private sector or in specified parts of it does not give the required information about the probable level of superprofits at the different levels.

V. It is probable that the wastes of generalized indeterminateness resulting from nationalization may not compensate the waste of oligopolistic competition which it proposes to eliminate.

Mr. Perroux's paper was discussed by Messrs. Abba P. Lerner, Jacques Dumontier, and the speaker.

THEORY OF CHOICE AND UTILIZATION OF RESOURCES: II

Saturday, September 13, at 2:00 p.m.

CHAIRMAN :

Harold Hotelling
Professor of Mathematical Statistics, University of North Carolina (United States)

THE OPTIMAL UTILIZATION OF NATIONAL RESOURCES

by J. Stafford
Chief Statistician, Central Statistical Office (United Kingdom)

1. INTRODUCTION

During the last fifteen years or so much has been written by economists about the economics of socialism. Influenced, one expects, in English-speaking countries by Prof. Pigou's *Economics of Welfare* and by the imperfections of an economy operating through unregulated market systems, something has had to be added to Adam Smith's philosophy of fortunate coincidence between private and public interest. Attention has been paid to the areas of divergence of the two interests, and individual conduct has been judged by economists in the light of its contribution to the more general good.

I expect that there are at any rate two quite different schools of thought concerning themselves with the action to be taken as a result of this judgment. There is a popular and empirical school which is prepared to take specific action to provide remedies in individual cases: to introduce price or profit controls, to "bust" trusts, to nationalize certain industries, to legislate against restrictive practices, and the like. The other and more scientific school, seeing, perhaps, more clearly the economic problem presented by these divergencies, has searched for rules of general applicability which may guide the economy and which may supplement or replace the present complex of rules, inhibitions and

restraints that govern so-called free systems. It is with the work of the second of these two schools of thought that this paper attempts to deal.

2. Full Employment and Investment

In some respects the rules that have been formulated by economists interesting themselves in the economics of socialism appear to me to be incomplete.

Perhaps I may give two examples.

Most people, I imagine, will set up as one of their ideal economic conditions a state of full employment; that is, perhaps, a state in which no man is unemployed who is willing to work at the highest wage rate his kind of skill commands and, in really Utopian conditions, where the attractions of leisure and income are in balance for each individual at the margin.

After the work of Lord Keynes, many of us have tended to take for granted the possibility of the attainment of such a state and to consider that the real problem is to use in the best manner the resources so employed. But we have not, I think, given sufficient thought to the problems of full employment or to the question of whether this state can be set up as the goal of economic policy. In practice, the lower the level to which one wishes to reduce unemployment, the greater is the pressure of demand which must be generated and maintained. The real question is what pressure of demand should be allowed to insure that resources are not wasted in idleness. To put the matter in another way, should an economy envisage 2, 4, or 8 percent of its labor resources as unemployed. The advantages of full employment are obvious—the disadvantages are not so apparent. Perhaps it is worth while to indicate some of them. In the first place, the lower the level of unemployment, the greater is likely to be the upward thrust on prices, the greater is the level of profits in the short run, the higher the rate of investment, and the more likely that future increases to sustain a collapsed demand will take the form of continued stimulus of investment in order that the immobility of resources shall not promote waste and induce future unemployment.

There are, I expect, other disadvantages. It is more difficult to bring about the free allocation of resources when the pressure of demand is high, for they are not pushed out of the relatively less desired kind of activity: they have to be drawn into the more desired by an offer of the appropriate incentive. The accounting of the monetary system is upset when there is marked instability of prices. This is particularly true in the field of costing where an important decision to be made by the entrepreneur is the appropriate allowance to be included in receipts

for the maintenance of capital equipment. I imagine, too, that in a situation in which everything that can be produced can easily be sold at prices that generously cover the cost of variable factors, there will occur a very considerable diminution in the urge to minimize the expenditure of resources to achieve a given end. There is perhaps, something in the old-fashioned view that in a boom the economy benefits from innovations, while in a slump it consolidates and learns to make the best use of the progress of the recent past.

The main attraction of a controlled economy to many economic writers has been the promise of releasing the system from the disruptive influences of fluctuations. Insufficient attention has been paid to the difficulties that may arise when a very high level of employment is maintained for long periods under controlled conditions. Too great a pressure of demand will impose severe strains at certain points in the economy, inducing considerable rises in prices which will be equally hard to tolerate in socialist or other systems, or will bring shortages into existence and tend to bring in their train the need for a consideration of priorities between the jobs or the needs demanding the scarcer resources. In other words, success in securing full utilization of resources may in practice only be achievable by a tighter and a more particularized control of the economy than most economists have tended so far to contemplate with favor.

Some of the advantages and disadvantages of full employment are of a like nature and theoretically it should be possible to make a rational choice between the benefits of a little more employment on the one hand and on the other, a little more friction that tends to reduce the efficiency with which resources are applied. In practice, however, it may be thought that the measurement of the defect from the optimum resulting from the high pressure of demand will be so difficult to make with accuracy that very little comfort can be taken from this consideration. And for the most part the advantages of higher employment may be of a different kind from the disadvantages, so that judgment must be made on qualitative rather than quantitative considerations.

The problem attending full employment that has perhaps the most direct connection with the optimal utilization of resources is that resulting from the upward pressure on prices. Instabilities of prices prevent both the individual and the business from making an appropriate allocation of the resources they command between the present and the future. There is no certain way of exchanging command over factors of production now for command in the future. This must distort the pattern of present expenditure from what it would be were there greater price stability. An individual desiring to save will, if he is rational, be persuaded to increase those present purchases which will be likely to put

him in possession of greater disposable resources at the time when he would normally have wished to dissave. For example, a man may so be induced to purchase a house or durable consumer goods that will reduce his future expenditure. He will be forced to reach his preferred position indirectly; or to protect savings from depreciation resulting from rising prices the individual may be forced into investments that carry a burden of risk that he would not normally wish to carry.

The securing of full employment and the advantages of substantial price stability simultaneously presents some interesting problems. Can it be that Marshall's proposals for a tabular standard promises something of a solution, and if so, what is the best standard to adopt?

Let us suppose, however, that an acceptable level of employment can be achieved with a tolerable stability of prices, allowing resources to be allocated through appropriate responses to the price system. There must, of course, be envisaged the possibility that demand will have to be supported by government spending of some kind. The possibility that many have in mind is one in which, when the normal or regular services of government have been provided, when consumption demands have been met, and the investment requirements of those carrying entrepreneurial responsibilities have been filled, there will remain resources that would be unused. The state may meet the situation in a variety of ways—by reducing taxation, by distributing some kind of supplementary dividend, by forcing the rate of private and public business investment, by public-works policies, or even by the purchase and destruction of unwanted supplies. Are there any rules to point to the most appropriate action, or is one to conclude that this is a matter to be decided on purely political grounds? I have nothing that is constructive to say on this matter, but there are one or two considerations bearing upon this subject that are perhaps worth mentioning.

First, there appears to be a curious lack of symmetry in the paying of so much attention to consumers' choice in respect of the allocation of resources devoted to the making of present utilities, if little attention is to be paid to the market choices of the public between present and future utilities.

Secondly, there may be practical dangers in meeting the potential excess of saving by deficit financing designed to allow tax remission that will raise the propensity to consume. A high level of private savings held in liquid forms constitute a potential source of dissaving at future, inconvenient times. If at future dates these liquid resources are pushed on to the market in order to increase the rate of spending, there may result, over a period of time, a tendency towards communal disinvestment. Or to avoid this difficulty, the timing of consumption will have to be dictated.

Thirdly, from the angle of expediency, it may be said that large variations in the volume and nature of investment are likely to be wasteful of resources. The promotion of instability in the make-up of aggregate demand is likely to bring open or concealed unemployment and what, from a long-run point of view, may well be judged an inefficient use of resources. According to the type of policy adopted for maintaining employment, skilled workers in the capital-goods industries may become labourers, for example, on civil-engineering projects, or become unskilled hands in some of the consumer-goods industries. It is not, of course, wise to determine economic policy by the immobilities and restraints of the system. The better course will be to do what is possible to reduce these immobilities. At the same time there is something to be said for a recognition of the institutional and social framework within which economic policy must be determined.

This suggests that there is an important economic rule that can be applied to the determination of investment levels— namely that investment policy should be drawn in such a way as to minimize the social cost of carrying through the investment of a period of years. Certainly one way of doing this would be to draw up an investment program that would be carried through at a steady rate and of a make-up that was phased from year to year to even the load on the various capital-goods industries. It cannot, of course, be pretended that such an approach would avoid very difficult decisions, but it is true that there would need to be emphasis on central decision rather than on the application of some general rule of conduct for the determination of private investment.

This kind of conclusion finds some support from another consideration. Presumably, among many others two important reasons for periodical reductions in business investment are the following:

(i) The uncertain immediate outlook that brings an overmagnification of risk. Investment, for example, may be postponed, not because it would fail to yield the necessary return, but because a balanced judgment of the future possibilities of the investment is not taken at that time.

(ii) Investment may be postponed, because though profitable at the moment, it is judged likely to become even more profitable later on. Prices, for example, may be expected to fall which will raise the rate of return in the future to a level higher than those present investments which have to be continued because legal and other commitments have been undertaken.

3. Maximizing Income and the Effect of Taxation on Incentives

The optimal utilization of resources as set out by economists postulates at least two conditions that under present circumstances make a heavy call on the budget. First, more nearly equal distribution of income

which may be provided by generous social services, and, if necessary, by the subsidized underpinning of living conditions. Secondly, the throwing upon the budget of those business losses which result from the expansion of increasing-return industries to levels that will insure equality between marginal cost and marginal social benefit.

This does not matter, perhaps, to any very considerable extent in wealthy countries where budgetary deficits year in, year out, may be an important means of insuring full employment of resources. But in less fortunate countries it may be of more consequence.

For a long time the fact has been recognized in an academic kind of way that high taxation may upset the balance between work and leisure. It has not been so generally recognized that it may lead to an underutilization of expensive equipment by reducing the intensity of work. The effect of taxation on incentives has been rather lightly dismissed on the score that, in the first place, there may be an important offsetting income effect that will promote rather than reduce the desire for income. Secondly, it has generally been felt that disincentives are strongest in the high income ranges where work is likely to be interesting and carried on for its own sake rather than for the income which it yields, and where there is unlikely to be a nice balance of advantage and disadvantage at the margin. Furthermore, more comfort has been drawn from the fact that so far as the income from risk bearing is concerned, both gains and losses will be shared with the government under the customary tax formulae.

At the end of two major wars, however, the budgets of some states are so large that heavy taxation has had to be imposed on those falling within the lower income groups, and therefore on those to whom work is not always of a very interesting nature.

The effect of this is likely to be twofold:

(i) Quite high marginal rates of taxation have been imposed on workers and on the smaller business and professional people. In the United Kingdom, for example, direct taxation at the margin on smaller earned incomes is over 40 percent, while of the remaining 60 percent, substantial amounts are taken by the state in indirect taxation on customary patterns of expenditure. Rates of this kind are bound to raise the question in the minds of individuals of whether it is worth while to do an extra pound sterling's worth of work for something under twelve shillings' worth of income.

(ii) The obvious disincentive effects of this high rate of direct taxation have unavoidably led governments to explore the possibility of thrusting some of the burden on to indirect taxes. The need for revenue is often so considerable that the indirect taxes imposed are likely to be designed as large yielders of revenue rather than as instruments of

paternalism that seek to turn the pattern of consumption expenditure into a wiser form. To put it more bluntly, it is to be doubted whether cigarettes are at present 3s. 4d. a package, and beer 1s. 4d. a pint in the United Kingdom, because nicotine and alcohol are regarded as vicious to that extent. The need for revenue that can be economically collected is so great that respect for consumer sovereignty has to go by the board.

Perhaps the moral is that economists should be more careful to recognize that there can be a serious clash between the maximization of income and the securing of the optimal utilization of that income, and should be more cautious in suggesting devices that throw even greater strains on national budgets. All this is the more important because of the fact that in some countries the damage of the war years may have to be rectified by financing capital expenditure out of budgetary surpluses because of an unfortunately high propensity to consume or an unfortunately low level of national income.

4. Some Difficulties in Applying Rules for the Optimal Utilization of Resources

In laying down rules for a controlled economy to achieve the optimal utilization of resources, certain implicit assumptions have been made that do not obviously conform to the facts of Western economies.

It has been recognised that there may well be imperfect competition between suppliers of products and a monopsonistic element in the factor market, and that some sort of action may be required to overcome these obstacles. But it is obviously true that there is far from perfect competition in the supply of factors in the most important markets. The wage structure, for example, has in many Western countries been evolved organically and historically under the influence of traditional forces and has been influenced by institutional developments. The result is that the social-opportunity costs of using labor in a given direction are by no means easy to measure.

Again, the rules implicitly assume a willingness to apply them once their beauty has been demonstrated.

Almost the whole of the analysis economists parade to reach the rules for a socialist economy is based on the assumption that over a large part of the field at any rate private judgments will prevail. The manager of socialist enterprises, final consumers, and private business people are to go their own way provided they conform to certain general procedures.

We are some way from that distribution of income which to those economists would give validity to the concept of consumer sovereignty and all that implies. At once this throws doubt on the relationship between need and demand. But this is by no means all. There is a

pretty wide field in which the state is prepared to act paternally in education, health, nutrition, housing, the provision of public utilities, and road transport to mention only a few that come readily to mind. It is not at all evident that society desires to leave the allocation of resources to reflect the choice of consumers. I suggest that there is a real intellectual difficulty here. Economists have wrestled with the problem of the utilities of irrationality. We have here something of similar character. People are, of course, members of society, in a number of capacities, and not only as consumers making nice decisions about preferred consumption patterns. They are members of the different microsocieties that go to make up the whole community, and in these capacities influence the allocation of resources not through money counters but through the ballot box. In these capacities, people do not settle economic problems in advance by decisions to apply in the future certain broad rules, but rather at the time and in the light of the ways in which different solutions affect them in their various capacities. Inevitably, therefore, government proceeds through a series of individual decisions, broadly influenced by the most general political and philosophical concepts.

Again, on another plane, there is here very real difficulty of policing the application of the rules of economists. It is a pretty good precept only to promulgate laws or rules that can be enforced or to which obedience can be checked. The rules that production should be carried to the point at which marginal social benefit and marginal private costs are equal and that marginal private cost should be equated to marginal social cost are by no means easy rules to enforce as those who have some knowledge of negotiating war contracts will know. Enforcement of rules of this kind demand elaborate cost data and cost analysis and even then it is a matter of judgement how nearly one can gauge the way in which the rules are being observed. I expect one of the main constituents of successful business in the correct interpretation of accounts; and one in which some intuition is clearly necessary is the assessment of marginal costs to allow for the frequently unknown results of change. I suggest that it is wrong to imply that these are simple rules. They are not difficult for the economists to formulate: they are likely to be both difficult and expensive for governments to apply.

The conflict between general and particularized control is perhaps seen most clearly in wartime or during some other period of comparable pressure on resources. Certain broad social ends may be in view—the winning of a war, for example, or the reconstitution of a devastated economy. Occasions do occur when reliance cannot be placed upon a market system to achieve the ends deemed imperative, however that price system may be influenced and shaped by accounting and fiscal procedures.

Résumé

I. *Des critériums d'après lesquels on juge l'efficacité de l'emploi des ressources nationales.*

1. Les économistes ont beaucoup cherché à développer des règles générales pour assurer le meilleur emploi des ressources nationales. Plusieurs économistes ont étudié les conditions qui doivent être remplies pour affecter les ressources aux usages les plus pressants.

2. Il paraît nécessaire d'étudier d'avantage ces questions: Quelle part du revenu disponible doit être investie et quelle part dépensée? Doit-on contrôler les investissements pour éviter une inflation ou doit-on réduire la consommation? Y a-t-il des principes généraux permettant de diriger les affaires, ou faut-il examiner chaque situation *ad hoc*?

3. Les réactions contre les efforts qu'ont faits certains états pour assurer l'utilisation optimum, n'ont pas été examinées d'une manière suffisamment approfondie. Les frais budgétaires de la guerre aussi bien que ceux de la rédistribution des revenus ont nécessité une forte augmentation des impôts. Comment peut-on balancer les avantages d'une meilleure répartition de revenu avec ceux d'un plus gros revenu?

II. *Comment atteindre l'utilisation optimum.*

4. Plusieurs empêchements à l'utilisation optimum n'ont pas retenu suffisamment l'attention: (I) La mesure imparfaite des "frais d'occasion" (opportunity costs) représentée par le taux du salaire; (II) La dèfiance qu'inspire le système du marché; (III) Le désir de contrôler les prix directement pour des buts sociaux; (IV) Les difficultés d'administration considèrables pour le contrôle des prix ou des dépenses.

5. Sous ce jour, la question de l'utilisation optimum des ressources change d'aspect; elle touche tant au domaine politique qu'au domaine économique.

III. *Contrôle du marché et planification.*

6. En général, les économistes ont discuté le problème d'assurer l'utilisation optimum des ressources au moyen de subventions et d'impôts. Il est très bien possible que ces instruments ne sont ni assez fins ni assez souples pour atteindre l'objectif voulu.

Il est probable qu'il aboutira à un contrôle plus direct au moyen de permis, etc. Cela peut amener à un équilibre intuitif dans le jugement des questions de nature à affecter l'action de contrôle.

7. De cette manière, l'utilisation optimum des ressources devient non pas une situation dont les divergences peuvent être mesurées par une méthode scientifique exacte, mais plutôt une affaire de jugement dont le pour et le contre sont réglés par ces échanges d'opinion qui dirigent les actions et la politique des états.

THE PRACTICE OF ECONOMIC PLANNING AND THE OPTIMUM ALLOCATION OF RESOURCES

by Oscar Lange
Delegate of Poland to the United Nations

Students of welfare economics and of the theory of a Socialist economy are very well acquainted with a whole literature on the subject that is concerned with developing the criteria of the optimum allocation of resources. This literature can be considered as a special branch of welfare economics that has developed the principles of an optimum allocation of resources in any type of rational economic organization. It has been developed largely between the two world wars by economists at a time where there was only one experience in planned economy, namely, that given in the Soviet Union. The literature was developed in a rather formal, abstract way, without reference to that experience. This situation is changed now, by the fact that in certain parts of Europe, particularly Eastern Europe, new types of a planned and socialist economy have been developed. We have got in Eastern Europe (particularly countries like, for instance, Poland or Czechoslovakia) a developed system of economic planning. The purpose of my address is to give first, a very brief review of the fundamentals of this system, and then ask questions as to how far this system compares with the criteria developed in the literature on welfare economics and of the theory of economic planning. I have to warn you in advance that I shall be able to ask certain questions, indicate certain problems, rather than give ready-made answers. The problem is a highly interesting problem for study, on which the proper research and investigation still have to be made.

The system of planning that has been developed in the countries of Eastern Europe is characterized by one feature; namely, that the economies in these countries consist of two sectors, one of which has been socialized and the other of which is a sector of private enterprise. Consequently, the plans that are made are a combination, first, of plans that are normative for the socialized section, and of certain previsions (forecasts) that are made for the private-enterprise sector. However, it should be understood that since in all these countries the State has the strategic positions in the economy, with nationalization of all the major and medium-sized industry and further nationalization of banking, the results of decisions in the private-enterprise sector are highly influenced by governmental policy. In this sense they are not entirely

automatic, but are partly a response to certain policy decisions of the State.

The approach to planning that is made, is made at this stage, I would say, in macroeconomic rather than in microeconomic terms.

The plan pursues two objectives: one of which is purely economic, and the other of which is social and political. The economic objective is an increase in the aggregate national income; the social and political objective is the achievement of a certain change in the social structure of the country and consequently also in the cultural pattern and type of civilization that is based on the social structure. It is a characteristic feature of the countries of Eastern Europe that the second factor is *a* basic, if not *the* basic factor of their economic planning. The major feature of that economic planning, therefore, is industrialization of the country. An industrialization which, again for social and cultural and political reasons, it is desired to achieve at the most rapid rate possible.

This objective of industralization determines the basic macroeconomic decisions of the plan, namely, determination of the fraction of the national income that is to be invested. Now, this fraction is determined by a kind of compromise of the objective of achieving the greatest possible rate of industralization and the objective of achieving it at a not-too-great sacrifice in the development of the production of consumers' goods.

For instance, in Poland it was decided to have at the same time an increase in the national income, and a diminution of the percentage of population engaged in agriculture, while at the same time having a steady growth of both total and per capita output of consumers' goods. Such a decision, of course, is a basic political decision which is not being made in the form that could be put in terms of any marginal analysis.

This is the first basic decision of a macroeconomic nature. However, it is not possible to avoid the microeconomic choices as to what goods and what quantities are going to be produced. I must say that, up to now, there has not been developed any theory of these choices. Nor is it possible to say that any of the existing theories has been in any conscious way used as a basis of a plan. On the other hand, the problem *does* exist. Those who are responsible for the planning are aware of the problem, and there actually has been some discussion.

At the moment, the problem does not enter into the conscious decisions for one reason. The reason is this: The planning is divided into two stages, which I would roughly describe as the stage of reconstruction and the stage of expansion. These stages are practically not quite so distinct as I put them now for our theoretical discussions; but still the distinction does exist and is rather basic. In Eastern Europe most of the countries suffered heavily from the war, both in terms of actual

destruction, and in terms of depletion of capital through lack of replacement. The first stage of decisions taken is simply one of restoration of the status quo. Factories are being rebuilt to produce what they did before; bridges that were destroyed are being rebuilt, and so on. Consequently, the problems of choices as to investment and production are very much simplified. They are simply solved by deciding to restore what was before.

The next step, after that, is the step of expansion. The two steps, of course, are not quite independent; and the question may be raised: Is it really a rational thing to decide to restore exactly what was before the war? Conditions have changed, conditions, directions of demand; and, consequently, is it at all possible to distinguish between such a stage of restoration or reconstruction and a stage of expansion? The question is quite legitimate; and the answer, *a priori*, would be that there is no justification for such a distinction. However, I think here a hypothesis can be made to rationalize and justify the approach that has been made in practice. The hypothesis would be this: that all decisions on expansion contain as a part of them the restoration of the existing old levels of output, and that, consequently, the choices to restore the old levels of output are made legitimately, insofar as they are an integral part of any program of expansion. I think this justifies the procedure followed in practice; but at the same time it really postpones the real problem of choices for a further future.

This being so, we have got a system of economic plans in most of the countries of Eastern Europe that are based on the restoration or reconstruction of the status quo, although, even in the first period, they do contain certain elements of expansion. The problem of choice up to now has been simplified further by the fact of the existence of an inflationary gap. The high investment program, which results from the fact that all these economic plans are connected with a political decision of industrialization at a rather speedy rate, produced everywhere an inflationary gap which is being solved in some countries by rationing, in different ways in other countries. On the whole, however, the problem has been solved by different means of monetary policy which do maintain a certain stability of the price level.

This makes for a constant pressure of excess demand in all markets, which, in turn, creates a situation that any increase in output, in whatever field, appears as a desirable thing. Mr. Stafford has mentioned in his earlier paper that the strict criteria of choice disappear or are thoroughly obscured under such a situation of full employment and inflationary pressure. Consequently, again the output levels are largely determined simply by limitations on the supply side, by the possibilities of increasing the supply that exists under given technical conditions, rather than by

a condition of the relative urgency and strength of the demand. The relative urgency and strength of the demand is taken into consideration, in certain respects, but insofar as the decision as to the division of the total output between the output of producers' goods and consumers' goods goes, the basic plan of industrialization, creates a certain system of priorities determined by public policy. To a certain extent, of course, the market demand effect on prices and on the calculation of cost and profit has repercussions on the plans of investment and expansion of different industries, though these appear in very crude forms which are not easily translatable into the precise forms of theoretical analysis.

Such is, roughly, the basic experience of economic planning in the countries of Eastern Europe. The questions that are raised are the following. The first is a critical question: From the point of view of our present theoretical knowledge, how efficient can this planned economy be considered? How near or how far are they from the social optimum as given by the criteria of modern welfare economics?

The second question is this: Is the practice of this planning such that it can be translated into the concept of modern welfare economics and marginal analysis? And also the question directed toward marginal analysis, a question which was raised in the preceding paper by Mr. Stafford, namely whether the criteria of welfare economics, particularly of the marginal analysis are operationally sufficiently clear so that they can be applied practically. If yes, what kind of development in the concepts and methods of public accounting do we require? If no, how have they to be changed and readapted, and by what criteria to be replaced, in order to give us a theory that would be sufficiently operationally definite in order to be applicable to the solution of our practical problems?

These are the problems that have to be answered. I do not claim to be able to give the answer at this moment. I can only make a few general observations, and these general observations would be the following: It seems that in all the planned economics that we know empirically, as in the Soviet Union, or the alternative type that we now have in Eastern Europe, there are certain elements of the economic plans, certain choices which are made on the basis of criteria that are not connected with marginal analysis.

The second general observation, I think, that can be made, is that marginal analysis requires much further elaboration in terms more operational than at present, to make it serve as a basis for practical decisions.

The question arises whether the planned economies in Easterr Europe, for instance, can be regarded as an economic success. I think that the question can be answered.

In order to make the question operationally significant it has to be put in the following form: How much more are these planned economies of Eastern Europe efficient than the prewar economic systems had been? I think there the answer is very simple. Through the adoption of planned economy and all the social changes connected with it these countries, which before the war were countries of chronic industrial and economic stagnation, have entered into a period of great and dynamic economic development, with a great rapid reconstruction, industrialization, undoubted increase in national income, diminution if not full disappearance of their large agricultural surplus population (a form of disguised unemployment), the disappearance of unemployment in industry which existed in the period between the two world wars or at least over a part of this period. The planned economies undoubtedly are an economic success. This economic success can be attributed largely to two factors: First, because these economies *do* operate on the basis of a full employment with a further plan to remove the element of disguised unemployment that exists in form of an agrarian surplus population, this surplus population is being absorbed by the process of industrialization. And, second, because there are no more the handicaps to the utilization of resources and industrialization, which existed before the war in the form of private industrial monopolies. Such monopolies were a very potent handicap of economic development of these countries. These are factors that both can be evaluated roughly in terms of quite simple economic analysis, terms that are quite sufficient to provide the foundation for basic political decisions.

As I have said, the more refined choices appeared less important because the need for such choices was very much limited by the fact that the choices were largely those of reconstruction or restoration of the status quo. But of course in all these planned economies the problems of more refined choices involved in the determination of the levels of the different outputs of the different industries, or that of agriculture, do appear and their importance will increase the farther we get away from the stage of reconstruction and enter into the stage of expansion. And in this case, I think the problem of a refinement of the method of analysis and of the criteria used for economic planning will be needed in order to solve the practical problems of economic planning.

I think that, for us economists, a study of these experiments in economic planning is a rather interesting subject. I think that, up to now, the literature on the theory of economic planning suffered very heavily from the fact that it was purely abstract without relation to the institutional set-up of the existing types of economic systems. And I do believe that by studying the recent experiments in economic planning, the practice of economic planning, as well as theoretical economics, can benefit mutually.

Résumé

Le but de la conférence est d'étudier la pratique des économies planifiées que se sont developpées après la guerre en Europe de l'Est (notamment en Pologne et Tchechoslovaquie) et de comparer la pratique de planification dans ces économies avec les théories de planification de l'économie du bien-être. Les plans économiques ont deux bùts: un but purement économique et un autre but social et politique. Le but économique c'est la reconstuction, le but social et politique c'est l'industrialisation. La décision d'industrialiser le pays est un décision politique. Le problème de reconstruction est simplifié par le fait qu'il s'agit de reconstruir des objets détruits et que la necessité de faire des choix nouveaux est réduite. Le problème de choix se pose pleinement quand on passe de la reconstruction à l'expansion économique. Mais même ici le problème est simplifié au moment par l'existence d'un excès de demande qui fait apparaître désirable une augmentation de production quelconque. Quant à la question d'éfficacité des économies planifiées en Europe de l'Est on doit remarquer le progrès rapide de reconstruction et developpement économique en contraste avec la stagnation économique de ces pays avant la guerre. Ce succès des économies planifiées est dû au plein emploi et à l'absence des restrictions de monopole. L'interprétation théorique de ce phénomène est donc claire. Avec le progrès de l'expansion économique des problèmes plus compliqués de choix se poseront. Pour résoudre ces problèmes on aura besoin d'une analyse théorique plus précise et souple.

Discussion

M. Perroux:

L'intéressante communication d'O. Lange fournit à tous ceux qui ont bénéficié déjà de ses précédents travaux, un motif supplémentaire de reconnaissance. Sa base même, ne peut, cependant, être acceptée par moi.

A—Je note que le théoricien socialiste a *complètement* changé de méthode. Il a autrefois essayé de montrer qu'une économie socialiste peut fonctionner *a peu près* comme une économie de marché, sur la base de *calculs isolés des unités* (règle du coût marginale). Il fonde aujourd'hui sa thèse sur les *macrodécisions* de l'Etat. Il le fait paradoxalement au moment précisément où tout le monde est d'accord sur la necessité du *breakdown of the aggregate quantities.*

B—Notre problème est: Comment l'Etat peut-il former des macrodécisions *économiques rationnelles*? Cela suppose que la Centrale collectiviste ne décide pas, préalablement, que ses choix coïncident nécessairement avec ceux des individus et ne se réfugie pas dans le démarquage de la politique même usitée par le capitalisme auquel elle succède.

C—O. Lange n'a pas discuté la rationalité économique des macrodécisions. *Aucune* des méthodes auxquelles on peut penser n'est praticable. Ni le calcul des effets *objectivement* utiles (rations "normales" en termes de laboratoire). Ni le calcul des surplus du producteur et du consommateur. Ni le calcul des maxima (exemple: la consommation qui maximise l'investissement). La *théorie des macrodécisions reste à faire.* Les plans collectivistes qui ont été concrètement élaborés et mis en oeuvre sont empiriques au sens *le moins* défendable de ce mot.

D—La distinction de la période de *reconstruction* et de la période *d'expansion* suggérée par O. Lange me semble illégitime. Il n'y a aucune raison, *tout au contraire*, de reproduire dans un plan *collectiviste* de 1947, la *structure passée* d'une économie de *marché* en 1938. Tout plan est une tension entre le passé et le futur. Si le socialisme répète les leçons de l'économie qu'il prétend remplacer, il avoue son impuissance.

E—La Théorie Economique s'est appliquée à des calculs économiques *isolés* et *marginaux.* Elle ne nous a presque rien dit sur un autre type de calculs économiques: les calculs *globaux* qui impliquent des *paris* sur l'opportunité des structures *neuves.* Si le socialisme théorique a quelques services à nous rendre, c'est en élaborant *cette dernière* théorie: je ne la trouve pas dans l'exposé de O. Lange.

Mr. Tinbergen:

I have been struck by the fact that there is much similarity between the way in which Dr. L a n g e puts his problems and the way in which we are accustomed to put our problems in some Western European countries. The planning in the Netherlands, which I may perhaps choose as an example, is rather detailed for the moment, in view of the extreme scarcity of foreign exchange and of other factors of production. It might be that in later years the planning would take the shape of a more over-all character.

But for the moment, as I said, there is a great similarity. And I quite agree with Dr. Lange that it is hardly possible to take decisions only on a rational basis. I should like to add, however, that in free economies also, there are a number of important decisions that are not rational, either. It seemed nevertheless possible, in some cases, somewhat to rationalize decisions.

The first example concerns investments. These were provisionally estimated on the basis of the total number of workers for whom employment has to be provided in the next five years and the average capital per worker that is customary in the Netherlands.

As a further example, I may perhaps add the example on the plan for building schools which was based on forecasts of the number of children to be expected and the need for trained workers.

Another example is in a similar program for hospitals.

It was possible to some extent to give a background to the rationing system of the government, by comparing rations to the budget statistics of prewar times, taking account of a number of corrections.

Dr. Lange also invited us to answer the question whether it would be possible to test the efficiency of this type of semi-planned economies. It is very difficult indeed, as you can surmise, to make such a test. One rather superficial but not uninteresting possibility, however, in the case of the Netherlands, seems to be the comparison with Belgium, a country whose economy is much freer than the Dutch economy is at present. And now it is interesting to state that the velocity with which the general index of production for industry has gone up in both countries since the very low point that was reached in the beginning of 1945 has been almost the same in both countries. Moreover, price control and wage control had far more success in Holland than it had in Belgium.

On the question raised by Dr. Lange, in what respect it would be possible to take more advantage of marginal analysis for advising the government on its economic policy, I should like to answer that, even if we did not succeed in finding very fine instruments of distributing resources, it might be not such a terrible accident for this transitory period

of great scarcity. Generally speaking, however, I think it is too early yet to answer this question. Therefore, collecting experiences as we are now doing may be useful for all countries involved.

M. Rueff:

Je tiens, en premier lieu, à rendre hommage au très remarquable exposé de M. L a n g e. Je voudrais seulement ajouter une question à celles qui lui ont été posées. Je voudrais qu'il nous indique les méthodes par lesquelles les prévisions du plan peuvent être transformées en réalité.

En effet, en régime planifié, si la conduite des producteurs et des consommateurs n'est pas déterminée par des prix tels qu'ils les incitent à vouloir spontanément accomplir les actes susceptibles de conduire à l'équilibre économique, il faut que ces actes leur soient imposés.

Le drame des politiques planifiées dans beaucoup de pays, c'est, qu'en fait, producteurs et consommateurs restent libres de se décider d'après les prix du marché; alors que ceux-ci ne sont pas ce qu'ils doivent être pour que les prévisions du plan soient satisfaites.

Je voudrais que M. Lange nous dise quelles méthodes pratiques doivent être employées pour que le plan soit, en fait, efficace.

Mr. Domar:

My remarks are theoretical. Assuming that our objective is to satisfy consumer wants with a minimum of centralized interference, the problem of resource allocation falls into two parts: how to determine the desirable pattern of behavior on the part of industrial managers, and how to make them behave in the desirable manner.

In either society, the first problem is solved in the well-known manner of equating marginal costs to prices. In a capitalist society with perfect competition, the second one is solved by allowing producers to maximize their profits. The emergence of monopoly (in its several manifestations) destroys this solution, and so far no other has really been found.

In a socialist society (according to Lange and Lerner), the second problem is solved by *instructing* the managers to equate marginal costs to prices. But what incentives will society offer to make them behave in this manner? And what method will be used to determine whether they so behave? Surely we cannot merely rely on their sense of duty. Nor is it practical administrative policy to make a continuous study of their accounting records: all decisions are made for the future, and perfectly honest differences of opinions are possible. I don't believe that either our economic theory or Mr. Lange's paper have answered these questions.

A similar question arises in connection with uncertainty. We don't want the managers to be foolhardy, nor do we want them never to take chances. So the punishment for a failure must be severe, but not too severe to inhibit experimentation; and the reward for success must be generous, but not too generous to encourage waste. I don't know how the socialist society will solve this dilemma, yet on its proper solution depends its whole economic development and progress.

For the United States these are academic questions. But they must be real and urgent in Europe, and I would like to ask the speakers whether much thought has been given to them in their respective countries.

Mr. Lundberg:

There is a fundamental difficulty with regard to effective planning that I think is on our minds, and is implicitly involved in the speeches that we have heard, not least in Professor L a n g e's lecture. But it has to be said explicitly. This difficulty refers to the disturbing fact of politics in a free democracy. The concentration of interest naturally tends towards the questions of income distribution on the cost of positive planning. I like to draw a parallel with the development of planning in Sweden during the postwar years.

One of the main problems with regard to the future development of the Swedish economy refers to the stagnation in population growth. There will hardly be any increase for twenty years in our labor force. That means great difficulty for a capitalist system to work. One of the ways to solve that problem is to get labor from agriculture where there are certain reserves. An agricultural planning commission has been working for four years with thirty members.

The economists in that commission made a dynamic plan for agriculture put into a framework of the general Swedish employment development. The solution implied a continuous rationalization of agriculture during a period of a generation without danger for our food situation. We could calculate the marginal cost of having more of agriculture than according to that plan, that is to say the real price for security in food supply. But when the politicians got hold of the plan their main interest was the equalization of income between agriculture and the other parts of the population. I will by this example only remind you of the risk that politicians in a free society will be much more interested in income distribution than in long-term planning because of the fact that they to a large extent represent different pressure groups. And this of course is very disturbing for long-term planning. My conclusion is that in order not only to have good plans made up by experts in different fields and consistent planning for the economy as a whole but also will and power to make the plans work, we have either to have

an enlightened dictatorship, if I may say so, or a people very united about the aims of economic policy.

If you have a free society, then you can have people united about the aims of economic policy under the condition that these aims seem predominating to all of us. That was the situation after the war in countries with large rebuilding and reconstruction problems such as Poland and the Netherlands, as Professor Tinbergen said. Then you can get a general understanding of and common interest in the plans. In Sweden the needs have not been urgent to the same extent and therefore we get too much of the income-(and power-) distribution problems about which people cannot easily agree with much room for politicians to play in. However, the Swedish economic policy has not been able to save Sweden from the dollar crisis (partly just because of its emphasis on the income-distribution aspects). Although we may regret it the new alarming difficulties, breeding urgent needs, will perhaps make better prospects for efficient planning in the future.

Mr. Kalecki:

I think that the basic mistake in this discussion is to compare what actually exists to something that has never existed. The present planned or controlled economies are being compared to capitalist economy as it was pictured in textbooks until recent times, where it was endowed with perfect equalization of marginal cost and prices, small degree of unemployment, optimum distribution of resources, and so on and so forth. In fact, it was an apologistic myth of the capitalist economy and nothing of the sort has ever existed. Prices were never equal to marginal cost; unemployment was considerable, and so on. It is from this point of view that the problems of the present discussion should be considered.

Take for instance the problem of the slow response to changes in the pattern of demand in a fully employed economy. It is true that in a normal capitalist economy this response is very quick; but this is due to the fact that usually a significant part of productive equipment and of available labor is unemployed.

Another problem considered in the present discussion was whether it would be possible in a planned economy to achieve equalization between prices and marginal cost. However, it is perfectly clear that in a capitalist economy prices are really not determined that way but by average prime cost and something like the degree of monopoly or by "full cost" principle. Does this assure the optimum distribution of resources? Why should we then complain that under planning or controls there is no exact equalization between prices and marginal cost if this has never been realized?

I should like still to add a word about devastated countries. In consideration of planned and controlled economies in devastated countries, you have to compare them with what would actually have happened if their economies should be "free". An extreme shift to profits from other incomes would take place in present circumstances and the demand for luxury goods would influence to a great extent the pattern of production. Now, however great are the difficulties of planning production in Eastern Europe, I still prefer the priorities established by it to those determined by the demand of black marketeers.

Mr. Zagorski:

The present state of economic planning in Poland, because of the enormous destruction has certain special characteristics. When Professor Lange said that we are in the stage of reconstruction, he did not mean what Professor Perroux tried to suggest, that our aim is to reconstruct all that was before, for it is not. Every decision as to reconstruction is before examiners who know of the changed economic conditions. What Professor Lange had in mind is this, that Poland is in this stage of intra-marginal decision which means every decision is a right one; when the time comes for extra-marginal decisions, and I hope it will be very soon, I hope that our distinguished counsellor will come back to our country and help us to get a most efficient and most rationalized and optimum economy. For such is, in my opinion, the intention of the Polish Government. However, in my personal opinion, I don't think that conditions of the optimum system elaborated by Professor Lerner and Professor Lange are practicable or are just. I don't think you can gather individual cost aggregates into social marginal costs, and I think it is social marginal costs that must be taken into consideration when you try to construct an optimum system.

As to the question whether we shall ever get an optimum system, or whether we shall get an efficient system, of course it is very difficult to answer. We can answer only in a comparative way. It is certain, to me at least, that you cannot get an optimum system in a capitalist economy. In order to get an optimum system in a capitalist economy, you must abolish monopoly profits, and abolishing monopoly profits means, of course, abolishing capitalism. In our country, as far as I understand it, the chief principle is to equate prices to average cost. There are no monopoly profits in Poland worth mentioning, at least in the nationalized sector. And I think, whatever be the ultimate conditions for optimum systems, this principle will get us nearer an optimum system than the capitalist countries.

Mr. Dalal:

The system of prices fixed freely in the markets to equate supply and demand performs, among other functions, that of shifting resources from the uses where they are less urgent to uses where they are more urgent, as reflected in the relative price of the different uses of the limited resources. This function of the price system can be called the dynamic function of prices.

In devastated economies or economies that are fundamentally disequilibriated, the price system fails to perform the dynamic function. It is often observed that even if the prices offered for the particular uses of resources rise, those resources fail to be augmented. The productive or distributive system ceases to be sensitive enough to the changes in prices. Under these circumstances, when the price system no longer performs its dynamic function, there is a logical and economic basis for controls and planning to achieve an optimum allocation of resources.

DE LA THEORIE DES CHOIX AUX BUDGETS DE FAMILLES

par René Roy

Inspecteur Général des Transports, Ministere de Travaux Publics et Transports (France)

La première partie de notre exposé sera consacrée à la présentation tangentielle de l'équilibre du consommateur. Nous ne ferons que résumer ce que nous avons développé, en premier lieu dans une brochure intitulée: *De l'Utilité—Contribution a la théorie des choix.* (Collection des Actualités scientifiques et industrielles, librairie Hermann, 6, rue de la Sorbonne—Paris 1942); en second lieu, dans la revue *Econometrica*, sous le titre "La distribution du revenu entre les divers biens" (Juillet 1947). Nous réserverons la seconds partie de notre exposé à la demande collective, en la considérant comme la somme des demandes individuelles.

I. La Demande Individuelle

A- Equations Tangentielles de l'Equilibre du Consommateur

1°—Pour un consommateur qui dispose du revenu en monnaie r et qui se trouve en présence d'un système de prix $\bar{P}(p_1, p_2, \dots, p_i, \dots, p_n)$ sur lequel il n'a aucune action, l'équilibre, c'est-à-dire le maximum de satisfaction ou d'utilité totale $U(q_1, q_2, \dots, q_i, \dots, q_n)$ est obtenu par le système des n équations ci-après, qui définit les n quantités constituant les composantes du "complexe adapté" $\bar{Q}(q_1, q_2, \dots, q_i, \dots, q_n)$ à la situation des prix $\bar{P}$ et au revenu r:

$$(A_1) \quad \begin{cases} q_1 p_1 + q_2 p_2 + \dots q_i p_i + \dots + q_n p_n = r \text{ (équation de budget),} \\ \dfrac{u_1}{p_1} = \dfrac{u_2}{p_2} = \dots = \dfrac{u_i}{p_i} = \dots = \dfrac{u_n}{p_n} \text{ (proportionnalité des prix} \end{cases}$$

aux utilités finales).

Dans ce système, u_i désigne le degré final d'utilité du bien de rang, $\delta u_i/\delta q_i$; la valeur commune des rapports u_i/p_i est égale au degré final d'utilité de la monnaie w.

2°—En supposant que le système A_1 soit résolu par rapport aux composantes q_i du complexe adapté, l'utilité totale ou "niveau d'existence" du consommateur peut s'écrire:

$$(A_2) \qquad U(q_1, \dots, q_i, \dots, q_n) = U(\bar{Q}) = \Phi(p_1, \dots, p_i, \dots, p_n, r) = \Phi(\bar{P}, r).$$

Dans cette nouvelle conception, l'équation:

$$\Phi(\bar{P}, r) = \text{Constante}$$

n'est autre que l'"equation tangentielle" d'une surface d'indifférence, et l'équilibre du consommateur est défini par le système des n équations:

$$(A_3) \qquad \frac{\varphi_1}{q_1} = \frac{\varphi_2}{q_2} = \ldots = \frac{\varphi_i}{q_i} = \ldots \frac{\varphi_n}{q_n} = -w.$$

Dans ce dernier système, φ_i désigne la dérivée partielle $\partial\Phi/\partial\pi_i$ et w se confond avec la dérivée $\partial\Phi/\partial r$ de cette même fonction Φ par rapport à r. Nous désignerons ce système (A_3) "équations tangentielles de l'équilibre du consommateur" par opposition au système (A_1) qui constitue les "équations ponctuelles." La fonction Φ est homogène par rapport aux prix et au revenu.

3°—Le système (A_3) permet d'obtenir les valeurs explicites des quantités demandées en fonction des n prix p_i et du revenu r qui forment un ensemble de $(n+1)$ variables homogènes. Nous avons ainsi:

$$(A_4) \qquad q_i = -\frac{\varphi_i(P_1, r)}{w(P_1, r)}.$$

Ce sont les équations des demandes individuelles, qui ne contiennent en fait que les n "prix relatifs" x_i répondant à la définition générale:

$$x_i = \frac{p_i}{r}.$$

L'utilité totale peut en effet se mettre sous in forme:

$$U(\bar{Q}) = \Phi(\bar{P}, r) = \Psi(x_1, x_2, \ldots, x_i \ldots, x_n) = \Psi(\bar{X}).$$

Si nous désignons par ψ_i la dérivée partielle $\partial\psi/\partial x_i$ de l'utilité par rapport au prix relatif x_i, nous obtenons pour la demande individuelle du bien de rang i:

$$(A_5) \qquad q_i = \frac{\psi_i(x_1, x_2, \ldots, x_i, \ldots, x_n)}{\Sigma x_i \psi_i (x_1, x_2, \ldots, x_i, x_n)} = \frac{\psi_i(\bar{X})}{\Sigma x_i \psi_i(\bar{X})}.$$

Le "coefficient budgétaire" concernant le bien de rang i a lui-même pour expression:

$$(A_6) \qquad \alpha_i = \frac{p_i q_i}{r} = x_i q_i = \frac{x_i \psi_i}{\Sigma x_i \psi_i}.$$

Cette dernière équation établit un lien entre les éléments observables (α_i, x_i) et les dérivées caractéristiques de la fonction d'utilité $\Psi(\bar{X})$ exprimée au moyen des prix relatifs.

B- Sens et portée de la présentation tangentielle

1°—La présentation tangentielle de l'équilibre du consommateur est plus objective que la présentation ponctuelle, puisqu'elle met en jeu des prix communs à l'ensemble des consommateurs et des revenus individuels susceptibles de caractériser les diverses couches sociales d'une population homogène. Elle donne immédiatement accès aux budgets de famille.

2°—Cette présentation a surtout l'incomparable mérite de permettre la description, non pas seulement d'un consommateur unique, mais d'un groupe qui prend les décisions d'achats par l'intermédiaire d'un représentant soumis à la résultante des sollicitations individuelles. C'est donc là une étape importante vers une conception élargie de la théorie des choix, conception qui tend d'ailleurs à placer le sujet dans son milieu social au lieu de l'en abstraire comme le fait nécessairement la théorie classique.

3°—Fournissant les valeurs explicites des quantités en fonction des prix et du revenu, la présentation tangentielle se prête particulièrement bien à l'analyse des déplacements d'équilibre qui mettent en jeu les coefficients d'élasticité. Voici quelques résultats particulièrement aisés à obtenir avec cette présentation:

a) $$(B_1) \quad \frac{E\,w}{E\,p_i} = -\alpha_i(\varepsilon_i + \mu) \quad \text{en posant} \quad \begin{cases} \varepsilon_i = \dfrac{Eq_i}{Er}, \\ \mu = \dfrac{Ew}{Er}, \end{cases}$$

b) $$(B_2) \quad \alpha_i\,\eta_{ij} + \alpha_i\,\alpha_j\,\varepsilon_i = \alpha_j\,\eta_{ji} + \alpha_i\,\alpha_j\,\varepsilon_j.$$

Cette relation n'est autre que la "condition d'intégrabilité de Slutzky," exprimée en termes d'élasticité avec les significations suivantes:

$$\begin{cases} \eta_{ij} = \dfrac{Eq_i}{Ep_j}, \\ \eta_{ji} = \dfrac{Eq_j}{Ep_i}. \end{cases}$$

c) Nous pouvons également définir "l'indice fonctionnel" des prix z par rapport à un vecteur-prix de base P_0 et pour le revenu r, au moyen des équations ci-après:

$$(B_3) \qquad U(\bar{Q}) = \Psi\frac{(\bar{P})}{r} = \Psi\frac{(z\,\bar{P}_0)}{r} = \Omega(\rho).$$

Dans ce système, ρ désigne le revenu réel r/z du consommateur. Nous établissons ainsi d'une manière absolument rigoureuse que le niveau d'existence du consommateur est défini sans aucune ambiguité par son revenu réel à la condition que celui-ci soit calculé au moyen d'un indice de prix répondant au type fonctionnel.

II. La Demande Collective

C—Notations et Hypothèses

L'objectif que l'on peut s'assigner lorsqu'on aborde la demande collective est de transposer sur le plan macroéconomique les résultats obtenus à l'échelle microéconomique dar la théorie des choix; mais si les prix du marché s'appliquent à la demande collective comme à la demande individuelle, le recours au revenu de la collectivité exige en revanche beaucoup de précautions.

Nous désignerons:

1° par r_k le revenu d'un consommateur envisagé en particulier et par r la somme des revenus individuels ou revenus de la collectivité;

2° par Q_i la demande collective du bien de rang i;

3° par β_i les coefficients de répartition des demandes individuelles:

$$\beta_i = \frac{q_i}{Q_i} = \frac{q_i\, p_i}{Q_i\, p_i}.$$

4° L'hypothèse fondamentale que nous prenons pour base du phénomène collectif il est l'existence d'une loi de distribution des revenus individuels qui reste permanente et qui, par conséquent, n'est pas modifiée quand le revenu de cette collectivité varie. Cette hypothèse revient à considérer que nous avons constamment:

$$(\mathrm{C}_1) \qquad \theta_k = \frac{r_k}{r} = \text{constante,} \qquad \text{c'est-à-dire,} \qquad \frac{dr}{r} = \frac{dr_k}{r_k}.$$

Nous pourrions à la rigueur nous passer de cette hypothèse particulièrement simple en reliant la variation de tout revenu individuel r_k à celle du revenu collectif r par l'intermédiaire d'un coefficient d'élasticité λ_k ; nous aurions ainsi:

$$\frac{dr_k}{r_k} = \lambda_k \frac{dr}{r}.$$

Mais l'introduction de ces élasticités λ_k exigerait que nous en connaissions les ordres de grandeur pour les diverses couches sociales, afin que leur emploi dans les formules concernant la demande collective ne se borne pas à une simple présentation théorique. S'il ne paraît pas impossible de connaître ces ordres de grandeur pour les couches fondamentales de la population, nous n'en disposons pas actuellement. Notre hypothèse peut d'ailleurs aussi se justifier par des observations portant aussi bien sur des situations monétaires stables que sur des situations instables.

D—Les variations de la demande collective dans leurs rapports avec celles des demandes individuelles.

1°— La variation relative de la demande individuelle du bien de rang i s'exprime ainsi

$$(D_1) \qquad \frac{dq_i}{q_i} = \Sigma_j\, \eta_{ij}\, \frac{dp_j}{p_j} + E_i \frac{dr_k}{r_k}.$$

2°— Si nous considérons la demande collective Q_i, sa variation relative a pour expression:

$$(D_2) \qquad \frac{dQ_i}{Q_i} = \Sigma_k\, \beta_i\, \frac{dq_i}{q_i} = \Sigma_k\, \Sigma_j\, \beta_i\, \eta_{ij}\, \frac{dp_j}{p_j} + \Sigma_k\, \beta_i\, E_i\, \frac{dr_k}{r_k}$$

3°— Considérons dans l'équation précédente les térmes du dernier membre, qui proviennent des variations de prix. Si nous sommons par rapport à l'ensemble des consommateurs, nous pouvons poser:

$$(D_3) \qquad H_{ij} = \Sigma_k\, \beta_i\, \eta_{ij}.$$

Le coefficient d'élasticité H_{ij} de la demande collective du bien de rang i par rapport au prix p_j apparait ainsi comme une moyenne pondérée des élasticités partielles η_{ij} en prenant pour coefficient de pondération les coefficients β_i définis ci-dessus.

En tenant compte de la permanence admise pour la distribution des revenus individuels, puis en posant:

$$(D_4) \qquad E_i = \Sigma_k\, \beta_i\, \varepsilon_i$$

nous aboutissons en définitive à l'expression générale:

$$(D_5) \qquad \frac{dQ_i}{Q_i} = \Sigma_j\, H_{ij}\, \frac{dp_j}{p_j} + E_i\, \frac{dr}{r}.$$

Dans cette expression, les coefficients d'élasticité H_{ij} et E_i ne dépendent que des prix p_j et du revenu global r, en supposant connus et invariables les coefficients θ_k les coefficients β_i résultant alors eux-mêmes de ces données.

L'équation (D_5) montre que la demande collective Q_i est une fonction des prix et dù revenu de la collectivité dans l'hypothèse faite précédemment. Nous avons ainsi

$$(D_6) \qquad Q_i \equiv Q_i\, (p_1, p_2, \ldots, p_j, \ldots, p_n, r) \equiv Q_i\, (\bar{P}, r).$$

Nous retrouvons donc la même forme que pour chaque demande individuelle; nous pouvons de plus observer que la demande collective est

homogène et de degré 0 par rapport à l'ensemble des variables $(\bar{P}, r)$. En d'autres termes, la demande collective ne dépend que des prix relatifs; cette propriété tient au fait que la distribution des revenus individuels est permanente et, par conséquent, le revenu de la collectivité varie dans la même proportion que chaque revenu individuel.

E—Relation entre les elasticités de la demande collective

1°— Pour le revenu de la collectivité, nous avons l'équation de budget:

$$(E_1) \qquad r = \Sigma_i Q_i p_i$$

2°— Nous pouvons définir également des coefficients budgétaires afférents aux divers biens de consommation, soit:

$$(E_2) \qquad A'_i = \frac{Q_i p_i}{r}.$$

Un tel coefficient global A peut être considéré comme une moyenne pondérée des coefficients individuels relatifs au même bien; nous avons en effet:

$$(E_3) \qquad A_i = \frac{Q_i p_i}{r} = \frac{\Sigma_k q_i p_i}{\frac{r_k}{\theta_k}} = \Sigma_k \theta_k \frac{q_i p_i}{r_k} = \Sigma_k \theta_k \alpha_i .$$

Il convient en outre d'observer qu'entre les coefficients β_i et θ_k s'établit la relation:

$$\beta_i A_i = \theta_k \alpha_i$$

Cette relation, valable pour un consommateur et pour un bien déterminé, s'écrit d'une manière générale:

$$\beta_k = \theta_k \frac{\alpha_k}{A_k}.$$

3°— L'équation collective de budget, conjugée avec l'homogénéité de la demande collective par rapport aux prix et au revenu global, permet d'étendre aux élasticités de la demande collective les trois groupes d'équations qui s'appliquent aux demandes individuelles. Nous avons ainsi:

$$(E_4) \qquad \begin{cases} \Sigma_i A_i E_i = 1, \\ \Sigma_i A_i H_{ij} + A_j = 0, \\ \Sigma_j H_{ij} + E_i = 0. \end{cases}$$

Ces dernières relations présentent manifestement de l'intérêt pour les applications pratiques: si nous considérons par exemple le cas simple de deux biens (alimentation et ensemble des biens de consommation),

isil exte un nombre relativement peu élevé de coefficients d'élasticité, liés par un nombre également restreint d'équations qui permettent de calculer certains de ces coefficients en fonction du coefficient budgétaire afférent à l'un des biens et de deux autres coefficients d'élasticité ($H_{1,1}$ et $H_{2,2}$, pour fixer les idées).

4°— L'extension à la demande collective de la condition d'intégrabilité de Slutzky (B_2) n'est possible que dans des cas très particuliers, nous ne faisons que mentionner ici celui où les élasticités de revenu étant toutes égales à l'unité, la condition de Slutzky se réduit à la condition d'Hotelling, c'est-à-dire pour la demande individuelle:

$$\frac{\partial q_i}{\partial p_j} = \frac{\partial q_j}{\partial p_i}.$$

Cette condition, étendue par sommation à la demande collective, s'écrit en termes d'élasticité:

$$(E_5) \qquad A_i H_{ij} = A_j H_{ji}.$$

Si cette condition est vérifiée, il existe une fonction d'utilité collective; mais l'hypothèse exprimée par le fait que les élasticités de revenu sont toutes égales à l'unité, aussi bien pour la demande collective que pour les demandes individuelles, revient à supposer que les sentiers d'expansion se confondent avec des droites issues de l'origine et que par conséquent, pour un système de prix donné, les accroissements de revenu sont également répartis entre les divers biens, situation exceptionnelle ou du moins assez rare.

Quant au sens qu'il faut attribuer à la fonction d'utilité collective, c'est là une question que nous ne saurions traiter ici, bornons-nous à observer qu'il ne faut pas oublier les deux hypothèses qui servent de support à ce cas très particulier: a) les revenus individuels varient tous proportionnellement au revenu global; b) les élasticités de revenu, pour les demandes collectives comme pour la demande globale, sont toutes égales à l'unité.

Résumé

I. The Individual Demand

A. Tangential Equations of Consumer Equilibrium. 1. For a consumer having at his disposal a money income r and finding himself confronted with a price system $\bar{P}(p_1, \dots, p_i, \dots, p_n)$ the equilibrium, *viz.*, the maximum of aggregate utility $U(q_1, \dots, q_i, \dots, q_n)$ is obtainable by

means of the n-equation system (A_1) which defines the n quantities constituting the components of the "adapted complex" $Q(q_1, \ldots, q_i, \ldots, q_n)$.

2. The aggregate utility for the consumer, or his standard of living, may also be formulated as (A_2), and the consumer equilibrium is then defined by the n-equation system (A_3). We describe the system (A_3) as "tangential equations of consumer equilibrium" as opposed to system (A_2) which constitutes the "punctual equations."

3. The system (A_3) enables us to obtain the explicit values of the quantities demanded as functions of the n prices p_i and of income r which form an aggregate of $(n+1)$ homogeneous variables (A_4), equations of individual demand which involve only relative prices $x_i = p_i/r$ and may be written as (A_5). The "budget coefficient concerning the commodity of rank i expresses itself as (A_6).

B. Meaning and Significance of the Tangential Presentation. 1. The tangential presentation of consumer equilibrium is more objective than the punctual plotting, since it elicits prices that are common to the whole community of consumers and individual incomes likely to characterize the various social classes of an homogeneous population. It gives immediate access to family budgets by establishing a relation between the coefficients q_i and the derivatives of the utility function. Lastly, it enables us to describe the behavior not only of a single consumer, but also of a group that decides on its purchases through the medium of a representative subject to the resultant of individual wishes.

2. Since it provides the explicit values of the quantities as functions of prices and incomes, the tangential presentation lends itself remarkably well to the analysis of such equilibrium shiftings as bring the elasticity coefficients into action. [See (B_1) and (B_2).] Definition of price index z and real income ρ characterizing the standard of living of the consumer is given in (B_3).

II. The Collective Demand

Assuming the law of distribution of individual incomes to be independent of the aggregate income of the population, it becomes possible to define for each commodity a law of collective demand as related to prices and total income. The changes in collective demand are themselves defined by means of elasticity coefficients which may be computed from individual coefficients.

The condition of integrability can likewise apply to collective demand, and proves correct in particular when individual elasticities of income are each and all equal to unity. In this very special case, one can define a function of collective utility which is nothing else than the real income of the population; this real income itself can be computed as a function of the individual incomes which characterize the various standards of living.

Discussion

Professor Wold:

Professor R o y' s presentation shows us that the classical theory of choice can be built up in a very nice structure, if we base ourselves on the notion of utility functions, or alternatively, as done by P a r e t o, on the notion of *indifference maps.* While these two approaches are mathematically equivalent, the latter has the advantage that the notion of measurable utility does not enter. More recently, however, two other approaches have been suggested; these are closely related to the former, but not quite equivalent. On the one hand, we have the H i c k s-A l l e n theory. This is based on the notion of *marginal substitution rates*, say $R_i\ (q_1, \ldots, q_n)$; its fundamental equations read[1]

$$\frac{R_1(q_1, \ldots, q_n)}{p_1} = \ldots = \frac{R_n(q_1, \ldots, q_n)}{p_n}; \quad p_1q_1 + \ldots + p_nq_n = \mu.$$

On the other hand, following C a s s e l, the theory may be based on the notion of *demand function*, say $Q_i\ (\mu, p_1, \ldots, p_n)$; these are here interpreted as referring to an individual consumer.

The theories based on marginal substitution and demand functions, respectively, stand out as more general, formally and logically, than the classical theory. In fact, in the Hicks-Allen and Cassel theories it may occur that the hypothetical behavior of the consumer is not compatible with the existence of an indifference map, the fundamental notion in the classical theory.

Examining the difference between the three theories, I asked myself a few years ago whether difference is really essential, and whether it might be possible to form a unified theory.[2] The pertinent point in the problem lies of course in the so-called integrability condition. In the marginal-substitution theory the integrability condition takes the form indicated by Professor Roy [eqs.(B_1),(B_2)]. In the demand-function approach, the integrability condition takes the form indicated by H. H o t e l l i n g. In each approach, as is well known, the hypothetical set-up is compatible with an indifference map if and only if the integrability condition is fulfilled. Now examining the logical foundations of the two approaches, it was possible to show that they are not more general than the indifference-map approach. In fact, if we only accept a few simple axioms (for example: the quantities $q_1, \ldots, q_n$ bought by the consumer at given prices $p_1, \ldots, p_n$ and given income μ form a

[1] The notations are those used in my paper "Synthesis of Pure Demand Analysis, I-III," *Skandinavisk Aktuarietidskrift*, 1943-44.

[2] See the paper already referred to, also for further references.

budget which he prefers to any other budget which he can afford) it turns out that the integrability condition must be fulfilled lest the hypothetical set-up be self-contradictory. The marginal-substitution and the demand-function theories, accordingly, are not complete without the integrability condition; as soon as this is accepted, however, they become equivalent to the indifference-map theory. In other words, the theory of consumer's choice has been unified, and the three approaches are merely three equivalent ways of formulating the theory. Each approach includes the two other ones.

May I add that I went further in the analysis, taking my starting point in equations precisely the same as Professor Roy's fundamental equations. Now there seems to be a considerable overlapping of our findings , but I shall not go further into detail in that respect, since the results seem to be so very similar.

Professor Hotelling:

The theory concerning which Professor R o y has spoken is more widely applicable than might be thought. Preference and demand functions and consumers' surpluses are commonly understood to refer to people's actual preferences and choices. Sometimes people do not make their choices rationally and consistently. There are situations in which the state, or a consumers' cooperative, or a consumers' information, research, and testing bureau can tell us what to consume better than we can judge independently. Thus in wartime a national food controller has at his disposal much information about the science of nutrition. He can plan the number of calories and the number of vitamin units of each kind that are needed, and can apportion them among the people in a way that will give them better health and welfare than as if each chose freely for himself.

If we are to consider such systems of planning and allocation, or the consumers' choices that would result from improved information on their part, then it is appropriate to take as utility or preference functions something based not on what consumers have been observed to do, nor yet on what they say when asked what they will do, but rather on what they ought to do if they were entirely rational and well-informed. It is worth noticing that the whole beautiful theory described by Professor Roy is equally applicable to this case.

This fact has a bearing on national planning schemes. Modernized measures of consumers' and producers' surplus, taking account of relations between commodities and expressed as line integrals instead of by areas between simple demand and supply curves for isolated commodities, may well be used as criteria as to whether this or that public investment will give the public the better return.

People will always insist on retaining wide freedom of choice, but they may well subordinate this in large areas of expenditure to group buying and guidance by central organizations, just as the various departments and foremen of a large business concern are not allowed to buy their supplies independently but must get them through a central purchasing agency that has much power to choose among different brands and types of goods, and may in the interest of over-all economy change the specifications sent in by individual departments for the articles they wish to buy. Furthermore, extensive areas exist suitable for public and joint expenditure, including the provision of roads, parks and many other things whose benefits to the public are far greater when free than when sold. For example it is better that roads and bridges be free of tolls and their location be decided by a central planning authority than that their provision be left to profit-seeking private companies. The proper criterion for the location of a bridge may be expressed as a line integral derived from technical and preference functions of the kinds considered in mathematical economics, and leads to decisions different from those designed to maximize toll-gate receipts minus costs.

Professor Koopmans:

I should like to raise a question regarding the remarkable result Professor R o y has achieved in this field of aggregation. It seems to me that his hypothesis that all incomes vary proportionally introduces a problem that has also troubled us in the work of the Cowles Commission. The situation, it seems to me, is somewhat as follows: Mr. Roy appealed to historical experience in support of his hypothesis; he has noted that in a certain period of time all incomes moved upward or downward in an approximately proportional manner. From such an observation one may deduce a certain hypothesis, go into an analysis, and draw certain interesting conclusions from it, which may then be applied in economic policy, or influence in some way the action taken by the government. It might then happen that as a result of the political action that is taken, the hypothesis might no longer be true. Could Mr. Roy give us his opinion on this particular aspect of his hypothesis? What were the reasons that this proportionailty of income was present during a given historical period? Can we expect this proportionality of income to remain in force in the future, and under what conditions?

Professor Roy:

I would in the first place like to answer Professor H o t e l l i n g who considered the case in which collectivity might be substituted for the individual in the making of certain decisions. I do not in any way question the legitimacy for those who represent the public power to

make decisions as regards the common welfare and the general interest. Naturally all the difference will remain if we are to differentiate between general interest and what we consider particular individual interest. As we differentiate between these two interests, arguments which have taken place here as regards the various economic doctrines have actually referred to this differentiation between general and private interest. I therefore will not come back to this question. Whereas I personally follow the microeconomic school in the field of consumption, I feel that the collectivity, the public power, has certain duties on which it alone can make decisions. Consequently, we can certainly admit that those who represent the public, administrators, judges, officials, etc., might in certain cases be called to make decisions for the collectivity of the whole. And thus they would be invested with a type of power that might give rise to a sort of field of choice that was particular to them in view of their particular functions. They would make these decisions which are optimum decisions in the general meaning of the term. Outside of these specific cases, once more, we must come back to the individual decisions as regards consumption.

As regards the question raised by Professor Koopmans I would answer that I am very happy that he agreed with me as to the very simple hypothesis which I made as to the permanency of the relationship between individual and aggregate income. In fact, we noted that this was frequently an acceptable hypothesis. But we can continue to apply the consequences of this hypothesis without having ascertained that it will always be a true hypothesis. This is a risk which is met in all countries in all questions that are based on a hypothesis. Only empirical observation can tell us whether the hypothesis remains valid. If I understood Mr. Koopmans correctly, he would wish us to confine ourselves not only to observation, but also to predictions; consequently, his question would be that of knowing whether in the light of past experience we could presuppose this quasi-permanence of the relationship between individual and aggregate income. In the past experience, that is to say, in the period before 1914, we did note a certain harmony between the increase of income of the various strata of the population. Can we in the future maintain this same structure in the distribution of income? This is not certain. It depends in various countries on the political concepts that will decide social legislation in those countries. It is quite certain that in many countries social legislation will lead to a certain amount of redistribution of individual income. I thought out what was necessary for income tax; this is a tax which would modify the distribution of income when the aggregate income increases. I think that consequently we might substitute for the simple hypothesis another hypothesis, a more complex one. This latter hypothesis

would be in relationship with the predictions that may be made as to the direction in which the income will be modified. I think that we might find an opinion as to the tendency, the direction in which an increase of certain percentage of the aggregate income would influence the income in various strata of the population. We could divide the population into strata, to study in each stratum the influence of an increase in the aggregate income that would not be the same in its relation to the various strata of the population. In the specific cases, naturally, we base our hypothesis on the influence of the modification of the aggregate income on the income of the different strata of population. With this hypothesis we can deal with problems of the microeconomic type if we considered a homogeneous subpopulation.

LES CHOIX ECONOMIQUES DANS UN MONDE ALEATOIRE ET LA NOTION D'ESPERANCE MARGINALE

par M. Massé*

Directeur de l'Equipement a l'Electricité de France

Résumé

Dans un monde déterminé chaque décision économique apporte une satisfaction définie U, le choix optimum correspondant évidemment à la maximation de U.

Dans un monde aléatoire, à chaque décision économique est attaché un ensemble de satisfactions éventuelles $U_1, U_2, \ldots, U_n$, de probabilités respectives $p_1, p_2, \ldots, p_n$. Nous postulons que le choix optimum correspond à la maximation d'une certaine fonction à définir $S(U_1, U_2, \ldots, U_n; p_1, p_2, \ldots, p_n)$.

Dans les jeux de hasard, S est l'espérance mathématique des gains éventuels; dans la vie économique, S sera en première approximation l'espérance mathématique des satisfactions éventuelles, la dissymètrie économique des gains et des pertes conduisant à la notion de prime de risque et expliquant certains paradoxes de M. Emile Borel.

En seconde approximation, on pourra prendre pour S une combinaison linéaire de l'espérance mathématique et de l'écart quadratique moyen des satisfactions éventuelles. Enfin, dans les cas particuliers où des éventualités désastreuses sont à craindre, la maximation de l'espérance peut être primée par la limitation du risque.

Chaque entité du système économique dispose à un instant donné d'un certain nombre de flux et de stocks.

A l'utilisation des premiers est attachée une satisfaction totale U et à la possession des seconds une espérance totale S.

L'optimum correspond alors à un certain nombre d'équilibres marginaux entre satisfactions et espérances.

Nous étudions particulièrement le cas où les flux successifs extraits d'un stock rendent des services indépendants et celui où ils rendent des services complémentaires.

Ces principes peuvent être appliqués au jeu des réservoirs hydroélectriques et conduire à des règles d'exploitation et d'équipement précises.

Ils peuvent également servir à la discussion de la *Théorie générale* de J. M. Keynes, spécialement en ce qui concerne le taux de l'intérêt.

* The author was not present to read this paper.

On a en effet:

$$\frac{\text{utilité marginale de la consommation}}{\text{prix de la consommation}}$$
$$= \text{espérance marginale de la monnaie}$$
$$= \text{espérance marginale des créances}$$
$$= \frac{\text{espérance marginale de l'investissement}}{\text{prix de l'investissement}}.$$

Ces égalités peuvent d'ailleurs s'exprimer sous la forme équivalente suivante.

taux marginal de la "time preference"
= prime marginale de liquidité
= taux de l'intérêt
= efficacité marginale de l'investissement.

Ainsi à l'équilibre le taux de l'intérêt est *à la fois* la rémunération de l'abstinence (Marshall) et la récompense de la renonciation à la liquidité (J. M. Keynes). On fait ainsi la synthèse des théories "capitalistiques" et des théories "monétaires" de l'intérêt (M. Allais).

Résumé

In a determined world, each economic decision brings a definite satisfaction U. The best choice evidently corresponds to the maximum value of U.

In a world subject to uncertainty, to each economic decision corresponds a whole set of possible satisfactions $U_1, U_2, \ldots, U_n$ with, respectively, the probabilities $p_1, p_2, \ldots, p_n$.

We assume that the best choice corresponds to the maximum value of a function $S(U_1, U_2, \ldots, U_n; p_1, p_2, \ldots, p_n)$ which is to be defined.

In games of chance, S is the probable value (mathematical expectation) of gains; in economic life, S will be, in first approximation, the probable value of satisfactions. The economic asymmetry between gains and losses leads to the idea of risk premium, and explains some of Mr. Emile Borel's paradoxes.

In second approximation, it will be possible to represent S by a linear combination of the probable value with the dispersion of satisfactions. Finally, when special and disastrous events are to be feared, the limitation of the risk outweighs the maximization of the probable value.

Each unit of the economic system has at its disposal a certain number of flows and stocks.

To the use of the flows is attached a total satisfaction U, and to the possession of stocks a total "*espérance*" S.

The best choice corresponds then to several marginal equilibrium between satisfactions and "*espérances.*"

In particular, we study the following cases:

The successive flows extracted from a stock are used for independent services;

The successive flows are used for complementary services.

These principles can be used to study the hydro-electric storage tanks and help to establish precise rules concerning operation and equipment.

They can also be used to discuss Mr. J. M. Keynes's *General Theory*, especially with regard to the interest rate.

In fact, we can say:

$$\frac{\text{Marginal utility of consumption}}{\text{consumption cost}} = \textit{espérance marginale} \text{ of money}$$
$$= \textit{espérance marginale} \text{ of loans}$$
$$= \frac{\textit{espérance marginale} \text{ of investment}}{\text{investment cost}} .$$

These equalities can be written under the following equivalent form:

Marginal rate of time preference = marginal liquidity premium
= interest rate
= marginal efficiency of investment.

Thus, at equilibrium, the interest rate is *simultaneously* the remuneration for abstinence (Marshall) and the reward for liquidity renunciation (J. M. Keynes).

A synthesis between "capitalistic" theories and "monetary" theories of interest (Mr. Allais) is thus achieved.

MEASUREMENT OF NATIONAL INCOME

Monday, September 15, at 9:30 a.m.

CHAIRMAN:

Herbert Marshall
Dominion Statistician (Canada)

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RECENT EXPERIMENTS IN SOCIAL ACCOUNTING: FLEXIBLE AND DYNAMIC BUDGETS

by J. Tinbergen
Director, Central Planning Bureau (The Netherlands)

and J. B. D. Derksen
Statistical Office, Department of Economic Affairs, United Nations

1. INTRODUCTION

In recent years increasing use has been made of systems of social accounts, and a variety of new types and new forms of presentation have been suggested.

The general features of such systems are:

(a) the division of an economy into a number of groups or economic entities, mostly themselves a complex of elementary units, and

(b) a summing up in the form of accounts of the transactions of various kinds between these economic entities.

As a rule, the economy of one country is investigated, implying that all other countries are regarded as one single group. The economy considered may be split up into a number of separate groups. It is customary to distinguish between consumers' households, business enterprises, and the government sector. If special attention is to be given to banking problems for example, banks will constitute a separate group.

Transactions are in a double sense two-sided. There are always two parties (groups) involved, the "paying" and the "receiving." At the same time, as a rule, one thing is being exchanged against another: goods

against money or financial claims against money. The first two-sidedness is reflected in the double entry of every item in the accounts; the second may, or at least should, be reflected in an exact statement of whether the money or the counterflow is the subject of the system. For gifts and similar "unilateral" transactions the latter two-sidedness does not exist.

Where the social accounts are used for purposes of planning, the system of tables relates to a future period. In such cases it seems appropriate to speak of a national "budget" instead of national (or social) accounts.

Transactions may be of different kinds, dependent on prevailing circumstances such as the forms of economic policy adopted. Consequently, there arise many alternative ways of constructing systems of social accounts. A first condition is that it should always be clearly stated on what definitions and principles a system of social accounts or a national budget is based.

2. Circumstances or Data Assumed

The figures in any system of accounts are determined by what are sometimes called the basic "data" of an economy such as the natural, technological, and institutional circumstances of the country involved. Examples are the weather conditions, the state of technology, and the tax system. Some of these "data" may be subject to decisions by the government or other powers, and may therefore not be known beforehand. It may be advisable to construct alternative budgets based on a number of different assumptions with respect to future government policies and other unknown data. The systematic survey of alternatives that is thus obtained is one of the main advantages of the social-accounting technique.

3. Flexible Budgets

In such cases it will be very useful to construct "flexible budgets" indicating in a systematic way how the figures depend on alternative assumptions.

The results may be represented in a rather primitive way by utilizing a number of columns, each of which corresponds to a given set of assumptions regarding the data. A technically more advanced technique would show not merely a set of columns but functional relationships. These functions would show the various items of the budget as depending on the values of certain "data." When choosing this latter representation, we are already midway between the numerical social accounts and what the econometricians call a system of equations.

In order to set up a flexible budget we need, for example, to know the relation between: (i) income and tax receipts; (ii) production and imports; (iii) income and consumption; etc.

Such relations may either be known beforehand as is usually the case with (i) and to some extent with (ii), or they may be taken from multiple-correlation analyses of time series, as in the case of (iii) and perhaps also (ii). In a war or postwar economy some of the relations may perhaps no longer be valid, since the government may, for example by rationing, simply prescribe a certain consumption level. The relationships prevailing in a free economy can, however, never fully be neglected. They may be essential in determining what reactions may be expected from the introduction of a government measure.

Examples of flexible budgets may be seen in the budgets for the postwar period prepared in Holland during the occupation, when it was not yet possible to foresee the exact numerical values of capital losses and hence labor productivity, world price levels, etc.

Another example is provided by the following comparisons between the Netherlands Central Economic Plan for 1947 and some alternatives considered in the First Memorandum on the Central Economic Plan 1947.[1]

TABLE 1

Alternatives in the Netherlands Central Economic Plan, 1947
(in billions of guilders)

	Plan	Alternative			Plan	Alternative	
		1	2			1	2
Net national product at factor cost	8.3	8.0	8.3	Consumption[a]	6.2	6.1	6.2
Reparations and restitution	0.1	0.1	0.1	Net capital formation	1.7	1.6	1.9
Foreign credits and sale of foreign assets	1.4	1.5	1.6	Public authorities[b]	2.7	2.7	2.7
				Corrections[c]	–0.8	–0.8	–0.8
Total available means	9.8	9.6	10.0	Total requirements	9.8	9.6	10.0

a Personal expenditure on consumers' goods and services.
b Expenditure of public authorities less transferred incomes.
c Subsidies less indirect taxes and similar items.

Alternative assumptions may be of two different kinds:

1. They may refer to unknown variables that are more or less outside direct government control, such as, for example, the future increase in labor productivity.

[1] Centraal Planbureau i.o., *Centraal Economisch Plan, Eerste Nota (Globaal Plan)* 1947, The Hague, September 1946.

2. They may refer to unknown variables that at least to a certain extent may be influenced by government policies, such as the amount of foreign assets to be sold abroad.

Alternative 1 in Table 1 is based on an assumed lower productivity of labor, and consequently a higher deficit of the balance of trade. Alternative 2 is based on higher imports of capital combined with the same productivity as adopted in the plan.

4. Distinction of Items According to Type of Money

In countries such as Holland a distinction must nowadays be made between the kinds of money needed for various types of payment. It makes an important difference whether certain imports have to be paid for in dollars, in sterling, or in "soft currency." To the individual, it makes a difference whether or not certain obligations such as taxes may be paid in blocked money or in free money only. This may be the basis for separate accounts in the system.

5. Choice of Price Level: Black Budgets

It is well known that the various commodity flows represented in a system of social accounts may be valued in different ways, *e.g.*, at factor cost or at market prices. The method of valuation sometimes influences in a rather unexpected way the results obtained by further computations. If, for example, net capital formation of a country is to be calculated, it makes a difference whether home investments are valued at factor cost or at market prices. Since foreign credits—which have to be subtracted to arrive at net capital formation—can only be estimated in one way, the result is sometimes very sensitive to the method chosen.[2]

Another example showing how the choice of price levels may influence the results is presented by black-market transactions. For the Netherlands it was attempted to estimate and present in the shape of an additional system of accounts these illegal transactions, an example of which is given in Table 2; for comparison Table 3 on legal transactions is added.

Black-market transactions as here understood may be of two different kinds, *viz.*: (i) the charging (by dealers) of higher prices than those legally permitted; and (ii) the selling of quantities illegally obtained or illegally withheld and not reported.

[2] *Cf.* for a discussion of this and a similar problem: *Report of the Subcommittee on National Income Statistics*, Appendix: Definition and Measurement of the National Income and Related Totals, Memorandum submitted by J. R. N. Stone (published by the United Nations), p. 39.

This applies to transactions in commodities and services. There may also exist a black market for services rendered by factors of production.

From Table 2 it appears that current black receipts of family households just seem to balance current black expenditures.[3] This is of course not necessarily the case; there might have been a net black saving or dissaving.

6. The Nature of Accounts

The principles that are the basis for the social accounts will vary in accordance with the purposes for which the figures are to be used. Often a distinction is made between cash accounts and operating accounts. Cash accounts show actual receipts and expenditures, whether they refer to transactions that took place in the period considered or not. Operating accounts reflect transactions, whether they have actually been paid for or will be paid for later.[4]

In a stationary economy there would be no difference between the two types of accounts. In a changing world, however, there is a need for making this distinction, and there may be use for other systems as well. It all depends on the problems one is dealing with. The problems of postwar reconstruction and the problems arising in connection with a full-employment policy may require the use of a third system of accounts based on orders. An example for the Netherlands Government is shown in Table 4. The relationship between the various items is further shown in Table 5. The numbers of the columns in Table 4 refer to those in Table 5. These very concise tables are examples of "dynamic budgets." They are characterized by constant or variable lags between orders, actual transactions, and payments. If lags were constant the figures in the order accounts would be repeated in future operating and cash accounts. Any discrepancies between deliveries and orders would point to changes in the volume of trade, or, if one prefers to formulate it that way, to hoardings or dishoardings by importers or foreign buyers. Similar systems of accounts may be set up for consumers'

[3] For a few other countries, estimates of income from black-market transactions have also been made. *Cf.* Dr. P. Kiranoff, *Le revenu national en Bulgarie*, Sofia, 1946; *Estimation du Revenu National Francais*, 1947, Commisariat Général du Plan de Modernisation et d'Equipement; P. J. Loftus, *National Income of Palestine, 1944*, Palestine, 1946.

[4] The definition of national income now used in most countries is, as a rule, based on the concepts of the operating account, *i.e.*, income is included when the productive service is rendered, not when the income is paid out. Exceptions are the estimates for agricultural income and income from trade, which for statistical reasons must sometimes be based on income paid out.

households and business enterprises. The dynamic accounts may be a useful instrument too for business-cycle research.

It will be clear from the foregoing that the balancing item in Table 4 has a different meaning for each of the three accounts. The amount of credit actually needed in the period under consideration is indicated by a footnote. The figure mentioned in column 1 indicates future credits needed, and that in column 7 the deficit of the operational account.

Relationships become more complicated when lags are variable. Even then, the dynamic accounts shown may be useful. For other purposes it may be desirable to show the order figures in a different manner, for example grouped by periods of delivery.

7. Practical Use of Complicated Systems

Some of the systems mentioned could become very intricate if they were worked out in detail. The question arises whether such complicated systems are expedient for practical economic analysis. Would it not be simpler to use time series for strategic variables when studying the problems of business-cycle policy? The authors are inclined to favor the latter approach. Nevertheless, the social accounts have certain methodological advantages which are not shared by any other method.

The amount of information that may be derived from a system is determined by the number of separate economic entities which are recognized as keeping distinct accounts. A system consisting of only two groups would constitute an oversimplification, as it could show only two money flows: from group 1 to group 2, or vice versa.

The number of "branch flows" between each pair of groups of economic entities increases rapidly with the number of accounts distinguished. The social accounts offer a convenient method to detect the interrelationships between various aggregates generally used, such as national income at factor cost, total government expenditure, gross capital formation, and consumers' expenditure, etc.

It is well known that national income may be defined as the aggregate of all income payments, as net national output, or as net national expenditure. These three approaches do not exhaust all information included in the social accounts. In general, in a complete system of n accounts there will be n identical aggregates for the national income.[5]

[5] *Cf. Report of the Sub-Committee on National Income Statistics of the League of Nations Committee of Statistical Experts*, p. 38.

As the items appearing in the social accounts will to a certain extent have to be based on estimates, the best estimate for the national income is obtained by making use of the information provided by all n aggregates.

The social accounts permit a systematic enumeration of all relevant money flows. This is an advantage of the method that becomes particularly important in all more complicated cases. The treatment of the problems of the credit ("money" and "capital") markets may serve as an example. In order to clarify the manner in which the interest rate may be influenced by the interplay of savings and investments and by the dealings in "old" bonds (by which are meant bonds issued in previous periods) we may draw up a list of all items concerned, *i.e.*, all amounts available for investment and all amounts needed for investment. Systems of social accounts are particularly useful as a tool in the analysis of complicated structures.

Only recently some attention has been given to the problems that arise if actual accounts must be compared with forecasts or plans made earlier. It is very important to present the results in such a way that it is easily seen in what respects the actual development differs from the forecasts. Discrepancies may be due to such factors as unforeseen developments in price levels abroad, a lower savings ratio than was assumed, etc.

TABLE 2

Illegal Operating Account for Family Households

(in millions of guilders)

Expenditures

Item	Reference	Description	Amount
		Consumption of commodities supplied by:	
108Z	126Z	families (ration tickets and coupons)	100
102Z	321Z	business enterprises	700
104Z	521Z	foreign countries	100
119Z	139Z	Total	900

Receipts

Item	Reference	Description	Amount
126Z	108Z	Sales of ration tickets, etc.	100
122Z	305Z	Remuneration of productive agents by business enterprises:	
		a. wages	450
		b. employers' income	350
139Z	119Z	Total	900

TABLE 3

National Budget, The Netherlands, 1947

Family Households

Operating Account

(in millions of guilders)

Expenditures

Item	Reference	Description		Amount
		Consumption of commodities and services supplied by:		
101	221	collective income recipients		105
102	321	business enterprises		7513
103	421	public authorities		94
104	521	foreign countries		114
105	425	Taxes		1160
106	221	Premiums to collective income recipients:		
		total	285	
		deduct item 101	105	
				180
107	621	Savings		163
119	139	Total		9329

Receipts

Item	Reference	Description		Amount
		Remuneration of productive agents by:		
121	205	collective income recipients		75
122	305	business enterprises		6950
123	410	public authorities:		
		a. wages	1116	
		b. interest	176	
				1292
		Transferred incomes of:		
124	201	collective income recipients		348
125	404	public authorities:		
		a. interest war debt	122	
		b. pensions and benefits	542	
				664
139	119	Total		9329

TABLE 4

An Example of Handling Dynamic Problems

(in billions of guilders)

Account	Order	Cash	Operating	Account	Order	Cash	Operating
	Orders	To be paid	To be delivered		Orders	To be paid	To be delivered
	1	4	7		1	4	7
Imports	3.0	3.1	3.2	Exports Balance	1.6 1.4	1.7 1.4 [a]	1.7 1.5

[a] Actual credit needed.

TABLE 5

The Relation Between Order, Cash and Operating Figures For Imports

(in billions of guilders)

To be ordered	Imports of 1946 to be paid for in 1947	Imports of 1947 to be paid for afterwards	To pay in 1947 $=1+2-3$	To import in 1947 and ordered in 1946	Ordered in 1947 to be imported afterwards	Imports in 1947 $=1\pm5-6$
1	2	3	4	5	6	7
3.0	0.8	0.7	3.1	1.2	1.0	3.2

Résumé

1. Le but que poursuivent les auteurs consiste à examiner quelques applications récentes de systèmes de comptabilité nationale et d'expliquer certains points au moyen d'exemples élaborés par un certain nombre d'institutions gouvernementales des Pays-Bas.

2. Il existe un grand nombre de manières différéntes de construire un système de comptabilité nationale et ces manières dépendent du genre de politique économique adopté, de la façon d'évaluer les transactions, etc.

3. Les chiffres de n'importe quel système de comptabilité sont détérminés par les données fondamentales de l'économie. Certaines de ces données peuvent dépendre de décisions gouvernementales ou subir l'influence d'autres facteurs qui peuvent ne pas être connus à l'avance. Dans pareils cas, il peut être utile d'établir des "budgets flexibles" indiquant d'une manière systématique comment les chiffres dépendent de différentes hypothèses par rapport aux données inconnues.

4. Les auteurs discutent des exemples de "budgets flexibles" dans le plan économique des Pays-Bas (*cf.* tableau 1).

5. On peut établir des comptes nationaux séparés d'après les types de monnaie dont on a besoin pour les diverses transactions.

6. Dans un système économique, on peut évaluer les diverses transactions en marchandises de différentes façons. Les auteurs présentent une tentative d'estimation de revenu provenant de transactions au marché noir et de dépenses totales faites au marché noir (*cf.* tableau 2).

7. On fait souvent une distinction entre les "comptes comptant" et des "comptes transactions." Les problèmes de la reconstruction d'après-guerre et ceux que soulève le maintien du plein emploi, peuvent exiger l'utilisation d'un troisième système de comptes basé sur les commandes (*cf.* tableaux 4 et 5).

8. Les comptes nationaux permettent de découvrir facilement les rapports qui existent entre différents ensembles d'élements, dont on se sert en général, tels que le revenu national au coût des facteurs, la dépense globale du gouvernement, la formation brute de capitaux, et les dépenses des consommateurs, etc.

9. La comptabilité nationale permet d'établir la liste systématique de tous les transferts importants de monnaies.

10. C'est seulement récemment qu'on a accordé quelque attention aux problèmes qui se présentent quand on désire comparer les comptes réelles avec les prévisions ou les "plans" qui ont été faits antérieurement. Il est très important de présenter les résultats de telle façon qu'on puisse facilement voir à quels points de vue les réalités s'écartent des "plans." Des divergences peuvent être dues à des facteurs tels que des modifications imprévues des prix à l'étranger, une proportion d'épargne inférieure à ce qu'on avait prévu, etc.

NATIONAL INCOME AND INDUSTRIAL STRUCTURE

by Simon Kuznets
University of Pennsylvania and
National Bureau of Economic Research

I

Industries are ordinarily differentiated by the raw materials they use, their productive processes, and the finished products they turn out. The extent to which the raw materials are organic, mineral, or synthetic; domestic or foreign; perishable or durable, puts a stamp upon the economics and sociology of an industry. That the productive process lends itself to large-scale machine operation or requires the personal effort of skilled craftsmen; can be handled by private enterprise or must be entrusted to public agencies; does or does not require large capital investment—are also factors determining the economic and social patterns by which an industry is guided. Finally, the characteristics of the finished product—the type of want it satisfies, its dependence upon foreign or domestic purchasers, the extent to which its sale can be left to private markets or must be regulated by public agencies, the manner in which demand responds to fluctuations in the purchasing power of buyers—may serve to distinguish one industry from another, despite common raw materials and similar although not identical production processes. A country's industrial structure may be defined as the relative distribution of its resources and total output among the several industries differentiated in the manner just suggested.

However, in considering the bearing of the industrial structure upon the measurement of a nation's total output, *i.e.*, its national income or product, differences among industries must be viewed from a plane somewhat broader than the purely technical one of materials, processes, and products. That one industry consumes an organic and another a mineral material does not, in and of itself, raise questions bearing upon the measurement of the two industries in estimating national income. Important questions arise only when the purely technical characteristics spell major differences in the nature of the economic institutions under whose aegis the industry functions. To illustrate: if, because of the characteristics of its material, processes, and product, one industry is carried on as an integral part of the family economy, largely self-subsistent

and relying to only a limited extent upon the market, while because of the technical conditions of production, another industry is carried on by business enterprises organized with exclusive orientation to markets, questions arise as to how the net products of the two industries are to be measured so that their contributions to national income can be compared. Similarly it is only to the extent that technical differences between two industries compel one to be conducted under government auspices and another under free enterprise that important problems arise in measuring the two industries in national income; or in defining national income for two countries differing in their industrial structure with respect to the relative importance of business and government.

Thus, industrial structure has a bearing upon the concept of national income so far as differences among industries are viewed as differences in the basic pattern of social and economic institutions under whose aegis the industries are carried on. Viewed in this light, three basic contrasts in industrial structure may be suggested. The first, between self-subsistent and market-oriented structures, is largely identical with the widespread distinction between industrialized and nonindustrialized countries. From the viewpoint of conceptual problems, the difference between industrial and nonindustrial countries lies in the fact that major portions of the productive activity of the latter are likely to take place within the family and the community, not in business enterprises working for the market. National income is not difficult to define for a country that is predominantly agricultural, but in which agriculture is organized on a business basis (*e.g.*, New Zealand). But for a country like China or India, whose major emphasis is also on agriculture but in which a great deal of agriculture (and related processes) is carried on within the family and rural community, it is difficult to define and approximate national income in a way comparable with that for industrialized or market-bound economies. The second basic contrast—between domestically oriented industrial structures and those heavily dependent upon foreign economies—also gives rise to problems in defining national income, problems residing largely in a proper delimitation of the nation as a unit of measurement. The third contrast—between privately and publicly organized industrial structures—creates obvious problems of comparison.

This paper is confined to national-income problems involved in measurement for countries whose industrial structures exhibit the first type of difference—that between a relatively self-subsistent family or communal economy and one operated primarily by enterprises oriented to the market place. I assume that the third type of difference—between private and governmental industrial structures—is treated in Mr. Smithies' paper; and neglect the problems arising in the

second type of difference among industrial structures, largely because the problems it raises seem of less interest.[1]

II

Problems in national-income estimation for industrial and pre-industrial economies[2] can be appraised properly only if we seek in national income some measure of the *real* net volume of goods produced, undistorted by duplication and unaffected by purely monetary differences in price levels. If we accept the formal accounting practices followed in the several economies and do not concern ourselves with what in fact happens under the money surface of economic circulation, we avoid many of the problems involved. The results, however, will be of limited use, since at most they give us the volume of pecuniary transactions, corrected for some types of duplication. They will fall far short of what is ordinarily wanted, *viz.*, a comparison of the real, unduplicated volume of commodities and services yielded by the productive systems of the two countries.

The concept of national income we, therefore, adopt is the net output of commodities and services flowing during the year from the country's productive system into the hands of ultimate consumers or into net additions to the country's stock of capital goods. National income, thus defined, must be measured for two countries so that, despite differences in industrial structure, the real net output of commodities and services can be fairly compared.

When, with this definition in mind, we inspect current estimates for countries differing in industrial structure, we are forcibly struck by the large disparity in per capita real income. Colin Clark's compilation, *Conditions of Economic Progress* (London, 1940), illustrates the point. For 1925-34 income is measured in international units, defined "as the

[1] An interesting recent illustration of this problem is provided by the estimate of income for Northern Rhodesia (Phyllis Deane, "Measuring National Income in Colonial Territories," *Studies in Income and Wealth*, Vol. 8, National Bureau of Economic Research, 1946, pp. 147-174). National income estimated as the yield of local productive factors entering the income of residents amounted, in illustrative figures, to about £7½ million; but if income of foreign firms operating in the territory and services rendered abroad by the colony's residents are included, the total is £13.0 million—almost double. How the national income of Rhodesia is defined is obviously of great importance to the resulting total—a situation that might be true of any colonial territory that is small (with respect to population or total output) relative to its "mother" country.

[2] These terms are used below to denote, on the one hand, an economy dominated by business enterprises, using advanced industrial techniques and ordinarily with a large proportion of its population in large cities; and, on the other hand, an economy in which a large part of production is within the family and rural community, a minor share of resources is devoted to advanced industrial production, and a minor part of its population lives in cities.

amount of goods and services which one dollar would purchase in the United States of America over the average of the period 1925-1934" (pp. 39-41). In these units the picture is (pp. 54-57):

Per capita income for four countries designated by Clark as Great Powers (United States of America, Great Britain, Germany and Austria, France) is 408.

Per capita income for pre-industrial countries (China; British India; Dutch Indies; Africa, excepting Algeria, Egypt, South Africa, Morocco, Tunis; Asia, excepting China, India, Japan, Palestine, Turkey, Syria, Cyprus; and Oceania, excepting Australia, New Zealand, Hawaii, and Guam) is 43½.

The former category includes over 290 million people or somewhat less than 15 percent of the world's population as estimated by Clark. The latter comprises over 1,100 million, or well over one-half of the world's population. An even more extreme contrast is that between the United States and China. For the former per capita income is about 500 international units (see pp. 54 and 56); for the latter, about 40 (see p. 46).

A ratio of some 10 or 12 to 1 between the per capita product of the most advanced industrial country and that of countries well behind in industrial development sounds plausible. Anyone who has seen, smelled, and touched the tangible industrial power of the United States and compared it with the physical apparatus of a pre-industrial country may legitimately feel that the ratio should be much greater. But if one is not too misled by purely visual or sensual contrasts and considers the figures more closely, elements emerge that justify incredulity or at least searching questions.

First, in following his definition of international units Clark attempted to raise the estimates for pre-industrial countries for several elements missing in the figures ordinarily derived. Thus, for both China and India, food output (and consumption) was estimated not at producers' prices in the country but at retail prices in an industrial economy like Great Britain; and at least for India substantial corrections for differences in prices paid for other types of productive service, between India and Great Britain, were made. In other words, the figures are literally what they are intended to express—the bundles of commodities and services that could be purchased in the United States during 1925-34 with 40 odd dollars.[3]

[3] Clark's adjustment brings China's per capita income close to that shown by the recent, more detailed estimate of Mr. Ta Chung Liu in *China's National Income, 1931-1936* (Brookings Institution, 1946). Mr. Liu's figure, adjusted for comparability with the United States, yields a per capita gross product of $41 (see p. 85), quite close to Clark's figure of $40 in international units.

Second, with respect to economic conditions during the decade, the comparison favors the pre-industrial countries. One of the most severe industrial depressions on record obviously affected industrial much more than the pre-industrial countries and was reflected most sensitively in the more precise national income estimates for the former. The figures for the pre-industrial countries can scarcely be said to reflect transitorily unfavorable economic conditions.

Third, and most important, an average income, particularly the arithmetic mean, substantially exceeds the incomes of most individuals, since the customary size distributions are skewed to the right. Furthermore, what we know about the internal structure of size distributions suggests that while there is some mobility, the majority of units in any size group tend to remain in that group for several years. This means, in terms of Clark's figures, that: (a) more than half the population of pre-industrial countries receive a per capita income less than 40 odd international units; (b) of this half a substantial proportion, say two-thirds (or one-third of the world total) are in the income brackets well below 40 international units per year for a substantial period.[4]

Now, if we ask, could people live in the United States during 1925-34 for several years on an income substantially below $40 per capita, the answer would be "yes," if they were sufficiently wealthy to have lots of possessions to sell, sufficiently lucky to have rich relations, or sufficiently bold to rob other people. The one-third to one-half of the pre-industrial population of the world would scarcely be in that position; and if we assume that all they have produced and could consume per capita was less than 40 international units for several years, the conclusion would be all would be dead by now. One is thus forced to infer that: (a) either that the estimates, even after the customary adjustments for comparability with industrial countries, are still deficient in omitting many goods produced in pre-industrial countries; or (b) in fact the whole complex of goods produced and consumed is so different that we cannot establish any equivalence of the type represented by Mr. Clark's international units. We shall see from subsequent discussion that neither suspicion is unjustified.[5]

[4] The discussion is in terms of income produced per capita. While savings are quite limited in pre-industrial countries, some proportion of national income is ordinarily saved. The arguments in the text could be applied to the distribution of income consumed with the arithmetic mean say about 5 percent lower than mean per capita income. However, the distribution of income consumed is less unequal than that of income received or produced.

[5] Mr. T. C. Liu argues for the plausibility of a $37 per capita consumption for China by referring to the data for 1935-36 for the United States, according to which small percentages (5 to 6) of farm families in some regions had a family income of less than $250. But this is a comparison of *average* values with the extreme of an income-size distribution and overlooks the fact that this extreme is composed largely of families that may have sustained entrepreneurial losses in this single year, not of families that are at this level for any length of time.

The form in which the question was raised—how it is possible for a large proportion of the population in pre-industrial countries to survive on an income that produced, for several years, less than the equivalent of $40 per year—obviously reflects my bias as a member of an industrial society. Personal experience and observation tell me that such an annual product is well below the starvation level. But were I a member of a pre-industrial society I might well have asked how it is possible for the majority of the population in the United States to dispose of as much as $500 per year, or whatever its equivalent would be in international units of rupees or yuan. Especially, on being told that of this huge income less than 10 percent is saved for net additions to capital stock, I might well ask how the population manages to consume so much—given the limited amount of food one can eat, clothes one can wear, or houses one can inhabit. And a suspicion similar to that voiced above could be entertained, namely, that these income figures for industrial countries must include many categories of items that are *not* included in income as ordinarily conceived in pre-industrial countries; and that the whole pattern of consumption and living in industrial countries is so different as to explain the ease with which these huge quantities of goods are produced and especially consumed.

Let me turn now to a more direct exploration, first of the categories that may be omitted from the national income figures for pre-industrial countries but included in those for industrial countries; second, of problems involved in the basic differences in consumption and production levels in the two types of country.

III

In a decentralized, agricultural, self-subsistence economy many productive activities take place within the family or the local community without finding overt expression on the market. The range of such non-market activities is extremely wide, extending from the production of primary food and other materials, through their fabrication, to the provision of all kinds of services—personal care, household operation, recreation, education, religion. Short of an intensive study of the households and of the agricultural communities for a year or longer, it is extremely difficult even to identify the contents of this productive performance outside the market sphere; and after its contents have been ascertained, it is even more difficult to assign values that would put these productive activities on a basis comparable with their counterparts in an industrialized market-bound economy.

One is, therefore, not surprised to find that in the estimates for pre-industrial countries the statistical allowances ordinarily made to cover

the value of such hidden nonmarket services are far from adequate. For example, in the case of India, Colin Clark allows for the retail value (at the English price level) of wheat and other grains, using flour prices for conversion (*op. cit.*, p. 43). This means that the only domestic manufacturing of wheat and other grains allowed for is its milling into flour. But what about further fabrication carried on in the domestic economy into final consumable goods? Likewise, in the estimates of untraded goods and services for Northern Rhodesia, Miss Deane includes "corn as meal, ground nuts after being shelled, and so on" (*loc. cit.*, p. 155), but does not allow for the services involved in further conversion, cooking, baking, etc. Even in the case of China, the excellent field studies of J. L. Buck, which provide many of the basic figures for prewar national-income estimates, do not include, and designedly so, all the productive activities carried on within the farm household. And in dealing with India's estimate, Mr. Clark excludes the services of women on farms in order "to obtain comparability with the figures of other countries"(p. 42).

While freely admitting the difficulty of including *all* the extramarket productive activities of a pre-industrial economy, I am inclined to argue that once a comparison between it and an industrial country is attempted, there is little justification in accepting the conventional rules of national-income accounting in industrial countries. In estimating income for the United States, we exclude the services of women on farms, as we do the services of urban housewives, partly because there is no good basis for valuing them, partly because they are governed by rules different from those guiding business enterprises, and partly because we assume that the omission is not too large as compared with what is included. But for a pre-industrial country the latter assumption is patently invalid; the acceptance of primacy of business enterprise is out of the question; and if national income is to be merely a measure of goods exchanged for money, an estimate had better not be attempted for pre-industrial countries at all.

Clearly the apparent consistency of applying the rules of national-income accounting in industrial countries to those in a pre-industrial economy is no consistency at all. For in scrutinizing the contents of the net output of industrial countries we find a surprising variety and volume of commodities and services that represent nothing but professional, *i.e.*, business pursuit of productive activities for which there is a clear counterpart within the family and community life of pre-industrial economies. The recent valuable publication of the United States Department of Commerce, *National Income* (supplement to *Survey of Current Business*, July, 1947), provides a wealth of data to illustrate the point. Table 30, pp. 41-43, gives details of the finished commodities and services purchased by consumers—a total that constitutes the overwhelming proportion of national

income (as defined by the Department of Commerce) in any except the war years. Each commodity category, except those that relate to such products of industrial civilization *par excellence* as automobiles and radios, represents activities for which there is a clear parallel within the family and community life of pre-industrial societies. Manufactured foods and tobacco, clothing, shoes, furniture—all commodities that are common to both industrial and pre-industrial economies—have market values in the former that embody a great deal of family work in the latter. And the same is true of various services. Thus, according to the Department of Commerce estimates, consumers spent in 1929 over half a billion dollars on cemeteries and funerals; and while these functions are presumably performed satisfactorily in India and China, I can not find any allowance for them in the estimates. The American consumers spent close to one billion dollars on life insurance in 1929. What about the value of such insurance provided by the family system of China, where the family comes to the succor of a member who may have been afflicted by one of the bad turns of fortune for which life insurance is supposed to compensate in industrial societies?

But let us grant that a pre-industrial country, in adapting its resources and skill to needs, manages to develop, within the family or the community, many productive activities that are taken over, if in modified form, by market-bound business enterprises of an industrial society. What can one practicably do to provide for a fair inclusion of these nonmarket activities, or in some other way attain proper comparability of measurement between the two types of economy?

That one should try, by intensive field study, to get an inclusive picture of nonmarket productive activities in pre-industrial society is good advice, too obvious to be stressed. It is, however, a long-run measure likely to yield results but slowly—given the difficulties of proper study of pre-industrial economies and the eventual problems of assigning some magnitudes to the activities, once they have been identified.[6] When and if such studies accumulate for any country to a point of providing an adequate basis for inclusive treatment, the way will be open to adjust for at least the major omissions in current estimates. Even with perforce arbitrary valuations, the inclusion of these extra-market activities will result in a smaller error than is inherent in the current estimates for pre-industrial countries which tend to omit them almost completely, with the apparently single exception of foods (in raw or semicrude form) retained for consumption.

[6] However, during recent years several interesting studies have appeared; they are listed in the Bibliography in J. B. D. Derksen, "On Comparability of National Income Statistics," these *Proceedings* (II, United Nations World Statistical Congress), pp. 267-271.

While waiting for such intensive studies, we might consider short-term expedients. As a tentative suggestion, advanced for discussion rather than as a tested recommendation, I would like to make two points. The first concerns activities closely connected with commodities whose market value in industrial societies enters the value of the finished goods flowing to ultimate consumers. In the case of pre-industrial societies primary and semifinished commodities flowing into ultimate consumption should be given the prices in industrial societies of the finished, fully manufactured products they enter, not of their exact crude or semifinished counterparts. The second point concerns services rendered directly to consumers, not embodied in new commodities; of these services in an industrial society, specific magnitudes would be included only for those categories that represent definitely much greater contributions to consumers' welfare in industrial than in pre-industrial society or vice versa, while for all those in which differences in relative supply are at all dubious a proportionate relation to other services would be assumed. Let me try to clarify each suggestion.

The first means that, *e.g.*, the amount of wheat produced and retained for domestic consumption in a pre-industrial country should be valued at the retail prices in industrial countries, not of wheat or of flour, but of the fully manufactured foodstuffs of which wheat is the component; and likewise with corn, rice, cotton, wool, hides, etc. This suggestion is practicable to the extent that whatever scanty statistics are available for pre-industrial countries usually cover the production of primary and semifinished commodities, and ordinarily their exports and imports; consequently, the flow into domestic consumption can be estimated. Also, for at least some industrial countries, *e.g.*, the United States, it is possible to calculate the total spread between the value of primary materials at the producer's door and the value of the finished products they enter, at the cost to ultimate consumers. But while practicable, is such treatment justifiable?

It obviously assumes that the relative weight of fabrication and treatment that intervenes between the material in its crude form and the product in its most finished form (that is, in the form in which it flows to the household in the *industrial* society) is the same for the two economies. Yet one might argue that, by and large, the relative extent of such fabrication and treatment is greater in industrial than in pre-industrial societies. For example, the way in which primary foods are treated, packaged, etc. before they are sold to an urban family in Chicago represents a much more extensive fabrication of wheat than the operations performed on wheat on a North China farm before the Chinese housewife proceeds to do with it whatever the Chicago housewife does with the wheat product she buys. However, a large part of such treatment

in industrial society is merely an offset to the disadvantages of the centralization of production. Food products must be treated, packaged, etc. because they are produced thousands of miles from where they are consumed; and in a pre-industrial society the efficiency of production is much greater in respect of distance between producer and consumer—a point to be discussed further below. Above all, we assume that once the products are eventually consumed by the individuals and households of a pre-industrial economy they have attained the same satisfactory state of "finishedness" as the final products of an industrial economy.

At any rate, the acceptance of the suggestion advanced here, and it is advanced only as a tentative expedient, must be decided by weighing the error involved in following it against the error attached to estimates that fail to follow it. The error attached in not following the suggestion is twofold: (1) included in national income for industrial countries is an element of commodity production that is gross, rather than net, being merely an offset to the disadvantages of the concentration of manufacturing in centers distant from the centers of raw materials and of consumption; (2) omitted from national income for pre-industrial countries are many productive activities concerned with commodities, which, being carried on to a great extent by market-bound enterprises, are included in national income for industrial countries. The error implicit in following the suggestion would be to exaggerate the national income of pre-industrial countries to the extent that *productive* treatment of commodities in the latter is relatively less than in industrial countries. Of the two errors, that involved in following the suggestion seems much smaller than that in not following it; and we urge an attempt to apply and test it as a practical expedient.

The second suggestion refers to services not embodied in new commodities. Of these there are definite categories of which we can be sure that the relative, per capita supply, is of greater economic magnitude in industrial than in pre-industrial societies, and vice versa. For example, the supply of qualified medical or educational services is definitely greater in industrial than in nonindustrial societies, in the sense that the tangible benefit to consumers, measured by any standard, is greater in the former than in the latter. But can we say the same of religious services, or of such services as are provided by funerals, recreation activities or barber shops and beauty parlors? Many of these are provided within the family or community in pre-industrial societies, while they are sold on the market in industrial societies. They are therefore likely to be included in national-income estimates for the latter, and omitted, in good part, from the estimates for the former. There is no basis for assuming that the per capita supply differs among the two types of

economy; and there are great difficulties in establishing any comparability between these types of activity in countries that differ greatly in their social organizations and patterns of life.

With respect to this category, which for convenience can be described as culture myth services, one of two practical expedients may be adopted. The first would be to omit them from national-income estimates for both types of country, thereby reducing the totals for industrial countries relatively more than the ordinary estimates for pre-industrial countries. The second would be to assume that the supply of such services in pre-industrial countries is in the same proportion to all other consumer services as it is in industrial countries—an assumption that perhaps results in too moderate an adjustment. The advantage of the second expedient is that, unlike the first, it permits us to leave the comparison between the two types of country on as inclusive a basis as is permitted by national-income estimates for industrial countries.[7]

IV

We have discussed so far extra-market productive activities, a substantial part of which is likely to be omitted from national-income estimates for pre-industrial countries while they are fully included in those for industrial countries. We are now ready to consider several categories, still within the area of the flow of goods to ultimate consumers, that are fully represented in estimates for industrial countries and yet are costs rather than final products, in the sense that they serve merely to offset some of the disadvantages of industrial organization.

The first category was suggested in the discussion above of the degree of fabrication of consumer commodities in industrial societies. A characteristic feature of the latter is that production tends to be concentrated in relatively large units, at some distance from the consumers who ultimately use the finished products. Thus, from the completion of commodities by producers to the time they reach the hands of consumers

[7] Throughout we face the choice between "inflating" national-income totals for pre-industrial countries to make them as comprehensive and as "gross" as the estimates for industrial countries; and "deflating" national-income totals for industrial countries to make them as restricted and as "net" as the ordinary estimates for pre-industrial countries. The usual choice in the national-income literature, and followed in the illustrative calculation in the Appendix, is to "grossify" —bring the estimates for pre-industrial countries *up* to the level of comprehensiveness and grossness of those for industrial countries. A more desirable but more difficult solution would be to raise the estimates for pre-industrial countries only for such elements of real productive activity as tend to escape measurement and to reduce the estimates for industrial countries (and to a lesser extent those for pre-industrial countries) by omitting such elements as are not net, *i.e.*, represent merely offsets to the disadvantages of industrialized urban societies.

there is a long chain of transportation and distribution, just as there may be one between the origin of the raw material with primary producers and its use in the manufacturing or construction establishment. This can be clearly visualized by assuming in an industrial society, a single shoe factory which, with the help of railroads and a whole network of trade, assembles raw hides from many livestock farms; then, with the help of transportation, trade, advertising, etc. manages to place the finished shoes at the disposal of the individuals and families that wear them. A hypothetical situation in an idealized self-subsistence economy is in sharp contrast: a handicraftsman residing in each village gets hides from the local farmers, converts them into shoes, and sells or barters them to local inhabitants—all without recourse to transportation, trade, advertising, etc. If the number of shoes and their quality are exactly the same, net product in the sense of the real flow to ultimate consumers is identical in the two situations. Yet in one, production, in the narrow sense of converting hides into shoes, accounts for merely a small part of the values of finished goods, whereas in the others it accounts for practically all of it. The transportation and distribution activities in an industrial society can thus be clearly seen as offsets to the disadvantages of large-scale, machine manufacturing, which, needless to say, are more than outweighed by its economies.

This problem is disposed of in current estimates for pre-industrial societies either by taking the finished consumer products at the *retail* prices of industrial societies, thereby allowing for the inclusion of all these transportation and distribution services (as Colin Clark does in the case of India or China); or by making a special adjustment for difference in the marketing structure of agricultural production (as T.C. Liu does for China). These are perfectly legitimate adjustments, and I have only a few comments.

First, the adjustments just described are *part* of the one suggested in the preceding section, in which not the finished product but the crude materials of pre-industrial societies are to be valued at the retail prices of the corresponding finished products of industrial societies. If the suggestion is accepted, the adjustments of the type made by Clark and Liu are automatically included. Second, even with the latter adjustments, revaluing to the price levels of industrial society involves "grossifying" the output of pre-industrial societies, to bring it on a par with the output of industrial countries. As already suggested, it would be just as valid to "nettify" the output of industrial societies to bring them on a par with pre-industrial societies, by omitting from national income all services embodied in the value of commodities that represent the extra transportation and handling. Third, while the adjustments discussed here are on the surface merely for differences in price levels,

they are in fact an application of the basic definition of national income; only when the latter is defined as the real flow of goods to ultimate consumers and of net additions to capital stock, does the need for the adjustments become apparent.

But there are several other categories of productive activity in industrial societies whose value does not enter the retail prices of consumers' finished commodities, and yet that are merely offsets to costs imposed by the organization of production. Such activities ordinarily enter income estimates for industrial countries in the form of direct services to consumers (rather than as the cost of commodities); and in some national-income concepts in the form of the purchase of commodities and services by the government (*e.g.*, in the Department of Commerce national-income total for the United States). Yet such activities are either absent from or present to only a limited extent in pre-industrial societies because their industrial structures impose no costs that have to be offset. And clearly the adjustment just discussed, of converting consumers' commodities to retail prices prevalent in industrial countries, does not dispose of the lack of comparability thus arising.

Three categories of such activities come readily to mind. The first is suggested by the fact that in industrial countries the dominant modes of production impose an urban pattern of living, which brings in its wake numerous services whose major purpose is to offset the disadvantages. A clear case is the transportation of employees to and from work—an activity that can hardly be said to constitute direct welfare to ultimate consumers and is merely an offset to the inconvenience that large-scale industrial production imposes upon the active participants in it. But what about the extra costs involved in providing urban consumers with the appurtenances of living? The costs are heavy exactly because the concentration of large numbers in limited areas raises geometrically the discomfort and the costs of offsetting it. For example, the Department of Commerce sets for 1929 the cost of space rent for urban and rural nonfarm dwellers at \$10·3 billion, which for a nonfarm population of 101 million works out to about \$100 per capita. For farm houses the total is \$829 million, which for a farm population of 30 million, works out to about \$27 per capita. Yet surely the real values of the two are scarcely in the ratio of 4 to 1. The costs of urban housing may well be high because of the technical problems created by dense aggregations of people.

The second category represents costs of participation in the complicated, technical, monetary civilization of industrial countries. Payments to banks, employment agencies, unions, brokerage houses, etc. including such matters as technical education, are payments not for final goods flowing to ultimate consumers, but libations of oil on the

machinery of industrial society—activities intended to eliminate friction in the productive system, not net contributions to ultimate consumption. And while identical or similar activities exist even in pre-industrial societies, particularly those in which the money economy has begun to spread, one might reasonably argue that their magnitude is much greater in the more complex industrial countries which make claims upon their members for a finer adjustment to the dictates of the market system.

The third category is represented by governmental activities. In any society the major part of governmental activity is devoted to preserving and strengthening the fabric of social organization and only to a limited extent to the provision of final services to ultimate consumers. The legislative, judicial, administrative, police, and military functions of the state are designed to keep society operating along accepted patterns, to create the conditions under which the economy can function, not directly to provide goods to ultimate consumers. The major yield of governmental activity is therefore indirect rather than direct goods, costs rather than net returns. Yet if we accept the concept at present followed by the Department of Commerce in this country (and the official estimates of Great Britain and Canada), the full magnitude of governmental outlay on commodities and services appears as part of the net output of society, of national income or product. It is quite likely that the necessary costs, which most of such activities represent, are absolutely and relatively much smaller in a pre-industrial than in an industrial society; consequently their full inclusion in national income introduces a greater element of grossness in the estimates for industrial countries.

In considering how to deal with the three categories of activities just noted, which may be interpreted largely as offsets to friction in the organization of economic society rather than as direct elements in net output, we are confronted with difficulties. The first is that activities of the three types described occur even in pre-industrial societies: in most of the latter, cities, a monetary and credit economy, and a central government are far from unknown. Hence if we are to omit some of these activities from the national income of industrial countries, we should be in a position to do likewise for pre-industrial societies. A more important difficulty is that in many activities the elements of net contribution and offsets to costs are inextricably interwoven, and can be disentangled only by intensive analysis. How much of the high price of urban housing is the high cost of offsetting discomforts of living in a densely settled community, and how much represents greater facilities and comforts? How much of the huge outlay on passenger automobiles in this country is an offset to the disadvantages of urban living

and how much a net contribution to welfare? What proportion of the cost of the telephone, telegraph, etc. is an offset to the obligations imposed by participation in a highly developed society, and how much a net contribution to the satisfaction of ultimate consumers *qua* consumers?

In the face of these difficulties it is not easy to indicate steps in the direction of attempting closer comparability. Yet three suggestions seem to be in order. First, such activities as beyond any doubt represent payments by consumers for services that are nothing but occupational facilities should be excluded from the estimates for both types of country. Clear examples are commutation to and from work and payments to unions and employment agencies; but one might add almost the entire gamut of what the Department of Commerce classifies as business services in its estimate of consumers' outlay (bank fees, brokerage fees, etc.). Second, where in industrial societies the costs of consumer services are inflated by the difficulties of urban life, some revaluation of these services by comparison with their costs in rural communities is in order. The magnitudes involved, especially in such an item as cash and imputed rent on housing are quite large. Finally, it seems indispensable to include in national income only such governmental activities as can be classified as direct services to ultimate consumers. This most important and inescapable step is urged here in full cognizance of the statistical difficulties, which are great. But if national-income figures are to retain any meaning as measures of the real flow of goods to ultimate consumers or to stock of capital, the huge duplication piled up by considering all governmental activity as a final product must be removed. Such a step is important and necessary even for intracountry comparisons over time; it is equally if not more important for comparisons between industrial and pre-industrial societies.

These three suggestions are a maximum program: their proper application requires information on and a functional analysis of the service sector of consumers' outlay and of governmental activities that are probably beyond the present supply of data and the present state of knowledge of the real contents of national product even for advanced industrial countries. As a more practical, if theoretically less satisfactory expedient, we may consider adjusting the national income of pre-industrial societies—as currently measured—for the elements of grossness that are present to a larger relative extent in the estimates for industrial societies. Thus, instead of excluding the service components of consumers' outlay that represent pure costs, revaluing inflated urban services, and reducing governmental activities to direct services to consumers, we can inflate the corresponding elements in the national incomes of pre-industrial countries to achieve a comparable level of

grossness. This is, in fact, the expedient adopted in the illustrative comparison of the national products of the United States and China in the Appendix. It has the advantage of being consistent with the application, to the commodity part of national product, of the raising ratio of finished products to crude materials derivable from the standard estimates of national income for industrial countries. But it is a temporary expedient; eventually it will be preferable to follow the suggestions in their original form, and to exclude from estimates for both industrial and pre-industrial countries such gross elements as occupational expenses of ultimate consumers, inflated costs of urban living, and intermediate product of governmental activities.

V

From the consumers' outlay component of national income, we turn to the treatment of capital formation.

a. Some of the elements omitted from national income estimates for pre-industrial countries and of grossness in the estimates for industrial countries characterize also the estimates of capital formation or investment. Thus a great deal of capital formation within pre-industrial economies takes place outside the market, *e.g.*, individual farmers' activities on improving the soil and buildings and communal construction activities. These are not likely to be covered fully in the estimates. Yet their relative share of total capital formation is likely to be larger in pre-industrial than in industrial economies.

Similarly, pre-industrial countries are likely to be characterized by a shorter distance between the producer and the user of capital goods (unless the goods are produced abroad). In an industrial economy labor is more extensively divided. Whatever we have said about consumer goods, in the illustration in terms of shoes, could be repeated in an illustration in terms of plows or farm carts. Here also, an advanced industrial organization may mean a considerable amount of extra fabrication, transportation, and trade that are not necessary in the simpler, decentralized structure of a pre-industrial economy.

However, quantitatively, such elements of both omission and grossness as tend to inflate the difference between totals for industrial and pre-industrial countries are likely to be relatively smaller for capital formation than for consumers' outlay. First, the real volume of *all* capital formation, whether market bound or not, is likely to be exceedingly small in pre-industrial countries living, as they do, close to the margin of subsistence. Second, in the case of capital formation, *i.e.*, construction machinery, and equipment, the relation between producer and consumer seems to be fairly close even in an industrial society. At any

rate, it appears to involve less of the cross-hauling, elaborate distribution, and advertising that tend to bring such large elements of grossness into the cost of consumers' commodities to ultimate consumers. And satisfactory use of at least industrial equipment and construction is not as subject to the inflation of costs by the difficulties of urban living as is true of the use of consumers' goods proper. These statements are particularly applicable to net capital formation, *i.e.*, if we exclude, for the purposes of the present analysis, the intermediate product of governmental activity. We thus deal only with net additions to the stock of capital goods, not with such gross volume as would include the contribution of governmental activities to the preservation and regulation of the society at large or of the economic system in particular.

b. What about such capital goods as serve only activities which, in the analysis of consumers' outlay, we characterized as representing offsets rather than net contributions to the flow of goods to ultimate consumers? If services of street cars and commuting trains are not contributions to the satisfaction of wants of consumers *qua* consumers, and, therefore, should be excluded from national product (the latter conceived as the sum of the flow of goods to consumers and capital formation), should we exclude from capital formation the additions to the stocks of street cars or of street railway trackage? Clearly, if the answer is "yes," a large proportion of capital formation in an industrial country will be omitted.

The answer, however, is "no." The addition to capital stock is part of national product properly defined, regardless whether the capital good in question will itself directly yield services to ultimate consumers in the future, or while not in itself capable of yielding such services, is still useful in keeping society going and thus avoiding future outlays. We consider a new blast furnace an addition to capital and a proper part of capital formation, even though the furnace in and of itself can not turn out final consumer goods. It will assist indirectly in turning them out and in its absence, a potential increase in the supply of finished products would be impossible without an additional outlay of resources. What is true of a blast furnace is true also of a street car, or of a battleship.

There is no inconsistency in excluding the *direct* services of capital goods from annual estimates of the flow of finished goods to ultimate consumers, and in including the tools that yield these services in capital formation. In measuring the flow of goods to consumers we are not justified in including goods that are wanted by consumers not as consumers but as producers. In measuring capital formation we are in fact estimating the future contribution—direct and indirect—of the goods in question to the ultimate satisfaction of consumers' wants. And so far as in

the technology of the economic system street cars are a useful and indispensable tool, we include them in capital formation.

VI

The last major problem of comparability, differences in the patterns of production in industrial and pre-industrial economies, is reduced by the suggestions advanced in the analysis above, especially by the omission of some commodities and services that in industrial society serve exclusively to offset disadvantages imposed by the productive system. But even with this and related suggestions pushed as far as possible, there will still be marked differences in the composition of the goods that constitute the national product of the two types of society.

The exact meaning of this difference for the problem of proper statistical measurement must be clearly seen. Assume that for all the goods that are in the comparison between two countries, A and B, prices can be established for each in both countries, even though some of the goods may not be produced or consumed in one. It would then be possible to estimate the total product of country A in prices of country B, and the total product of country B in prices of country A. Though all problems would not be solved thereby, the assumption serves to illuminate two points important in the analysis. First, the analysis can best be handled by dividing it into two parts; one is the difficulty or impossibility of securing prices in country B (A) for such goods as are produced only in country A (B); the second remains even if prices in both countries could be secured for all goods in the comparison. The second point is that the difference in the goods patterns can be discussed only in connection with the relative price patterns of the two countries.

The difficulty created by the fact that for goods produced only in pre-industrial countries it is often impossible to get a price in an industrial country (and vice versa) cannot in fact be resolved, short of a close analysis of the function of the good in question, finding a functional counterpart in the other country, and then finding a price for it by analogy. While in certain classes of goods (—how could one find the functional counterpart in the United States of, say, shark-fins soup or of the services of a Chinese fortuneteller?—) this may seem a counsel of despair, for simpler types of goods the task is not impossible (*e.g.*, for certain classes of food or clothing). But it is important to remember that the comparability to be established, the counterpart to be found, is not that of scientifically established physiological or medical service—but of position in the economic scale. At the present stage of our knowledge of industrial and pre-industrial societies, it is difficult to extend the range of price comparisons; and we have to accept the fact that prices will be found

only for such goods as are used in both types of country. This means that in practice price comparisons are established for only such goods as are common to both types of economy; and that the ratios are applied to the over-all totals with the implicit assumption that the price relations for the goods omitted are the same as those for the goods covered.

This resolution, by assumption, may be the only practicable. But it brings into comparisons of national income for industrial and pre-industrial countries a potentially large bias. Comparable prices in general can be found only for goods whose qualitative characteristics are easily recognized and comparable—commodities rather than services; simple crude materials rather than complex fabricated articles. And as between two countries comparable prices are most easily established for crude commodities that move freely in international trade, not between commodities, no matter how crude, that are peculiar to one country alone. But commodities that move in international trade are likely to show relatively narrow price differentials: were the differentials wide, foreign trade would tend to reduce them. Consequently, the selectivity of price comparisons, in their emphasis upon crude commodities with international markets, has an important bias—*understating* the price differentials between the two countries. How considerable the understatement is depends upon the factors that produce the price differentials. When one country is industrial and the other pre-industrial, the understatement can be large indeed.[8]

This observation applies to price comparisons for identical goods, at identical levels of fabrication and circulation. The bias is, therefore, over and above any of the other elements of disparity already discussed. Hence, it is not disposed of by the adjustments suggested, and cannot be mitigated except by extending the range of goods for which comparable prices can be found. Such extension, as already indicated, can be made only by dint of further analysis of the two types of society, and by a search for more common denominators than are evident on the surface. This is only one more argument for more intensive study, particularly of pre-industrial societies.

If we assume that prices can be found for all goods in both countries, the national product of country A can be valued in prices of country B and compared with national income of B; and the national product of country B can be valued in prices of country A and compared with the national income of A. Differences in the patterns of goods of the two

[8] See some observations in this connection in the Appendix. The examples given there could, I suspect, be easily multiplied by anyone who would take the trouble to compare prices first for internationally traded crude commodities, and then for nonexportable (or nonimportable) types of commodity and service.

countries then cease to make comparison impossible. But they introduce an entirely different type of difficulty, *viz.*, they give two measures of what is essentially one difference: for the ratio of national income of A to B, when measured in prices of A, may be different from the ratio of national income of A to B, when measured in prices of B.

It need not be labored here that the two ratios would differ only if the relative quantities of goods in the two countries differed. Were the goods structure of national product, *i.e.*, its percentage distribution among the various goods (including those with zero weight, *i.e.*, absent) exactly identical, then no matter how the price structure (*i.e.*, relative prices of goods) differed, the ratios would be the same in the two indexes. Similarly, were the goods composition of the national product of two countries different, but the price structures identical, the two ratios would be the same. In fact, in comparisons of industrial with pre-industrial countries, both the goods and the price structures are likely to differ materially; and as a result the ratio of the national products of the two types of country will differ as we weight the quantities by prices of the industrial or of the non-industrial economy.

Thus, given differences in price structure, those in the goods composition of the national products inevitably result in a lack of determinateness of the difference between the national products of the countries. Only the upper and lower limits are set—the ratios of the two national products weighted first by the price system of one country, then by the price system of the other. At present, we do not know how far apart the limits are; but further studies in the field would be well worth while.

One can do no more than suggest the direction of the bias involved in using as base the price system of one or the other type of country. In general, relative price and quantity differentials tend to be correlated negatively: if a good x is priced much more highly than a good y in one country, other conditions being equal, the quantities of x produced and consumed will be in a smaller ratio to the quantities of y in that country. In other words, there is some adaptation of the goods structure of a country to the relative price structure. This means that when we revalue the quantities produced in a pre-industrial country in prices of the industrial country, we tend to assign too high a set of price differentials to goods with relatively large quantity weights, and too low a set of price differentials to goods with small quantity weights. This tends to impart an *upward* bias to the national-income totals of pre-industrial countries; and since they are in general much lower than the totals for industrial countries, the ratio between the two tends to be reduced. Per contra, when we revalue the national product of an industrial country in the prices of a pre-industrial country, we impart an upward bias to the

national-income total of the former and thus tend to magnify the ratio between it and the national income of the latter.

Consequently, the common practice in current national-income literature of revaluing the national product of pre-industrial countries in prices of industrial countries tends to impart an upward bias to the former and to reduce the disparity.[9] In this sense, the bias is in the right direction in that it serves to reduce the downward bias implicit in confining the comparison of price *levels* to internationally traded crude commodities. But with limited coverage of the price comparison, the differences in the price *structures* of the two countries are also underestimated; so that the upward bias due to using the price structure of the industrial economy as a base is minimized. One may, therefore, reasonably argue that in current practice, the downward bias in the evaluation of national product for pre-industrial countries due to the limited coverage of price comparisons, is much greater than the upward bias resulting from using the price structure of the industrial country as the base.

The range between the limits within which the ratio of the national products of the two types of country falls is likely to be increased as the variety of goods for which price comparisons can be made widens. In other words, as the difficulties of proper comparison between the national products of industrial and pre-industrial countries, due to lack of comparable prices, are overcome, the second type of difficulty—associated with differences in price structure—is likely to become more prominent. This is as it should be: as our knowledge of both types of economy becomes more adequate, the problem of establishing unequivocal quantitative comparability should become more complex. As such knowledge accumulates, it will be seen that, by accepting the valuations implicit in the price system, we are in fact accepting two yardsticks which, each applied separately, naturally produce different results. The eventual solution would obviously lie in devising a single yardstick that could then be applied to both types of economies—a yardstick that would perhaps lie outside the different economic and social institutions and be grounded in experimental science (of nutrition, warmth, health, shelter, etc.).

9 The discussion is in terms of the price *structure*, *i.e.*, of relative prices differentials among identical goods, not in absolute price levels. Prices of identical goods are in general much higher in industrial than in pre-industrial economies, higher than the official conversion rates of currencies indicate (with the exception of the highly complex products of industrial civilization). The adjustment for the *level* of prices, therefore, almost uniformly reduces the spread between the national products for the two types of country when measured in official currencies and converted by means of official exchange rates. It is the effect of the price *structure* that is different as we take the price system of the industrial or of the pre-industrial economy as the base.

This consideration brings us beyond the plane of intellectual discourse on which national-income estimates at present rest. But it is not irrelevant that the ease with which national-income comparisons, among countries with differing industrial and social structures are currently made, may largely be due to the shallowness of our knowledge and to our willingness to stay on the surface of social phenomena. As knowledge increases, it may be more rather than less difficult to make effective comparisons within the present frame of reference.

VII

In applying the above suggestions to an illustrative comparison of the national incomes of China and the United States (see the Appendix), we followed the path of least resistance. Rather than employ selective reduction and inflation, we raised the estimates for China to compensate for both possible omissions in them and the elements of grossness that are peculiar to the estimates for the United States. The purpose of this calculation is not to provide conclusive or even semiconclusive results, but to test the feasibility of the suggestions and to get some idea of the size of the adjustment involved.

In this comparison, which uses Mr. Liu's gross-national-product estimates for China for 1931-36, the adjustments applied to secure greater comparability with the United States figures, yield a per capita product for China of $73 (U.S.); and a per capita consumption of $65½ (U.S.). The latter figure can be compared with that established by Mr. Liu of $37 (U.S.) per capita after his adjustments for differences in price levels, in the marketing structure of agriculture, in the supply of unpaid domestic services, and in the ratio of consumption to gross national product. While the application of the adjustments suggested here raises the per capita figures by almost 80 percent, the calculation in the Appendix still takes no account of the downward bias implicit in the price comparison (we accepted Mr. Liu's estimate for this item); or of other sources of lack of comparability that might raise the figure for China even higher. Taking these into consideration could easily bring per capita consumption in China to over $75 (U.S.), or over twice Mr. Liu's adjusted figures. The experimental calculation yields, therefore, two significant conclusions: first, the adjustments suggested above are feasible and can be applied even with the present limited supply of data; second, the adjustments are sufficiently big to affect markedly comparisons between industrial and pre-industrial countries, and change materially the results of comparisons that have been made in the last two decades.

While the discussion, and the calculation, have so far been in terms of national-product totals alone, the points raised are relevant to every

important type of distribution. Thus, the usual industrial allocation of national income is affected by the fact that many extra-market activities in a pre-industrial economy elude measurement: these activities are in the nature of either manufacturing (or construction) or the provision of personal and other services. So far as they are omitted, and to an extent presumably greater than similar activities in agriculture, the industrial distribution of national income for a pre-industrial economy would show too large a share for agriculture and too small a share for other industries. The overinclusion of certain activities in national incomes for industrial countries would exaggerate the shares of some industries, *e.g.*, transportation, distribution, housing.

The distribution of income by size is also modified. Even pre-industrial countries have upper income groups that tend to be heavily concentrated in cities. In a national-product estimate that follows closely the conventional rules of industrial countries, the incomes of urban population are likely to be more completely covered than the incomes of rural; which means that there is more complete coverage of upper than of lower income brackets. Any more inclusive treatment of extra-market activities in pre-industrial economies or adjustments for the elements of grossness in the estimates for industrial countries are likely to shift the income distribution by size in favor of the groups at the lower end, thereby reducing the inequality of the income distribution as shown in unadjusted or incompletely adjusted distributions.

The effect on the percentage allocation of national product between the flow of goods to consumers and capital formation is somewhat different. As suggested above the elements of omission and grossness that affect comparability may well be relatively small for such items of capital formation as industrial construction or industrial machinery and equipment. If so, the adjustments advocated here, when applied in a specific rather than a crude wholesale fashion, may raise the flow of goods to consumers sector of national income of pre-industrial countries by a greater relative proportion than they would capital formation. Thus while the absolute magnitude of capital formation in pre-industrial countries and its ratio to capital formation in industrial countries may be raised, the ratio to the flow of goods to consumers within pre-industrial countries may well be lowered.

This suggestion applies to the real volume of capital formation or investment. A somewhat related point concerns the distinction between outlay and savings. In all countries, even advanced industrial, some categories of consumers' outlay include elements of savings in the sense that the purchase is guided, at least secondarily, by the utility of the good as a storage of value (luxuries that tend to have stable values). In pre-industrial countries, with the prevailing limits for safe productive

investment, such purchases of consumer goods, which in fact represent hidden savings, may well loom much larger, relative to total consumers' outlay, than they would in an industrial country. So far as they do, the volume of savings exceeds the volume of domestically financed productive investment or capital formation.[10]

These brief comments suffice to indicate that the attempt to introduce greater comparability between the national-product totals of pre-industrial and industrial countries affects also the comparisons of internal distributions by industrial source, by size classes, or by type of use. This is natural since closer analysis of the contents of the national products for the two types of country reveals differences that have to be recognized in a proper comparison, differences that have a differing impact upon the industrial, size, and use classifications, traditional in industrial countries and often applied without modification to pre-industrial economies.

VIII

In conclusion, I would like to stress what it is that we are *not* trying to measure by means of national-income estimates, and indicate why. Such comments may prevent misunderstanding as well as suggest lines of exploration other than those stressed here.

National income, as we conceive it, measures the flow from the productive system, but not the inclusive consumption totals for the economy. There is a significant difference between the flow of goods to consumers and what Joseph S. Davis calls the consumption level.[11] The latter includes, in addition to the current flow of goods from the productive system, the yield of goods owned by the consumers; and excludes from the current flow to consumers goods they do not actually consume during the period. With the much greater stock of goods in the hands of individuals and households, the consumption levels per capita in an industrial economy may well show a greater relative excess over per capita consumption levels in a pre-industrial economy than might be revealed by

[10] This is of importance for the analysis of the savings-potential of pre-industrial countries in connection with plans for industrialization. Since conditions of political security and extension of productive investment opportunities are involved in any effective industrialization, the savings potential is suggested not only by the past flow of savings into productive investment but also by such elements of consumers' outlay as would become unnecessary with the progress of industrialization. The latter comprise the purchases of luxuries intended largely as a storage of value; and expenditures closely connected with traditional pre-industrial culture (funerals, feasts, religious observances, etc.), whose practice is likely to be greatly reduced by the secularizing influence of industrialization.

[11] See his presidential address, "Standards and Content of Living," *American Economic Review*, Vol. 35, March, 1945, pp. 10-15.

a comparison of the flow of goods per capita—no matter how fully the latter is covered.

Nor are we trying to use national-income estimates to measure what Professor Davis calls the level or plane of living, which includes, in addition to consumption, working conditions, cushions against major and minor shocks, freedoms of various kinds, and other spiritual constituents of social life. These ingredients of living are extremely important in spelling happiness and unhappiness; and it is easy to conceive of situations where the consumption level rises yet the plane or level of living declines; *e.g.*, when the rise is attained by sacrifices in working conditions or by loss of freedom of a kind highly prized by the population.

Finally, we have not tried to push the analysis of national income estimates in the direction already mentioned, *viz.*, of gauging the degree of satisfaction of wants ascertainable by experimental and scientific methods and in disregard of purely economic valuation imposed by society. Thus, we accept the valuation of foods as provided by the markets, attempting only to make both terms of the comparison (*i.e.*, in an industrial and pre-industrial country) equally inclusive, and employing, for identical foods, the same prices. We are not trying to convert the foods into vitamin equivalents and thus translate physical quantities into vitamin content, completely bypassing the market valuation. Nor are we trying to do the same for clothing, fuel, shelter, and the like.

The refusal to extend discussion in these directions—of fuller coverage of consumption levels, of levels of living, and of experimentally established functional equivalents—is not due to the possibly low yield of such explorations. On the contrary, they promise results of great value. They might explain, more satisfactorily than can be done otherwise, the basic differences between industrial and pre-industrial economies, and the conditions which favor or disfavor industrialization. As already suggested, they might provide a more effective basis for comparisons and help overcome the difficulties imposed by differences in the goods composition of national product. Studies of nutrition indicate unmistakably that pre-industrial economies manage to obtain the basic vitamin supply at much lower economic costs, and hence at much lower prices, than a price comparison of *identical* commodities would indicate.

That we have paid little attention to these aspects of the comparison is due largely to a feeling that study has not advanced sufficiently to permit abandonment of the more traditional approach, via the customary definitions of national income or product. At any rate, I did not feel competent to discuss the problems that would emerge in any direct consideration of consumption levels, planes of living, and functional equivalents. It does seem, however, that as customary national-income estimates and analysis are extended; and as their coverage includes more

and more countries that differ markedly in their industrial structure and form of social organization, investigators interested in quantitative comparisons will have to take greater cognizance of the aspects of economic and social life that do not now enter national-income measurement; and that national-income concepts will have to be either modified or partly abandoned, in favor of more inclusive measures, less dependent upon the appraisals of the market system.

We can view national-income comparisons among countries in the light of an entirely different set of basic criteria. Rather than concern ourselves with national product as a flow of goods to consumers present and future, we can view it as a measure of a nation's power—defined broadly as power to impose upon the rest of the world conditions which, for one reason or another, are considered favorable to the given nation. Whether we further specify such power to consist of the ability to provide security, or to expand the area of sovereignty, both the concept and comparison of income between industrial and pre-industrial countries will differ widely from those used in the analysis above. Many elements of the industrial economy we considered gross because they represented an offset to extra costs of urban civilization are not gross from the viewpoint of national power; for in armed conflict, the crucial weapon in the exercise of such power, many appurtenances of urban civilization can be temporarily sacrificed and the resources used for them diverted into other channels. Many elements of pre-industrial economy whose inclusion in national product we urged because they contributed to the flow of goods to consumers should perhaps be excluded from national income as a measure of national power because these decentralized extra-market activities cannot be effectively mobilized or controlled by the state in case of an armed conflict (even though they may be immensely useful in passive defense). Indeed, for many aspects of national income as a measure of national power, the relative disparity between industrial and pre-industrial countries is very much greater than the customary estimates of national income indicate, even before the adjustments suggested above.

We did not touch upon this line of approach for two reasons: its application to income measurement has not reached a point where its potentialities and problems are clear, and it differs so sharply from the customary approach that to include both within the bounds of a single paper would be impossible. But it seemed important at least to mention the approach, to invite attention to its implications, and to suggest that the ordinary impressions of the vast relative difference between the economic performance of industrial and pre-industrial societies may well be colored by vague thoughts concerning differences in national power, rather than in supplying goods for the satisfaction of consumer wants.

These comments on the potentialities of explorations in the direction of a better analysis of the contents of living or of relevance to national power should not be interpreted as minimizing the importance and usefulness of national-income analysis on the more orthodox level discussed here. Granted that from the viewpoint of contents of living (or national power) national income, as ordinarily measured, stops halfway. It is a compromise in the sense that it accepts the valuation of the market place, with some adjustments, but without probing too deeply beneath the surface of economic phenomena; and if a national-income estimate for one country is a compromise, a comparison for an industrial and pre-industrial country is a compromise of compromises. Nevertheless, it is an enormously useful device for measuring, if proximately, the magnitudes of performance of the economies, and providing a quantitative framework within which the weight of significant sectors can be gauged. Nor does the realization that we deal with compromises free us from the necessity of looking closely and assuring ourselves that on the level of comparison accepted, the scope and basis of valuation are as truly comparable as they can be made. The major burden of this paper is that there is room for improvement even in such proximate measures as currently defined national-income totals, improvement for purposes of comparing the totals and their components for industrial and pre-industrial countries. These opportunities for improvement can and must be pursued, before analysis can be extended in any direction that transcends or differs materially from the level of current national-income estimates.

Appendix

Illustrative Calculation of the National Products of the United States and China on Comparable Bases

1. *General Plan*

We chose the United States and China largely because recent estimates for both are available, and particularly detailed ones for the former. The two main sources are: for the United States—the supplement to the July, 1947, *Survey of Current Business* (referred to below as *DCS*) and Ta-Chung Liu's *China's National Income, 1931-36* (Brookings Institution, 1946)

For the United States, gross national product is divided into: (a) commodities; (b) services not embodied in new commodities. For the former we calculate the over-all ratio of the final cost of finished commodities to the value of raw materials consumed, at producers' prices. Among

services not embodied in new commodities we segregate groups that are comparable with the measurable service performance in a pre-industrial economy like China. For these several groups, whose magnitudes are approximated in the United States estimates, we assume reduction ratios reflecting the extent to which such services in a pre-industrial economy are either carried on within the family and community or to which they are superfluous, being in fact but offsets to the disadvantages peculiar to a highly developed industrial society.

The two ratios derived for the United States—of finished commodities to raw materials and of the gross volume of services to a net volume representing comparable net coverage in a pre-industrial economy—are available for application to the data for China. Neither ratio allows, however, for differences in the price level between the United States and China for *identical* raw materials or *identical* services.

Our treatment of the data for China consists, therefore, of: (a) adjusting the raw-material flow for differences in prices; (b) raising the latter by a ratio of finished products to raw materials; (c) adjusting comparable services not embodied in new commodities for differences in prices; (d) raising the result under (c) by a ratio of gross to net services.

2. *Allocating United States National Product between Commodities and Services*

For 1931-36, the gross national product of the United States is estimated to be $68.26 billion per year (*DCS*, Table 2, p. 19). Of this total, net foreign investment, averaging $133 million is not easily allocable between commodities and services; we therefore omit it, reducing national product to $68.13 billion per year.

Commodities account for by far the major part. The commodity sector comprises (annual average for 1931-36): durable and nondurable products flowing to ultimate consumers—$31.92 billion (see *DCS*, Table 2, p. 19); gross private domestic investment—$4.14 billion (*ibid.*); and the commodities purchased by government. The latter can be approximately set at the amount governments purchase from business enterprises: these may include some services but probably relatively few. Government purchases from business enterprises are estimated to be $3.93 billion per year (see *DCS*, Table 9, p. 23). Thus the commodity total, per year, for 1931-36 is $40.0 billion. The service total comprises the service sector of consumers' outlay, averaging for 1931-36 $22.64 billion (*ibid.*, Table 2, p. 19); and payments by governments for services of employees —$5.45 (*ibid.*, Table 9, p. 23)—a total of $28.1 billion.

3. *Calculation of the Ratio of Finished Commodities to Raw Materials*

The raw materials that flow into finished commodities are the products of agriculture and mining. Their basic components for 1931-36 are, therefore, approximated by taking the gross income for these two industries (*i.e.*, gross sales and products retained, adjusted for intra-industry duplication alone). The annual averages involved are $8.3 and $2.3 billion, or a total of $10.6 billion (see Simon Kuznets, *National Income and Its Composition*, National Bureau of Economic Research, 1941; pp. 543 and 551).

However, some of these raw materials may go into exports and not become embodied in finished products purchased by consumers, by government, or by business enterprises (for capital formation); and, per contra, some of the finished commodities may be from imported materials. We must, therefore, adjust the total just derived for imports and exports. Imports of raw materials, raw foods, and semifinished manufactures averaged for 1931-36 $1.16 billion (see *Statistical Abstract of the United States, 1944-45*, Government Printing Office, 1946, pp. 532-533); exports, $0.99 billion (*ibid.*), the latter amount to be scaled down 10 percent to the level of producers' prices. The net balance of imports over exports is therefore $0.27 billion ($1.16 – 0.89). Hence, the crude-materials total, which forms the denominator of the fraction we are trying to estimate, is, for 1931-36, $10.9 billion per year ($10.6 + 0.3).

The numerator is the commodity total of $40.0 billion, derived above, also adjusted for imports and exports. Exports of manufactured foods and finished manufactures in 1931-36 averaged $1.07 billion, a total that need not be adjusted for transportation and distribution charges since the latter are part of the spread we are attempting to calculate. Imports of finished products averaged $0.67 billion (*Statistical Abstract, 1944-45*, pp. 532-533). The net balance of $0.4 billion added to the commodity total, $40.0 billion, yields a numerator of $40.4 billion. The ratio is, therefore, 3.71.

This ratio is for a commodity total that includes the elaborate items of producers' durable equipment and construction as well as highly fabricated consumers' durable products (automobiles, radios, household electrical equipment, etc.) that find little counterpart in pre-industrial economies. The ratio we need should have been calculated only for the sector of raw-materials and finished-commodity flow that comprises the simple types of product—foods, clothing, other nondurable goods, furniture, etc. A more elaborate calculation of this type, more directly relevant to our purposes, is beyond the scope of this paper. We therefore arbitrarily reduced the ratio from 3.71 to 3.25 to allow for the inclusion

of these more complex commodities specific to an industrial country alone. This is a fairly generous scaling down, in view of the fact that consumers' durable commodities (not all of which should be omitted), construction, and producers' durable equipment (averaging per year \$4.8+2.1+2.9) amount to only \$9.8 billion, less than one-fourth of the comprehensive commodity total of \$40.0 billion. The reduction of the ratio by about one-seventh associated with a presumptive exclusion from the total of somewhat less than one-quarter means that the ratio for the excluded part is much larger than that for the nonexcluded part (indeed, the implicit ratio for the durable part is 6.6).

Both to check on the commodity ratio just derived for 1931-36 and to demonstrate what a large proportion of the difference between crude materials and final cost of finished products is due to functions other than manufacturing proper, we calculated the components of the difference. As already indicated, total finished commodities average \$40.4 billion per year 1931-36; the cost of crude materials, \$10.9 billion. The absolute difference, \$29.5 billion per year, may roughly be accounted for as shown in Table 1.

TABLE 1

	Annual value per year, 1931-36 (\$ billions) (1)	Ratio (2)	Total (\$ billions) (3)	Source *DCS* Table (4)
A Net income originating in				
1 Manufacturing	11.27	1.0	11.27	13
2 Contract construction	1.35	1.0	1.35	13
3a Trade	8.08	0.8	6.46	13
b Rr. transportation	2.25	0 8	1.80	13
c Highway, water, pipe line & related transportation services	0.80	0.8	0.64	13
d Business, banking, legal, engineering services	1.90	0.8	1.52	13
B Depreciation & capital consumption	7.58	0.8	6.06	4
C Business & sales taxes	2.52	0.8	2.02	8
Total	35.75		31.12	

The deduction of 20 percent for items A3, B, and C is intended as a rough allowance for the part relating to crude materials proper and entering their value, or to services not embodied in new commodities and hence not relevant to commodity-ratio calculation.

The total for comparison is \$31.12 billion (col. 3). No precise check is attempted here, nor is one feasible. But the rough congruence of the totals shows that the huge difference between the value of crude materials

and the cost to ultimate users of the finished commodities they enter can easily be accounted for by the totals of net income originating in the industries handling them and the additional items of depreciation and taxes.

Of more interest in the present connection is the fact that of the total spread, production activities proper account for little more than half. If of the depreciation total (item B) we allow about one-half as chargeable to products of manufacturing and construction (before they are transported and distributed), the strictly fabricating functions account for $15.65 of the total, $31.12 billion. The rest is associated with transportation, trade, and other services. These figures make it easier to see that the very high commodity ratio characterizing a highly developed industrial economy is due in only small part to more elaborate fabrication; a great deal of it is accounted for by extensive transportation and intensive handling in distributive, credit, and other service channels.

4. *The Analysis of Services*

Of the total volume for services, $28.1 billion, 1931-36, the major item is services flowing to ultimate consumers, $22.6 billion. The details in *DCS* Table 30 suggest the following rough functional classification.

The first group are services for which there is a clear counterpart in a pre-industrial economy, but of which a large proportion is carried on within the family and the community and perforce escapes measurement —domestic service, personal care, recreation, religion, funerals, etc., care of clothing and furniture, etc. For this category, labeled A, a rough calculation (for 1931-36) suggests that they amount to 21 percent of all services flowing to ultimate consumers. Allowance must be made for the fact that many of them are performed within the family economy and are not reflected in orthodox estimates; and others (such as religion and recreation) are of a type, as suggested in the text, in which no clear case of greater per capita supply can be made between countries differing widely in social pattern. For purposes of comparison with a pre-industrial country the value of the services can at least be halved.

The second category, B, is the part of service flow to ultimate consumers that represents chiefly adjustments to the money economy. The foremost example is the large group of business services in the Department of Commerce classification (brokerage and bank fees, union dues, employment-agency fees, insurance, foreign transaction, etc.). This category accounts for roughly 17 percent of total consumers' outlay on services; and the reduction for the extent to which it represents costs of an industrial economy rather than net returns must be large. We set the reduction at eight-tenths of the total.

The third category, C, comprises expenditures of urban populations on services whose value is grossly inflated by the extra problems and difficulties of urban life. The most conspicuous example is urban rents (cash or imputed). Others are commutation, communication costs, and the like. This category accounts for 50 percent of total services to consumers, and it should be cut at least in half if the real-net-product element in it is to be comparable with that in an essentially nonurban society—still disregarding price differentials for *identical* goods.

The fourth category in consumers' outlay on services, D, comprises services that contain no element of grossness, and that, on the whole, are likely to be as fully recorded in the ordinary estimates for pre-industrial countries as they are for industrial. The foremost examples are medical services or services of education. This category accounts for roughly 12 percent of total outlay by consumers on services, and no reduction should be made in it.

Finally, we come to services purchased by government rather than by ultimate consumers. As argued in the text, a major part of governmental activity is intermediate rather than final product; hence this category, E, contains elements of grossness not unlike those in category B. However, similar elements of grossness may also be included in the estimates for pre-industrial countries; and while in any comparison between the two some reduction may be in order, it should be fairly small—about one-fifth.

TABLE 2

Service category	% of total flow to consumers, 1929, U.S. (1)	Reduction ratio suggested (2)	Col. 1 reduced (3)
A	21	0.5	10.5
B	17	0.8	3.4
C	50	0.5	25.0
D	12	0.0	12.0
E	24	0.2	19.2
Total	124		70.1

The following items were included in each category (the numbers for A–D refer to those shown for various service groups in *DCS*, Table 30).

A: II–5, 6, 7, 8, 9, 10, 11; III–2, 3, 4; V–10, 11, 12, 26, 27, 28; VI–7, 9, 12, 13, 14, 15, 16, 17; IX exclusive of all commodity components; XI.

B: VII–3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18; XII–1 excluding 1c.

C: IV–1, 2, 4, 5; V–20, 21, 22, 23, 24, 25; VIII–1d, 1f, 1g, 2, 3.

D: II–13; IV–3; V–13; VI–3, 4, 5, 6, 8, 10, 11; X.

E: *DCS*, Table 9, compensation of employees.

The ratio of the total flow of services to that considered truly net and comparable is as of 124 to 70.1 or 1.77. For purposes of comparison, any standard estimate of services for a pre-industrial country like China should be raised by some such ratio, even if price differentials for identical service items are disregarded.

5. *Recalculation of the Estimates for China*

We may now apply the results to the estimates for China, taking advantage of the data in Mr. Liu's book and the similarities in the concepts used by him to that of the Department of Commerce.

Crude materials flowing into domestic consumption can be estimated first. The average per year, 1931-36, of the gross value of agricultural products and of mineral and metallurgical output, for the 22 provinces amounted to 16.89 billion yuan (for the former see Liu, Table 11, pp. 35-40; for the latter, Table 19, p. 51). With the allowance of 11 percent for the missing provinces, the total amounts to 18.75 billion yuan per year. Let us assume further that no raw materials were imported and that all commodity exports were raw materials—an assumption that tends to minimize the value of raw materials flowing into domestic consumption and hence the value of the national product. The average annual total of exports, including the adjustment for undervaluation, is 771 million yuan; deducting 10 percent for the adjustment to the level of producers' values (a patently small deduction) the figure becomes 694 million yuan per year (see Liu, Table 28, p. 69). Subtracting it from the already derived annual average output of crude commodities, 18.75 billion yuan, leaves 18.06 billion per year.

According to Mr. Liu's calculation, the difference between prices of identical commodities in China and the United States, estimated from a comparison of crude commodities alone (rice, wheat, other grains, beans and peas, sweet potatoes), 1931-36, resulted in undervaluing Chinese commodities 47 percent (see pp. 73 and 75). Before any conversion by means of the customary exchange rates, the value of Chinese crude material in yuan must, therefore, be raised 47 percent. The next step is to allow for the ratio calculated above of finished commodities to crude materials, 3.25. Hence, for an estimate in yuan, directly convertible by means of official exchange rates, the value of finished commodities in China's gross national product must be derived by multiplying the value of 18.06 billion yuan per year, first by 1.47, then by 3.25. The result is 86.28 billion yuan.

The service component of China's gross national product can also be derived from Mr. Liu's figures. The average annual value of professional and domestic service, 1931-36, is approximately 3,438 million yuan (see Table 27, p. 66); of value added by governmental and educational

institutions, 882 million yuan (see Tables 23 and 24, pp. 55 and 58); of imputed house rent (farm alone), 1,620 million yuan (see Table 11, pp.35-40). For 22 provinces the total of these services not embodied in new commodities is therefore 5,940 million yuan per year; and with the 11 percent increase for omitted areas, becomes 6.59 billion yuan.

This total must be raised to adjust for the differences in prices for identical services in the two countries; and further for the ratio calculated above, of the gross to the net element in the services. Mr. Liu has no data for prices of services, and in fact employs the price differential derived from a comparison of crude commodities, For lack of information we follow his practice. The adjustment then consists of multiplying 6.59 billion yuan, first by 1.47, then by 1.77. If the result, 17.15 billion yuan, is added to 86.28 billion, the total derived above for the commodity component, annual gross national product is 103.43 billion yuan.

These are yuan that are directly convertible to United States dollars by the official rate of exchange: 1 yuan = $0.2886. The total $29.85 billion, can be compared with that derived by Mr. Liu as a result of his adjustment for differences in price levels, in the marketing structure of agricultural production, and in the extent of of unpaid family services. With these adjustments Mr. Liu raises China's gross national product, 1931-36, to $16.68 billion per year (see p. 85). The present adjustment thus raises Mr. Liu's adjusted total by $13.2 billion, or another 79 percent.

In passing from gross national product to consumers' outlay, we may accept Mr. Liu's figure of 10 percent for gross savings (see pp. 86-87). Consumers' outlay per year amounted, in terms of equivalent purchasing power in United States dollars to $26.87 billion. With a population taken at Mr. Liu's figures of 410 million, per capita consumption is $65.5, rather than the $37 derived by Mr. Liu. For the same period, consumers' outlay per capita in the United states was estimated by the Department of Commerce to be $433.

6. *Concluding Comments*

The experimental calculation above is admittedly susceptible to criticism, and particularly to revisions entailed by a more specific and elaborate application of the basic assumptions. But if the latter are granted, one is justified in claiming that the magnitudes assigned to the adjustments are moderate. The ground for increasing them, thereby reducing the difference between the national products of United States and China even further, are:

a) The coverage of crude materials in the estimates for China may well be less complete than in those for the United States, partly because

some agricultural and mining crude materials escape measurement in China to a greater extent than in the United States.

b) The ratio of finished products to crude materials, 3.25, may be on the low side. An increase would add proportionately to the commodity sector, and to national income.

c) The price differentials between China and the United States are probably underestimated, largely because the figure used by Mr. Liu is heavily dominated by basic foods freely entering international trade. The crude materials that do not move as freely, either because they are too perishable or bulky or because they supply local demand primarily, are likely to exhibit much greater price differentials. Even in Mr. Liu's five agricultural commodities, those moving in international trade—rice, wheat, other grains—show price differentials of from 27 to 50 percent of the price in China; whereas beans and peas and sweet potatoes, which are of more local use, show differentials as large as 80 and 118 percent. A simple recalculation, in which the combined differential for beans and peas and sweet potatoes is given the full weight of all agricultural products except grains, would raise the five-commodities price differential from the 47 percent calculated by Mr. Liu to 70 percent. This adjustment alone would raise the national product for China 15 to 16 percent beyond the $29.85 billion established in our calculation.

d) The price differential for services is likely to be greater even than the 70 percent just suggested. Mr. Liu estimates per capita income for professional services to average roughly 105 yuan per year (Table 26, p. 65). This, for a family of six, works out to 630 yuan per year, or at official exchange rates, to about $180. In the United States per capita compensation of employees in professional activities would average well over $1,000 and of entrepreneurs in a field like medical service well over $2,000. It is difficult to assume that the quality differential is such as to bridge the difference between some $300 ($180 × 1.7) and say $1,500 to $2,000.

These considerations suggest that further analysis might bring the per capita estimates for China and the United States even closer.

Résumé

1. L'examen des comparaisons courantes du revenu national pour les pays industriels et les pays pré-industriels, même après l'ajustement pour assurer la comparabilité des estimations, montre que les chiffres des pays pré-industriels sont trop bas. Les estimations pour les pays

pré-industriels excluent une grande partie de ce qui devrait être inclu; celles des pays industriels incluent une grande partie de ce qui devrait être exclu; lorsque leurs capacités respectives ont été rééquilibrées, il devient difficile d'établir un dénominateur commun.

2. Une partie importante du travail productif de l'économie pré-industrielle est effectuée au sein de la famille ou de la commune, indépendamment du marché. Ceci peut comprendre des travaux aussi apparents que la fabrication de matières premières, ou des services aussi effacés que ceux d'un système familial étroitement uni pour protéger ses membres individuels contre les désastres, ou leur donner des secours relatifs à leur bien-être spirituel. Il est douteux que les calculs puissent tenir compte de la pleine valeur de ces services qui n'ont aucun rapport avec les marchés; dans l'économie industrielle, cependant, ils dépendent presque tous des maisons de commerce et des éléments destinées au marché et doivent être compris dans les estimations du revenu national.

3. Dans l'économie industrielle, une grande partie du travail est dirigée en vue de parer aux désavantages de l'organisation économique et sociale, inconnus dans l'économie pré-industrielle. Le produit de ces travaux est plutôt intermédiaire que final et devrait être exclu de tout calcul du produit national net employé aux fins de comparaison. On peut distinguer les catégories suivantes: (a) la fabrication au dehors, le transport, les services de distribution rendus nécessaires par la concentration géographique de l'industrie et des distances à couvrir entre les producteurs et les fournisseurs de matières premières et entre les producteurs et les consommateurs; (b) les frais supplémentaires que la vie urbaine impose aux consommateurs, ceux-la constituant l'accompagnement indispensable de l'organisation productive de l'économie industrielle (déplacements réguliers, transport, coût de la vie plus élevé, etc.); (c) les frais extraordinaires occasionnés par la participation à une économie monétaire et de crédit compliquée indispensable à une structure industrielle progressive (frais de banques et autres, cotisations à verser syndicats, et autres dépenses plus ou moins en rapport avec les affaires); (d) le total du revenu national qui comprend toutes les dépenses gouvernementales pour les produits et les services, les frais gouvernementaux extraordinaires causés par la gestion et la préservation de la structure économique complexe de l'économie industrielle.

4. Il est impossible d'obtenir les prix comparatifs dans les pays industriels et les pays pré-industriels lorsqu'il s'agit de marchandises qui ne sont produites et consommées que dans un seul type de pays. On peut ordinairement résoudre ce problème en cherchant le prix de quelques produits identiques et en appliquant aux autres produits le rapport ainsi obtenu. Les produits dont on peut facilement établir les

prix comparatifs sont ordinairement des produits bruts avec marchés internationaux très vastes; il s'agit des produits pour lesquels la différence de prix est maintenue aussi petite que possible par l'influence compensatoire du commerce international. Il s'ensuit que les différences de prix entre les pays industriels et les pays pré-industriels sont ordinairement sous-estimées et que les comparaisons basées sur cette technique amplifient l'inegalité des revenus totaux et des revenus par tête d'habitant. Même s'il était possible d'obtenir le prix de toutes les marchandises produites dans les deux types de pays, le rapport entre les revenus nationaux évalués d'après les prix des pays pré-industriels différerait du rapport de ces mêmes revenus nationaux évalués d'après les prix des pays industriels. Ainsi la comparaison des deux revenus nationaux ne donnerait pas un seul chiffre, mais se trouverait en quelque sorte placée entre les limites constituées par les deux rapports.

5. Des calculs expérimentaux sont faits avec les données pour les Etats-Unis et pour la Chine, afin d'illustrer comment on peut faire disparaître quelques-uns des éléments de non-comparabilité avec les renseignements disponibles.

THE EFFECT OF THE ROLE OF GOVERNMENT ON INTERNATIONAL COMPARISONS OF NATIONAL INCOME

by Arthur Smithies
Assistant Chief, Fiscal Division
Bureau of the Budget (United States)

There are four types of national-income comparison that can usefully be made:

A. Money-income comparisons

(1) Comparison of changes over time in the composition of money incomes—or, more significantly, national products of different countries.

(2) Comparison of the composition of money national products of different countries at the same time.

B. Real-income comparisons

(1) Comparison of changes in real income of different countries over time.

(2) Comparison of real incomes of different countries at the same time.

A. Money-Income Comparisons

Attempts to compare total money incomes as between countries could only be valid if changes in official exchange rates were proportional to changes in the relative price indexes that should be used to convert money incomes into real incomes. Official exchange rates in general do not satisfy this requirement. Attempts to convert money incomes by using an index of relative purchasing power are identical with real-income comparisons discussed under B(2).

Consequently, so long as money income alone is considered, we must confine attention to the distribution of national product.

During the war, study of the national amounts became an important source of intelligence. The degree of mobilization in the various countries was measured by the proportions of their national incomes they were devoting to war production.

In the same way after the war, the national-income amounts will be important indicators of policy, from the point of view both of changes in policy and of what a country is doing at the present time. I suspect that the ratio of military expenditures to national income will be one of the more important statistics in international politics.

Finally money-income figures may be useable to assess contributions to international organizations. I shall discuss this later.

(1) The simplest problem is to compare changes in the composition of the national product over time. Despite all defects in the figures and concepts, I think it can safely be said that if the proportion of Government expenditures to national income as officially measured is increasing faster in one country than another, the Government is in fact bringing under its direct control a relatively greater part of the economy of the former country.

(2) If we want to compare say the proportion of total economic activity in the control of the Government, it is necessary to make certain adjustments to the national-income figures in order to achieve comparability.

(a) National income should be computed at factor cost, that is indirect taxes and subsidies should be deducted. It may also be necessary to consider some profits taxes as indirect if they can be shown to be shifted.

(b) Government expenditures for goods and services which should be considered to be intermediate rather than final products must not be counted twice. Depending on the purpose of the investigation they should be deducted from the total expenditures of Government or from the value of the final products of private industry.

(c) The question of profits causes the greatest difficulty. There is no item in the Budget for the payment for government services of entrepreneurship analogous to private profits. In the United States as distinct from most other countries, there is no estimate in the national-income figures for the "profits" of Government business undertakings. To achieve comparability therefore it is necessary either to impute profits to Government activities or to eliminate private profits from the national income at factor cost.

My preference would be to eliminate private profits since it is by no means clear what profits should be imputed to Government, and it is also not clear that private profits are payments to a factor of production. In fact, there are some indications that corporate-profits policy could most appropriately be treated as excise taxes and losses as subsidies privately imposed. Part of noncorporate profits should of course be considered payments for management. To exclude private profits would have the added advantage of avoiding difficult problems related to the different degrees of monopoly in various parts of the economy.

It may turn out that money-income comparisons are the most satisfactory way to deal with the question of international contributions. If national incomes are comparable from the conceptual point of view an equal percentage levy would mean each country was required to devote the same proportion of its current production to the international purpose. If it is desired to make the system of assessment progressive,

it may be possible to do this purely on internal evidence. For instance, if one country saves a greater fraction of its income than another, that could be regarded as prima facie evidence that it could stand a higher percentage rate of contribution. Other internal indicators can be found. This method could only be justified, however, if exchange rates are in equilibrium from the balance-of-payments—not necessarily the purchasing-power-parity—point of view.

B. Real-Income Comparisons

When real incomes are compared, we necessarily have to deal with a welfare question with all its difficulties. A sensible meaning can be given to the statement that real income has increased or that real income is greater in one country than another only if we are entitled to say that an increase of real income means an increase of welfare.

Strictly, real income can only be used as a welfare index if purchasers of final products are free to buy any quantities they like at the prices at which they are offered. It is only then that relative prices are equal to marginal rates of substitution. If that condition is fulfilled the Laspeyres and Paasche indexes of quantity between them can be used in most cases to indicate the direction of changes in well-being. The usual process of deflating money national income by a price index is to be regarded as an approximation to a quantity index.

Government activities necessarily mean that this condition is not fulfilled. In the first place, the Government as a purchaser does not act as the collective agent of the public. Although it exercises preferences when it imposes taxes and spends on national defense, we cannot say that the relative prices of national defense and other things indicate relative marginal utilities. Second, services provided to the public through the Budget are fixed as to price (taxation) and quantity from the point of view of the consumer. On the other hand, a Government business undertaking can fulfill our condition if it is prepared to sell as much as consumers want to buy.

In view of these difficulties, what validity can be attributed to a measurement of real-income that includes a large Government contribution. What was the significance, for instance, of real-income computations over the war period, when Government purchases absorbed almost half insofar as they measure potential civilian production. If the figures used for tank and military-aircraft production and prices indicate the automobiles, trucks and civilian aircraft that could have been produced, the figures are useful and meaningful. In the same way services provided to consumers by the Government can be included in the estimates if it is not unreasonable to think of consumers buying those services if they had freedom of choice.

Thus, if the patterns of production and consumption are determined by the Government for a long enough period, real-income estimates may lose any validity they once had, since reference back to a free-choice period becomes increasingly difficult. On the other hand, however wide the range of Government activity, the estimates may remain useful provided the Government allows freedom of choice over a wide enough field.

The question at issue is essentially one of how far the validity of an index number depends on the accuracy of the weighting system. Since the prices and quantities of the components of national income are not highly correlated, we do not have to be too rigorous in applying the theoretical requirements to indexes of real income.

Let us now consider the specific questions posed above:

(1) Comparison of rates of change of real income of different countries. If we can satisfy ourselves that the index of real income is valid for each country, there is no objection to comparing their rates of change notwithstanding the difference in the roles of Government. If, for instance, the real-income index of country A has increased by 20 percent over a given period, while that in country B has increased only 10 percent, we may conclude that economic well-being is increasing more rapidly in A—depending of course on the statistical error in the indexes.

(2) When we attempt to compare the levels of real income in different countries, our difficulties increase. There are three possible situations in general.

(a) The scale of preference in the countries in question may be assumed to be the same. In this case the same index-number methods used for comparison between different times in one country can be used.

(b) It may be necessary to assume that the scale of preferences differs between countries. In this case it is possible only to apply the prices of one country to those of another and to infer, for instance, that if country B had the same preferences as country A, economic well-being in one or the other country would be greater.

(c) The physical conditions of production and climate may differ so greatly among countries that the same commodities have very different significance. For instance, services of central heating have functions in the United States that is very different from, say, the Philippine Republic. Does the United States have a higher real income because it has to incur costs to keep warm? Where there are differences of this kind, any attempt to compare real incomes becomes hazardous.

In all these situations the part played by Government increases the difficulties of making significant estimates. The problem does not differ in kind from that of intertemporal comparison in one country, but the possiblity of error is much greater.

It may be that the objectives of a State-controlled economy are not to increase well-being in the sense of economic satisfactions to individuals. A comparison that asssumed the economic objectives of such a country and those of one that did aim at maximum satisfaction were the same would naturally have very little significance.

Nevertheless real income comparisons will and should continue. In some cases they will be meaningful. In others they will be little more than a fiction that is used for want of something better. About all one can say in defense of the weaker comparisons in practice is that they put no undue strain on the imagination.

Résumé

Les comparaisons des revenus nationaux peuvent consister en comparaisons de revenus en monnaie, ou de revenus en nature.

A. Comparaisons des revenus en monnaie.

Les différences relatives dans la composition du revenu en monnaie au cours d'une période peuvent servir à indiquer les changements dans le rôle du gouvernement dans la vie économique de divers pays.

Avant de comparer la composition du revenu en monnaie de différents pays, il est nécessaire d'apporter certains ajustements aux chiffres. Il faut, par exemple, éviter le double emploi des services gouvernementaux qui doivent être regardés comme produits intermédiaires de l'industrie. Des ajustements sont nécessaires aussi pour tenir compte du fait que les chiffres gouvernementaux ne comprennent pas des profits imputés.

B. Comparaisons des revenus en nature.

La conception du revenu en nature est compliquée par les activités gouvernementales. Un indice du revenu en nature doit être conforme aux demandes d'un indice de bien-être. A ces fins, il faut que les prix utilisés comme poids reflètent les préférences des consommateurs. Quand le gouvernement fixe tant le prix que la quantité des biens, cette condition nécessaire n'est pas toujours satisfaite. Cependant, puisque les prix des biens qui sont compris dans le produit national ne sont pas en corrélation étroite avec leurs quantités, il n'est pas nécessaire de répondre trop rigoureusement aux exigences théoriques.

Même si des difficultés initiales peuvent être surmontées, les comparaisons des revenus en nature entre pays demeurent hasardeuses, sauf quand les préférences individuelles et les conditions physiques dans les pays comparés sont semblables.

THE MEASUREMENT OF NATIONAL WEALTH*

by G. Findlay Shirras

Professor Formerly of Economics and Dean of Faculty, University College, Exeter (United Kingdom)

I. Object of Paper

The object of this paper is twofold. In the first place it suggests that each country should prepare not only annual estimates of national income but also estimates of national wealth at least every five years or, if practicable, annually. In the second place it attempts to estimate the national wealth of Great Britain and Northern Ireland before the war and in the last year of the war.

By national wealth is meant the total goods within a country owned by the inhabitants in their individual or corporate capacity. It includes the foreign possessions of nationals and their Governments but excludes the possessions belonging to the inhabitants of other countries. It excludes the free gifts of nature, such as climate, waterfalls, and the fish in the sea, and also living capital or the value of people which Sir William Petty and his followers in "political arithmetic" attempted to value. The expression "national capital" is sometimes used as an alternative to "national wealth" whether the capital produces a monetary return or not. The monetary system of the country is the general standard of measurement, The principal way in which national wealth increases is by abstinence from enjoyment.

Statisticians of eminence in almost all the chief countries of the world have made estimates of national wealth at different times, especially when circumstances were such as to excite interest in questions relating to the resources of a country or countries. The justification for such calculations is the information on a nation's economic and social position. When national debts are large as they are today the concept of national wealth is important. National debt is a charge against the whole property of the community including the funds themselves in favor of the same community.

The recent emphasis given to national-income estimates should be extended to national-wealth measurements. Before and during the war the concept of national income and the statistical estimates of its magnitude in various countries have been at the basis of much current economic discussion. Such estimates have been used to great advantage

*With the consent of Professor Shirras, his original paper has been condensed to about half its length by the volume editor.

to measure war burdens by statistics giving the percentage of national income devoted to war in different countries. Since 1941 in Great Britain an analysis of the national income and expenditure is presented to Parliament at the time of the annual Budget, showing the structure of the national income and containing estimates of the growth or loss of capital and the effect of taxation on the distribution of income. Although interest has reawakened in national-wealth measurements, the number of up-to-date treatises and papers on the subject is small.

Scattered studies of national wealth in various countries have appeared from time to time during the last half century. Let us hope that the International Statistical Conferences now taking place in Washington will consider the necessity of national-wealth estimates in addition to those of national income.

II. Uses of National-Wealth Estimates

There are five main uses of such estimates:

(1) As a test of progress between different years in the same country. Is the country growing more prosperous? Changes in the value of money, *i.e.*, the rise or fall in the price level and the rate of interest, must be considered in this connection. Anything that changes prices must change the valuation. Changes in prices between two different dates where the changes are considerable must be allowed for in estimating the growth of national wealth. The distribution of wealth among private persons, corporate bodies (educational, medical, religious, etc.), local authorities, and the State has to be considered.

(2) As a test of relative prosperity or resources between countries, either as a whole, or per head of population. The rate of increase in wealth as compared with population should not be overlooked. The United States, for example, has the largest national wealth in the world but only six percent of the world's population. India, on the other hand, is only a little more than half the area of the United States but its population of 400 million is three times the American total and its ability to build up its national wealth is small in comparison with that of the United States.

(3) For a comparison with national income. A nation should endeavor to increase its income in relation to its capital. Capital should be made to go further than previously. There are, however, great difficulties in relating national income to national wealth for a number of countries, in regard to the methods employed in compiling the estimates, the available statistics as well as the concept itself varying considerably between countries. It is difficult in the same country to obtain accurate measurements of the capital used in industry.

(4) For the purpose of a capital levy and examining the tax structure as a whole. Detailed information in regard to the national wealth is of much value in the imposition of the burden.

(5) For reparations. As the recent discussions have very clearly shown, statistics of national wealth are of value in fixing reparations, although this cannot be the sole test. Other factors, such as national income, the distribution of income, and potential wealth, have to be considered. Great care must be taken not to exaggerate national wealth.

III. Methods of Estimating National Wealth

There are four main methods for the measurement of national wealth: (1) The income method. This is the capitalization of actual income accruing to private persons individually or corporatively at the average prospects of the continuance of that income. This is supplemented by estimates of public property not yielding an income as well as of that yielding an income. (2) The estate or probate method. This method attempts to ascertain the proportion between what passes to successors in a year and the total wealth of the community, or in what number of years the same estate will be recharged to duty on the average. (3) The inventory or objective method which aggregates each kind of wealth. (4) The census method. In this method the individual fills up a form showing his wealth and income.

(1) There is a mine of information in the income-tax statistics of the United Kingdom and in the view of the present writer it is the most satisfactory method for this country in spite of the criticism by supporters of the estate method that it tends to exaggerate profits. The Board of Inland Revenue's Annual Report publishes five schedules of income-tax assessments, three of which are invaluable. Schedule A gives the income from the ownership of lands, houses, and other buildings; Schedule C the income from British, Dominion, and Foreign securities where such income is based at source; and Schedule D the profits from business, professions, and certain interest. The accepted method of ascertaining the national wealth depends on the number of years' purchase or the rate of interest used to capitalize the annual income or profits. It does not register the true accumulation of capital because an increase may be exhibited where no actual outlay was made. It capitalizes future prospects. In Table 1 the national wealth is the aggregate of public and private property, less capital belonging to people abroad (gross wealth), and less the national debt and local authorities' debts (net wealth). This treatment of national debt has not received universal acceptance and is, in fact, still in dispute. Our treatment may reduce the net total to an unnecessary degree but on the whole it seems best to deduct it to get a true net wealth.

TABLE I

THE NATIONAL WEALTH OF THE UNITED KINGDOM (GREAT BRITAIN AND NORTHERN IRELAND) BEFORE AND AT THE END OF THE WAR

(in millions of £)

Item	1937–38	1944–45
1. Real property		
a. Land	950	1,080
b. Houses and other buildings	5,181	5,298
2. Farmers' capital	475	712
3. Capital of industry, transport, commerce, and finance		
a. Stock-exchange value of assessed profits	6,896	12,477
b. Retail distribution	1,415	1,058
c. Finance, professions, and other profits	1,947	3,049
4. Railways in Gr. Britain and N. Ireland	800	1,100
5. Interest		
a. British, Dominion and Foreign securities (sch. C) and war securities not taxed by deduction at the source, deposit and other interest (sch. D)	7,110	11,562
b. Other dominion and foreign securities and possessions	1,514	1,441
c. Value of savings certificates not assessed for income tax	386	1,511
6. Unrevealed values, loss and evasion	500	250
7. Capital of nonincome-tax-paying classes	100	100
8. Movable property yielding no income—furniture, motor cars, wireless, works of art, plate, etc.	1,500	1,400
9. Government and local property	1,400	1,600
10. Total	30,174	43,538
11. Deduct capital belonging to people abroad	500	750
12. Gross Wealth	29,674	42,788
13. Deduct debt charges	9,706	24,108
14. Net Wealth	19,968	18,680

National wealth per head, national income per head, and national wealth expressed in number of times national income

	1937–38	1944–45
1. Population in millions	47.7	45.2
2. Total wealth per head (£)		
1. gross	622	946
2. net	418	413
3. Total national income (millions of £ at market price)	5,741	10,090
4. National income per head (£)	120	223
5. Wealth expressed in number of times of national income	5	4

Table 1 summarizes the investigations into the national wealth of Great Britain and Northern Ireland for the years 1937-38 and 1944-45. The detailed explanation of each step has to be omitted from this condensed version of the paper; the most important considerations are in respect to (1) the capitalizing of industry, transport, commerce and finance, and (2) the capitalizing of interest. The main items, 14 in number, except 2, 8, 9, and 10-14, have been arrived at by a careful examination of the income-tax schedules of the Board of Inland Revenue.

The results for 1945 are of special interest when compared with those for 1938. While the national income per head doubled, the national wealth per head certainly did not, and this is not to be wondered at when the destruction of houses and other buildings during the war years amounted to £1,500 million, of shipping and their cargoes to £700 million, and internal and external disinvestment brought the total loss of national wealth to £7,300 million.

(2) Next as to the estate or probate method. This is a favorite method of calculating national wealth in France and in some other countries such as New Zealand. It deals with only part of the total property, *viz.*, that belonging to private persons above a certain sum, £100 in Great Britain. The proportion of property in private hands passing with death is determined and the reciprocals are then used to multiply the value of estates passing in those years. The chief difficulties connected with this method, apart altogether from its incompleteness, are these arising from evasion and undervaluation of an estate, especially in times of changing prices, and also those connected with changing vital statistics. Then there is the question of gifts *inter vivos* which keep down the amount of private property passing at death in each year. I hold that the estate method omits so much that is included in the income method that, *ceteris paribus*, comparison is impossible even after deducting from gross wealth arrived at by the latter method the capital value of corporate and other property not included in the estate method, which, be it remembered, deals only with individual estates.

(3) The inventory method, which has been used by many countries, notably the United States and Canada, aggregates each kind of wealth—land, houses, agricultural products, machinery of all kinds, factories, mines, ships, transport, etc. The statistics used have usually been compiled for other purposes and are unfortunately divorced from profit earning. It is not always possible to avoid overlapping or to prevent omissions or to be certain that the wealth belongs to individuals in the country.

(4) The census method requires the filling up by each individual in the country of a form relating to his whole income and wealth. The

best-known example of this method is, perhaps, Knibbs' *Report of the War Census of 1915*, published in 1918. The drawback of the method is in accuracy, since individuals fear that the information may mean heavier tax burdens. Additions have to be made for public and social wealth.

Table 2 attempts to give some recent statistics of national wealth for countries other than the United Kingdom.

TABLE 2

ESTIMATES OF NATIONAL WEALTH FOR VARIOUS COUNTRIES

Country	Source	Year	Unit	National wealth	National wealth per head
United States	Conference Board	1938	Million $	309,430	$2,376
	Doane (*Anatomy of American Wealth*)	1938	,, ,,	388,421	$2,982
Canada	Canada Year Book	1933	,, ,,	31,275	$2,412
Australia	Official Year Book 1933	1929	,, £ A	3,351*	£ A526
New Zealand	Official Year Book	1941	,, £ N.Z.	905	£N.Z.566
Palestine	Official	1945	,, £	281†	£161
Norway	Official	1939	,, Kr	31,372	Kr 10,319
Italy	Gini	1939	,, Lire	700	L 15,051
	Coppola d'Anna	1938	,, ,,	672	L 14,467
Netherlands	Official	1939	,, Fl	34,950	Fl 3,882

*Private wealth

†Excludes the value of urban-land buildings and improvements and the value of public fixed assets.

For want of space I have not been able to put forward all the details connected with the method of calculation of the United Kingdom national wealth, but I hope to be able to do this on some future occasion. Also owing to lack of time at the Washington Conferences, I was unable to put forward some points which I regard of importance in all future inquiries. We are now equipped with statistical data that governments did not have some decades ago. We ought to be able to indicate in national-wealth measurements (a) the depletion of natural resources and (b) the extent of capital formation. The former is very important.

Finally, I should like to acknowledge and pay tribute to the technical assistance that I have received from the Director of the Statistics and Intelligence Branch of the Board of Inland Revenue, London (Mr. F. A. Cockfield, C.B.) and from Mr. F. R. Althaus, Partner, Messrs. Pember and Boyle, Stockbrokers, Princes Street, Bank, London. I

have received valuable material from the Colonial Office; the High Commissioners for Canada, Australia, New Zealand, and South Africa; from Dr. Per Jacobson of the Bank for International Settlements; and from Mr. Sargent B. Child, Director of the American Library, American Embassy, London.

Résumé

1. Les objectifs de cette communication sont les suivants:

(1) de proposer que chaque pays prépare non seulement des estimations annuelles du revenu national, mais aussi des estimations, annuelles si cela est possible, mais au moins tous les cinq ans, de la fortune nationale.

(2) de discuter les différentes méthodes par lesquelles on peut mesurer la fortune nationale ou le capital national.

Avant et pendant la guerre, la notion du revenu national et les méthodes d'estimations statistiques de son volume dans les divers pays étaient la base de nombreuses discussions économiques. On s'était servi des estimations du revenu national avec beaucoup de succès pour mesurer le fardeau financier de la guerre. Dans quelques pays, on se sert de ces estimations comme point de départ ou base d'une politique gouvernementale.

On pourrait, pareillement, se servir des estimations de la fortune nationale ou du capital national des manières suivantes: pour évaluer le progrès entre les différentes années dans un certain pays; pour comparer la prospérité relative et les ressources des différents pays, soit en montants globaux, soit par tête d'habitant; en tant que comparaison avec le revenu national; comme une source d'information préalable à l'imposition d'un prélèvement sur le capital; et comme un des éléments relatifs aux réparations.

2. Il y a plusieurs méthodes de mesurer la fortune nationale. Premièrement, il y a la méthode basée sur le revenu effectif, gagné par les individus ou par les entreprises, capitalisé à la prévision de la continuation de ce revenu. Les statistiques des impôts sur le revenu dans le Royaume-Uni forment une riche source d'information à cet effet; elles peuvent être combinées avec les données disponibles concernant la propriété publique et la propriété privée qui ne donne pas un rendement en argent—telles que l'estimation de la valeur du capital des fermiers, des meubles, des objets d'art etc. Cette méthode est appliquée

depuis longtemps dans le Royaume-Uni. Deuxièmement, il y a la méthode basée sur la statistique des successions. Cette méthode est appliquée sur tout en France et dans certains autres pays. Chaque année, des propriétés privées changent de mains à la suite de décès, et la proportion par rapport au total de propriétés privées doit être déterminée, et les réciproques sont employées ensuite pour multiplier la valeur des successions changeant de mains pendant ces années. La méthode se servant de l'intervalle de dévolution et celle se servant du taux de dévolution sont discutées. Troisièmement, la méthode objective ou d'inventaire est employée dans beaucoup de pays, y inclus les Etats-Unis, le Canada, la France, l'Allemagne, etc.

Finalement, on peut aussi se servir de la méthode du recensement. Cette méthode prévoit que chaque individu de la communauté remplisse une feuille declarant tous ses biens et son revenu. Un bon exemple de cette méthode est donné par feu Sir Georges Knibbs, Statisticien du Commonwealth d'Australie, dans: *A Report on the War Census of 1915.*

3. La méthode à suivre dans chaque pays dépendra des circonstances. Pour certains cas, plus d'une méthode pourra être employée. Mais en faisant des comparaisons, il faudra ne pas oublier les limitations de chaque méthode.

Discussion

Mr. Gilbert:

In the short time available, I should like to provide an introduction to Messrs. Tinbergen's and Derksen's paper on social accounting, and to direct a few remarks to the administrators of statistics-collecting agencies.

One might ask why we have turned to social accounting when we were getting along so nicely with the national income; why complicate life with all these elaborate tables? What meaning have they from the standpoint of economic and statistical analysis?

The first point I want to make is that social accounting was developed, not as an intellectual exercise, but through attempts by practicing economists to do their job more effectively. Let me cite my own experience, if I may. I was formerly the editor of the *Survey of Current Business*. Each January, I had to try to explain what had happened to the economy during the year just ended. I would look at the 2000-odd statistical series the magazine carried, and think hopefully, "When the national-income estimates are completed, they will provide the key to a synthesis of all these statistics—they will show the really significant developments of the years."

Then we would receive the current national-income tabulations, which at that time were limited to a breakdown by distributive shares, and they did not explain what had happened to the economy at all. In order to give an intellectually satisfying explanation, we found that we had to refer to all sorts of other data as well: data on inventories, on construction, on public fiscal policies, etc.

The reason, I think, was that the national income had been designed as a kind of yardstick for the economy, and not as a basis for analytic description of economic events. It was the possibility of broadening its scope, of building into an integrated system the national income and all these other elements needed for an analytic description, that attracted me in the National Income Division.

The development of the national income into such a system of social accounting has meant that the data now are organized in a way that suggests explanations of the economic events. And I think that this has given organic structure and more definite significance to the economic analyses which appear nowadays in the *Survey of Current Business*.

The second point I want to make is that social accounting carries a message for data-collectors, as well as for data-interpreters. I have frequently puzzled my students by asking this question: Suppose a given country has no statistics whatever; knowing the general characteristics of its economy, how would you decide what statistics should be collected to meet the needs of economic anaysis?

Most assuredly no student would base his answer on the considerations which usually guide the collection of statistics in actual fact: that certain data are required for the administration of certain laws, and that some bureau chiefs are more energetic than others. Even if the question were answered on the basis of the statistics needed to provide a measure of the national income, I submit that the answer would be incomplete. But if one conceives a national statistical program in terms of providing some integrated system of social accounting, then the statistics sought will be those actually necessary and sufficient for economic analysis.

It is only by having such an all-embracing framework that one can establish any order of priority in statistical collection. Otherwise the national "program" of data-gathering tends to be like the program of a variety show; the first act is an acrobat, the second is an Irish tenor, the third is a tap dancer, and so on, with no connection between them. The parts of the program do not make up a whole drama or tell one story.

The third point I should like to emphasize about social accounting is its role in insuring statistical accuracy. As long as the statistics of employment, prices, production, etc., are kept separate from one another, no one can judge whether they are mutually consistent. When we brought our statistics together in setting up our system, we discovered some amazing things about them. As you know, the system should balance; the credits should equal the debits, but they seldom do. And you are forced, thereby, to re-examine all the component elements in order to see where the errors lie.

If I were administrator of a data-collecting program, I should require every item collected to be so defined as to fit into such a system. If an agency set out to measure the profits or wages of the economy, it would measure not the profits or wages of 227 firms, but *total* profits or wages. There is no other way of knowing whether the data collected are meaningful. I think that all our statistics on quantity, price, and value must be fitted into this system. We have relied too long on payroll figures that do not equal employment times pay rates, and on value figures that do not equal quantities times prices. What meaning have these "basic data" which cannot be reconciled with one another?

I turn to Professor Shirras' paper on national wealth for one additional comment. Without intending criticism of what is a model paper on national wealth as such, I nevertheless have the feeling that it is in a sense a prewar model. Professor Shirras, too, is interested in devising a yardstick for the national economy. My own experience with yardsticks, notably the prewar national-income estimates, suggests that these measures fall short of their full usefulness when considered as entities in themselves. If we are to engage in research on the national

wealth, I think we should try to fit it into our system of social accounting. Instead of measuring a pile of something called national wealth, we need to develop balance sheets that correspond to and are consistent with our income accounts for the various sectors of the economy. These are what are most urgently required by practicing economists, who are typically concerned, *e.g.*, with investment-savings relationships and their impact on creation of income. Such purposes cannot be served by old-style national-wealth estimates any more than by old-style national-income estimates.

Mr. Clark:

Together with Professor Kuznets and Professor Shirras, I speak as one of the representatives of free enterprise in the national-income field. While welcoming the vast amount of new information collected by governments during recent years, at the same time there is much of which I am deeply suspicious. Why do Britain and some other countries publish aggregates only, making it impossible for any outside statistician to judge either their sources or their methods? This bureaucratic suppression of criticism is followed logically by the next step — alteration or suppression of figures for political purposes — and if anyone thinks I am going too far, let him look at what has happened to cost-of-living index numbers in so many countries, including the U.S.A., where the Commerce Department has to point out (as soon as the war was over) that the Labor Department index numbers were quite incompatible with other known data of money and real values of consumption.

Professor Kuznets, in a paper of fundamental importance, has pointed out that:

(i) *Many services which we have to pay for out of our money incomes are, in a simpler society, provided domestically.* Norway and Hungary attempt to meet this problem by including in national income a valuation of unpaid as well as paid domestic work.

If, however, we refrain from expressing national income in the aggregate or per head of population, and express it always *per man-hour of work done*, we can, in effect, get round this problem.

(ii) *Certain expenses, the need for which does not arise at all in a simple community, are difficult or in some cases (as with expenses incurred by Government departments) impossible for us to avoid.*

Transport and distribution of food is the most important of these items. I value food consumption in China at American retail rather than wholesale prices and obtain a real income per head of 65 international units (approximately dollars of 1941 purchasing power), while

Professor Kuznets, by a very different method, obtains a figure of 60—but more than half of my result consists of "imputation."

Professor Kuznets also has in mind expenses of travel to work, and much of what is included in Mr. Milton Gilbert's consumption entry for "personal business" (the work of insurance agents, trade-union organisers, lawyers, bankers, *et hoc genus omne*).

Germany and Sweden have attempted to distinguish between government departments that really add to the welfare of the community—*e.g.*, health, education, national parks—and those whose activities are a necessary cost to society without which our present social and economic life could not be carried on—*e.g.*, police, highways, and—shall we say—statistics?

Long and interesting researches lie before us in determining how many of these services really add to our welfare and how many are unavoidable costs arising out of our present structure of society.

To allow for the cost created by urban congestion, I have some data which I hope may be published shortly in which, by examining the way the populations distribute themselves through urban areas, I can express density of settlement in urban areas as a function of real income and of transport cost. This means that we can get some measure of the "disutility function" of congestion, which appears to vary with the cube root of the density. For big cities the aggregate disutility comes to a very large figure. It may be as much as 60 international units per year per head of population (total population, not working population); or a net 100 units or more if we include the cost of travel incurred in addition to the disutility of congestion. That can be put down as an unavoidable expense of urban civilization.

We can examine further the extra administrative costs of large aggregations of urban population. I refer you to Lomax's article in the *Journal of the Royal Statistical Society*, 1943, where he finds that the additional cost of administering large cities as compared with cities under 150,000 population comes to at least 2 percent of the national income. It follows that the deconcentration of urban population may become one of the most important issues in the next few years, and the existence of the atom bomb may have the effect of hurrying us up along the road that we ought to have followed in any case. There may be some contra-entries; *e.g.*, for some unrecorded[1] esthetic advantages which some people at any rate achieve from living in large urban centers. But, generally speaking, the balance will be one of a large deduction.

[1] *I.e.*, which do not find expression in our present national-income measurements (cultural activities that are paid for, whether from private or from public funds, are of course already included).

But I would conclude by coming back to the fundamental issue; our research into real income is an attempt to measure our ability to satisfy our desires. That is to say, our real and justifiable desires, not the immoral or antisocial desires that all of us sometimes may entertain. Applying this principle, a first point to be made is that in measuring the enjoyment we get from something, we should not try to include in our measurement any allowance for the additional enjoyment that we may get because we know that other people lack it. In the same way, we should not make any allowance for our failure to enjoy some goods because we know that other people have more. Those are the two desires of pride and envy respectively which are of their nature irrational and insatiable, and we should ignore them for our purposes. We are trying to consider our ability to satisfy rational and justifiable desires. Is there, as some philosophers have thought, some fatal flaw in our social life that inevitably compels our desires to increase faster than any possible means of satisfying them, so that the wealthy communities end up unhappier than the poor communities? If that is true, as I fear it may be, that is not our fault as economists, insofar as it is our business to see to the supply and distribution of goods and services and not to examine the motives for which people demand them. But the economist is responsible, I think, insofar as any actions in any part of the field supervised by him are responsible for *creating* desire rather than attempting to satisfy it. We have got quite enough legitimate desires that we cannot satisfy, to make the artificial creation of desire by means of advertising and salesmanship a false and dangerous proceeding. In measuring economic welfare (though I know that I am speaking terrible heresy in the United States), I must venture to discount, or even to deny altogether, the supposed satisfaction obtained from gratifying desires which do not exist naturally, and have been artificially created for the purpose of making a profit out of supplying them.

Mr. J. R. N. Stone:

I think it will be useful to draw a distinction between two quite separate purposes for which national-income investigations can be used. I shan't attempt any very exact definitions; but it is clear that the sort of purpose that Professor Tinbergen described is different from that of Professor Kuznets. Perhaps I may say that Professor Tinbergen was putting himself in the position of a government adviser who has to provide facts about the working of the economic system; whereas I think it is true to say that Professor Kuznets is more interested in getting down to the fundamental problems of the comparison of welfare.

Now I think these two purposes are quite different, and that one is asking a different set of questions in either case. I don't think there

is any doubt that in dealing with the first set of questions we can get along from the practical point of view by adopting a number of conventions in much the same way that business accountants do in keeping the records of individual firms. They may not be the same, but in using them we shall always be guided by the same fundamental idea that we have got to collect figures and therefore to adopt methods such that those figures are, or at least could be, provided in our economy. If we develop definitions and concepts that are non-operational in this sense, then as government statisticians we shall certainly fail. I think it is important to grasp this point, because there are a lot of problems that can be answered satisfactorily with the use of conventional definitions.

Following along this line of thought, we try, in portraying the structure of the economic system, to build up a set of social accounts in such a way that they reflect the different sorts of transactions between different parts of the system. We thus link up with Professor Tinbergen's idea that a statement of these transactions in a single period of time is not sufficient. In order to link different time periods together we need to know something about the way in which our society behaves, either as consumers and producers, or from the standpoint of the influence of the legal system or the state of technology. And so we develop a system of relationships that will connect up the transactions in our accounts and the other economic, legal, and technological variables that enter into the world that we are trying to explain.

Now for a few remarks about the other main use of national income studies. One of the problems in any work is to find out not only what questions we think we would like to have answers to, but also what questions we have any hope of answering. When I think of this whole question of an attempt to measure welfare, whether over time or between countries, I ask myself whether it is possible to give any concrete meaning to this term, which we can put down in statistical terms, other than the kind of meaning that we are driven to in developing the conventional accounting systems that I have spoken of. Just because we put our information in the form of an accounting system, of course it says nothing whatever about how that accounting system is constructed except that it would have certain formal properties of a very simple kind. But is it really possible to develop accounting systems that do not rely largely on ordinarily accepted conventions but seek to go behind these to something more fundamental? I don't know the answer to this question but I hope that Professor Kuznets in considering these problems himself will not reject the accounting approach because he doesn't like the conventional basis of existing accounting studies. The point is that an accounting approach will help, even when we are trying to measure

welfare, because it will enable us to see inconsistencies and the implications of what we are doing. I feel sure that an attempt to introduce, say, the family household as a sector in an economy would form an illuminating study.

But there is another aspect to this whole matter. Why do we want to compare the United States with, say, China or India ? What possible interest is there in it ? Everybody knows that one country is, in economic terms, very rich and another country very poor; does it matter whether the factor is thirty or fifty or what? I suggest that in default of having solved the intellectual problems, we should content ourselves with comparisons of a rather simple kind; and furthermore that we should not always expect to be able to sum up the relative position in a single figure. For I do not think that when we have made these comparisons we have really done a great deal. The figures in themselves may be useful; what I am drawing attention to is the question of comparing them and of making use of the comparisons. I feel that from the scientific point of view we should concentrate our energies on the attempt to solve the intellectual problems and in the meantime leave those in charge of affairs with no illusions about the exactness of the comparisons which can at present be made. In order to act, we don't need to know whether one country is just so much better off than another; rather we need to know a great deal about the situation of the two countries in its various aspects. The problem then is to make up our minds in terms of values, which are not matters that can be reduced to statistical facts, just what we would like and what we feel it is possible to do about the situation. Thus while I do not expect a very rapid resolution of the intellectual problems of making welfare comparisons between widely different communities, I do not think that as statesmen or as civil servants we need be unduly depressed on this score.

M. Perroux:

I. Le très intéressant exposé de Professeur J. Tinbergen et de M. Derksen est une contribution à la théorie appliquée du *Budget national*. Celui-ci à mon sens doit être considéré centralement comme une *comptabilité de gestion* à l'échelle de la nation. La comptabilité de gestion (par opposition à la compatabilité de caisse ou à la comptabilité financière) se définit: le relevé détaillé et convenablement groupé de tous les éléments propres à fonder les décisions d'une politique économique poursuivie par une unité quelconque, qui se propose de porter au maximum un résultat jugé avantageux. Une simple firme envisage, même lorsqu'elle ne les matérialise pas en plans *divers* systèmes de décisions, correspondant à des *hypothèses* diverses (chiffrées si possible).

La même méthode peut être employée à l'échelle de la nation; elle tend à assurer la *cohérence* des décisions et à reduire les divergences entre plans de l'Etat et plans des particuliers dans un mond dynamique. Cette méthode n'a pas été étrangère aux préoccupations des auteurs du Plan Monnet (*cf.* le *premier* rapport officiel sur le Plan).

Elle n'apparaît utilisable que que si l'économie n'enregistre pas de trop sensibles variations du niveau général des prix, et des niveaux des groupes de prix relatifs. La possibilité mme de dresser des tableaux alternatifs est liée à cette condition. Les quantités globales inscrites dans un budget national *ex ante* expriment l'adaptation présumée d'offers globales et de demandes globales pour un certain niveau des prix. En France, l'ètat actuel de nos statistiques sur le revénu ou produit global rend très problématique l'application détaillée des budgets flexibles. La stabilisation des salaires et des prix et la compression des coûts sont des tâches incomparablement plus importantes que l'alignement de quantitès globales jugées souhaitables. La reprise de l'investissement *privé* serait plus décisive que les indications sur l'investissement global *en l'absence* d'une politique *cohrente* de *contrôle* de l'économie et de *financement*. L'utilisation pratique et fructueuse des budgets flexibles dépend présentement de circonstances dont la conjonction est rare: une excellente comptabilité sociale, un *planning* cohérent, un contrôle efficace de l'économie, une *articulation correcte* des organismes techniques qui proposent les systèmes de solutions et des organes politiques qui poursuivent la réalisation d'un de ces systèmes.

II. J'ai examiné avec intérêt et profit la communication de S. Kuznets et suis heureux de lui dire publiquement l'admiration que les spécialistes français vouent à ses travaux.

J'avouerai n'être pas tout à fait convaincu par les conclusions de sa présente étude.

a) Je marquerai d'abord qu'une extension de la comptabilité nationale dans les pays préindustriels (suivant sa définition) présente les plus grands avantages, *quand bien même* elle ne permettrait *aucune comparaison.* Fût-elle incompléte et rudimentaire, elle permet une mise en place et une mise en oeuvre de la politique économique. Ella doit donc, en tous cas, être intensifiée et étendue.

b) Pour le surplus les *comparaisons* entre le revenu national des moins évolués des pays préindustriels et le revenu national des pays industriels, me paraissent *sans résultats utilisables.* Il n'en est pas ainsi seulement pour des raisons opérationnelles (manque de statistiques) mais pour une raison théorique et fondamentale: les comparaisons n'ont de sens qu'entre populations et économies de structures au moins grossièrement comparables. Bien entendu, l'ingéniosité statistique peut toujours produire des chiffres et la méthode préconisée par S. Kuznets est la

plus ingénieuse que je connaisse. Elle consiste à ajouter le produit hors marché au revenu des pays préindustriels, à retrancher du revenu des pays industriels divers coûts imposés par la structure économique et sociale, à évaluer les quantités ainsi obtenues en termes de prix des pays industriels et de prix des pays préindustriels: l'évaluation en prix, —la seule dont nous disposions—, ouvre des difficultés classiques (non homogénéité des prix et des régimes de prix), même quand il s'agit d'un même pays industriel ou de la comparaison entre deux pays industriels de structures très analogues. Mais que signifie, économiquement, l'appréciation que les habitants de l'Afrique Equatoriale Française, par exemple "*feraient*" de tous les biens et services nets obtenus pendant une période "*Si*" ces biens et services "*venaient sur un marché*" ? Que signifie l'apprèciation que les citoyens des Etats-Unis "*feraient*" des biens et services consommés ou investis, pendant une période, par les noirs de l'Afrique Equatoriale Française, si ces citoyens américains avaient à apprécier ces biens? Ce sont des extrapolations de cette envergure qui sont implicites dans la méthode préconisée.

On est sur d'avance, en l'employant, d'arriver à *réduire* l'invraisemblance constatée au début de la communication (§4). On est moins sur, me semble-t-il, d'atteindre à des résultats *économiquement significatifs*. Entre l'*abstraction statistique* et la traduction de la *réalité par la statistique*, la différence est le plus souvent difficile à repérer. Il y a lieu de craindre, dans notre cas, que nous donnions de bons examples de la première. Pour apprécier le bien être d'un pays préindustriel peu développé, pour le comparer à celui d'un pays industriel, pour apprécier les potentiels de production ou d'effort collectif, dans les deux cas, la comptabilité nationale, même perfectionnée paraît encore d'un bien faible secours.

c) Elle a, je le répète, un immense intérêt, *indépendamment* des comparaisons et c'est pourquoi à l'Institut de Science Economique Appliquée à Paris, nous nous efforçons d'attirer l'attention des pouvoirs publics sur l'opportunité d'améliorer (ou de créer) la comptabilité nationale de la France d'Outre-Mer.

III. Je regrette de ne pouvoir analyser de façon détaillée l'intéressante communication de notre collègue et ami, G. Findlay Shirras. Je voudrais seulement mentionner un point. La distinction de la *richesse* nationale et du *capital* national (capital de production à l'échelle de la nation), bien qu'elle soulève des difficultés statistiques et théoriques très connues, est fondamentale. En France (et dans plusieurs autres nations du monde) elle *n'a pas* fait l'objet d'une discussion théorique *modernisée* et approfondie et *n'a pas suscité de regroupements statistiques souhaitables*.

IV. Quant à la comparaison de la contribution des gouvernements au revenu national, que soulève M. Smithies, je me limiterai à deux

remarques: 1) Contrairement à une pratique fréquente une distinction même sommaire de la participation de l'Etat d'une part au produit final, d'autre part, aux produits intermédiaires me paraît indispensable; même quand elle n'est pas adoptée à titre principal, elle devrait être présentée sous forme d'alternative account. 2) Une théorie de l'Etat comme monopoleur de la *contrainte publique organisée* (cette dernière étant analysée comme un *bien économique*, substituable et complémentaire par rapport à d'autres biens) semble capable de rajeunir la discussion sur le produit net de l'Etat. Je me réserve, en cette matière, de développer ailleurs, sous forme d'article, le contenu de cette suggestion.

Mr. Lieu:

I do not pretend to be an expert on the measurement of national income. What I want to say concerns China's national-income statistics, which question has been raised by more than one speaker today. There are altogether twelve estimates of China's national income, most of which are perhaps not available to foreign readers, because they are written in the Chinese language. The range of these estimates is between 12 billion Chinese dollars and 68 billion. That is a very large difference which cannot be accounted for by price variations, because the estimates refer to periods during which price variations were not so great. Of these twelve estimates, four give detailed figures. Dr T. C. Liu has one on national income, and another one on gross national product; both of which are published in English in this country. Mr. P. S. Ou has an estimate of national income in Chinese, but I understand an extract of it has been published in English. The last of the four is made by myself. The range between these four detailed estimates is from 18 billion to 35 billion. That shows that when details are given, the differences become smaller. However, even then, there is some possibility of underestimating.

The largest portion of China's national income consists of income of the agricultural sector. For that sector, all three of us utilized the sample studies of Dr. J. L. Buck; and Dr. Buck's sample studies were made in parts of China where his university, the University of Nanking, had better contacts, and where they could get more details. There might be some bias due to the choice of regions. For instance, I made a statistical study of the agricultural economy in the county of Wuhsin in Chekiang, which is not in the regions covered by Dr. Buck, and income of the farmers there is much higher. There are also other sample studies of this nature that show different results.

A second difficulty is this: For the manufacturing industries Mr. Ou and I both used my 1933 census of manufactures, because that census

was comparatively complete. Even so, that census did not cover Manchuria, which had already been occupied by Japan. There were also no statistics of foreign factories, because they refused to supply figures. We tried to make up for this incompleteness by estimates, which might be considerably below the actual figures. The incomes of handicraft and other workers can only be guessed at.

Another important item for which we cannot get good data in China is the income of wholesale and retail trade. I believe that it produces a fair proportion of China's national income, because there are wide margins between the prices of goods received by the producers and those paid by the consumers. Of course, a part of such differences represents transportation costs. We do not have adequate data to estimate these two kinds of figures.

Mr. Evelpides:

Gentlemen, we frequently have a tendency to measure social welfare on the basis of the degree of national income, due account being taken, naturally, of the distribution of national income. But national income may be noticeably increased following certain factors such as inflation, increase in taxes, and customs duties. These factors lead to an increase in the cost of living and lead consequently to social misery. The method of dividing the net income by the index of the cost of living very often increases this basic error. It seems to me we can have a much better idea of the increase — or the lack, of increase—of social welfare if we compare national income not with indices, but either with the relative increase or decrease of such indices or with their trend. If national income increases more rapidly than index numbers, then social welfare increases, provided the distribution of national income, naturally, remains constant. But if index numbers of the cost of living increase more rapidly than national income, then there is a decrease in the social welfare prevalent in that country. The comparison must be made from year to year, and we must take into consideration the increase or decrease of all these factors, always calculated on the basis of 100. A double curve of increase of national income and of the index of cost of living may facilitate an easy understanding of the expounded method of the measurement of social welfare. We have applied this method over a period of years, roughly twenty years, in Greece, where fluctuations of the national income as well as of the index number of the cost of living were extremely pronounced; and we reached very satisfying practical results. I must say that an eminent economist arrived in Greece, studied the question of national income there, saw that there was an increase in the cost of living but a decrease

of production, and he concluded that everything was fine since the national income remained constant, or was even slightly increased; whereas, in fact, in the situation there was a decrease in the social welfare of the community. We must, therefore, be extremely cautious in this type of calculation and measurement; and that is why I gave you this example.

M. Divisia:

Je veux apporter mon accord d'ensemble, et mes compliments, aux très remarquables exposés que nous venons d'éntendre, auxquels j'associe celui de M. Derksen. J'y ajouterai quelques observations générales.

J'ai été frappé par la grande importance de cette notion de revenu flexible, que M. Tinbergen a conçue dans des circonstances particulières, mais qui a une portée universelle. Je crois, moi aussi, qu'il n'y a pas une notion unique de revenu national: tout calcul de revenu national n'est qu'un moyen, et doit donc être pensé en fonction du but poursuivi; or les buts peuvent être divers. J'en dirai autant de la notion de richesse nationale, rejoignant ici M. Shirras, et, au surplus, estimant qu'on ne doit pas séparer les notions de revenu national et de richesse nationale, pour cette raison que, dans un ensemble renouvelé, on ne doit jamais séparer le flux du stock.

La méthode des revenus flexibles me paraît essentielle, aussi, en raison de son caractère inductif. Et je voudrais, en passant, appeler votre attention sur la portée très générale de cet aspect, qui vise les diverses estimations d'une donnée statistique quelconque: Toute évaluation statistique est toujours le résultat d'une inférence, laquelle, à mon avis, ne peut jamais découler uniquement et objectivement d'aucune méthode purement logique; elle implique donc toujours une responsabilité d'homme d'action, qui oblige à se rendre compte de la gravité des conclusions qui sortiront des calculs. Ainsi, j'estime essentiel de connecter toujours les évaluations statistiques avec les problèmes qu'elles aideront à résoudre.

Notamment, M. Derksen nous a parlé de la comparaison et de l'unification des méthodes sur le plan international. Un des interêts de cette très importante question me paraît être de nous pousser à considérer les problèmes pratiques en jeu, et à préciser leurs buts, et même les buts de ces buts: Pourquoi unifier les méthodes? Apparemment, pour pouvoir comparer les résultats des divers pays. Mais pour quels problèmes prècis veut-on faire cette comparaison? Les évaluations à faire sont-elles bien les mêmes pour chaque problème, et aussi, le degré d'èxactitude requis?

La structure des calculs de revenu national, ou de richesse nationale, a donné lieu à des discussions interminables, que je crois sans issue sur

un plan purement conceptuel; au contraire, je pense qu'il doit être facile de s'accorder sur le caractère adéquat de telle structure pour tel problème posé.

Dès lors, je suggérerais volontiers, comme méthode de travail, de dresser une liste aussi complète que possible des problèmes pratiques impliquant le calcul du revenu national ou de la richesse nationale, et je pensè qu'il pourrait y avoir là l'objet d'une action internationale particulièrement intéressante.

J'aimerais voir dressèr aussi une liste des problèmes théoriques conçue dans le même esprit, car, tout en écoutant les éxposés de ce matin, j'ai senti une fois de plus, combien ces questions mettent en jeu des conceptions théorique qui ont besoin d'être encore approfondies, touchant notamment à la mesure de l'utilité et au revenu psychique introduit dans la science par Irving Fisher. Et, puisque je suis amené à prononcer le nom de ce grand Maître, qu'il me soit permis, en terminant, amené à d'adresser un hommage à sa mémoire.

Mr. Tinbergen:

In view of the short time available, I think that I will only try to answer a few of the most important remarks that have been made.

I have been asked by Professor Shirras how the figures on savings in the national account have been found.

Savings are composed chiefly, at least in my country, of three items, namely:

(i) business savings which we are able to estimate more or less satisfactorily,

(ii) savings by social security agencies and similar institutions which we know accurately, and

(iii) savings by families, particularly in the higher income brackets. These are very difficult to estimate, but for the moment they are almost zero, We are planning to organize some extensive sort of budget statistics for those income brackets, indicating only consumption as a whole, taxes as a whole, and savings in a few components.

I am in full sympathy with Professor Divisia, if he asks for the aims with which all these statistics and systems are being made; and in full agreement that we need a list of the problems and a list of the theoretical questions.

I do think that some contributions have already been made this morning to such a list of questions; particularly Mr. Stone, I think, has very clearly and frankly described, at least for my own work, what the purposes are. I need not go into that matter therefore now. Indeed, it is quite true that much could be done still to clarify it.

I may add one more remark, perhaps, as to the usefulness of national-wealth statistics. I think we have to attempt to establish production functions, and there are some of the components in the estimate, namely the active capital components, that are of importance to that type of work. We discussed that the other day after Mr. C l a r k's contribution.

Mr. Stone as well as Mr. P e r r o u x asked some questions about the flexible budget.

Mr. Stone was quite right in suggesting that they are something half-way between common-sense guesses and a treatment with the help of complete models, Indeed I think the use of flexible budgets is some sort of a safeguard against introducing contradictory figures. We are at least prevented from sinning against the balance equation, although we have no guarantee that we are not sinning against the other relations of a complete model. Sometimes we do not know these other relations very exactly; and until we know, we have to proceed in this more practical way.

I may add one example of the use we made of flexible budgets; they were useful, in my country, to the discussions on the measures to be taken in view of the dollar scarcity.

Mr. Kuznets:

This reply deals, necessarily briefly, with only some of the points raised by Messrs. S t o n e, R a o, and C l a r k.

1) I have no quarrel with the social-accounting approach as an instructive description of the working of selected economic institutions. I do have a quarrel with the approach if: (a) its conventions, borrowed from business accounting, become criteria governing evaluation of national income or national product (I use the terms as synonyms) as approximations to real flows; (b) practices of social accounting, as they have been followed, are not recognized for what they are—compromises geared to the analysis of short-term economic problems in Western countries; (c) followers of social accounting don't recognize that the selection of accounts often reflects an unconscious mixture of welfare with institutional considerations. Thus, illegal activities are not included, presumably because they do not satisfy any recognized ultimate wants; but other activities, of problematical relevance to such ultimate wants (*e.g.*, war expenditures) are fully included.

2) The paper has been criticized as pursuing the mirage of fundamental content, and as thus departing from canons ordinarily followed in empirical science. To my mind, it is just this pursuit of fundamental content that is the driving force in all scientific investigation, and empirical science has rarely progressed by keeping to the observable surface

of phenomena and forbearing from inquiries that lead to more searching analysis.

3) Questions of the type raised in the paper are of use beyond help in sharpening the tools of statistical analysis; directly or indirectly they can be of use also in problems of public policy. For example, in considering policies relating to industrialization, it is never safe to lose sight of the contributions made by the nonmarket elements in pre-industrial economy, an oversight altogether too likely with the conventional methods of measuring national income. The consequence may easily be policies involving needless and premature destruction of the domestic economy, the latter viewed as a producing institution—before the benefits of the "industrial" sectors that are being nurtured become available.

4) I fully agree with the suggestions of Dr. Rao and Dr. Clark of the need of further exploration in this field and that care must be exercised in classifying various economic activities viewed as contributions to some end-purposes. The aim of the paper was to raise questions that might stimulate such further exploration, rather than provide definitive answers. It is important to press such questions upon the attention of students in the field, even if no fully defensible answers are available. The overemphasis, if it be such, is distinctly preferable to underemphasis, since, with the latter, these questions are answered implicitly rather than explicitly—and not necessarily more satisfactorily because implicitly. The paper suggests a few modifications of the customary approach which are feasible with already available data and which should permit more significant comparisons between pre-industrial and industrial countries. What is even more important, they open avenues of further work that may yield results contributory to analysis and policy consideration of long-term, as distinct from short-term, economic problems.

Mr. Smithies:

I would just like to make one comment on the question of a single figure. It seems to me that at this stage of the proceedings to say that you should not have a single figure is very definitely locking the stable door after the horse has got out. It seems to me that one of the major problems of the present time is to steer the horse in the right direction. These figures have been produced and people use them. They will continue to be produced, and people will continue to use them. If we were starting afresh, I would have a great deal of sympathy with what has been said about not using a single figure, and not even producing one. But the way the thing stands now is that

in every governmental problem where a multiplicity of regions or countries is involved, national-income figures are used. Anyone who has had anything to do with the Federal system of government will know that real national-income figures enter into a multitude of discussions. And every international organization that has been formed has used national-income statistics in one way or another. Therefore, I think the statistician cannot bury his head in the sand in this matter. He should know the practical politicians will use his results and probably will misuse them. And therefore I do believe that it is imperative to make the best single figure that is possible and to use a few very simple rules for its application. I have no optimism that it will not be misused, but I think the statisticians still have a responsibility to assist in this rather important aspect of international negotiation.

Mr. Shirras:

I should like to say in the first place how grateful I am that the paper was received without too many comments on definitions, which rather pleased me. I am at one with everything said by Mr. Milton Gilbert, of whom I am very envious with his huge staff that gets out all sorts of detailed figures, and Professor Divisia and Mr. Colin Clark and others. What I assume now is that we should get down to a common definition of both national income and national wealth. Each country should collect data for each of the components of this definition so that whether any particular country accepts the definition or not it will always be possible to make comparisons between countries by using the same components. That in my view would be a great advance. The measurement of national wealth has again come into fashion and I am anxious that countries should undertake this measurement with the least possible delay. It would be exceedingly useful if all four methods were used for comparative purposes. Moreover, I agree with Mr. Milton Gilbert and I should like to see in national-wealth measurements careful estimates of gross and net capital formation. One of the most urgent matters for the International Association for Research in Income and Wealth is, in my view, the laying down of principles of measurement of national wealth for the guidance of countries. Some countries use the income method, others the estate-duty method, and others the inventory method. No attempt has yet been made to evaluate these methods and to set out the connecting principles. Mr. Smithies is perhaps rather optimistic when he says he is a great believer in army clothing because it fits everybody but nobody in particular. But especially in the field in which I have been most concerned, on the measurement of the burden of taxation, I think we ought to be

very careful when we use such terms as "national income" and "national wealth" to state exactly what we mean.

Mr. MacGregor (sent in after the meeting):

From the days of Sir William Petty estimates of national wealth have been complementary to those of national income, but scholars have emphasized now one and now the other. Since the end of the nineteenth century the emphasis has been swinging from wealth to income, probably because valuations of wealth made at rather long intervals become less representative of physical volume in a period of widely fluctuating prices and interest rates. Meanwhile the sources of information relating to income have been improving steadily. In common with many others I have favoured this change of emphasis and in a review written some years ago I criticized the Dominion Bureau of Statistics for continuing to publish estimates of the wealth of Canada. Since that time I have altered my opinion in some respects, and the purpose of these remarks is to support but at the same time to modify Professor Shirras' argument by stressing the usefulness of special-purpose estimates of certain branches of wealth.

It now seems clear that the valuation of wealth deserves careful attention for at least three reasons in addition to the study of production functions already mentioned by Professor Tinbergen. These are for: (i) estimates of capital formation, capital values being necessary for estimates of depreciation, maintenance, and repair, and for estimating changes in investment in circulating capital; (ii) independent estimates of capital formation by the inventory method where that is appropriate; and (iii) studies of depletion of natural resources.

The difficulty of securing meaningful valuations, which is the perennial objection to estimates of capital value, is not as serious for the first two of these as for other branches of national wealth. Thus, in the case of depreciation in manufacturing, accountants' estimates are available and may be used at least as a starting point despite the presence of bias. In the case of dwellings and farms, however, the statistician will probably have to depend on less technical estimates of capital values and apply his own rates of depreciation.

As to measuring capital formation by comparing inventories of capital values, this method must still be used where production figures are inadequate. It may be added that gaps or weak spots in statistics will doubtless remain as long as the interests of men prevail over the interests of book-keeping and tax-gathering, and it may always be necessary to have recourse to simpler and perhaps less accurate methods for some sectors of an economy, or for some countries.

The point which I want to emphasize is that the uses of capital values mentioned above do not call for a complete enumeration of national wealth nor do they involve trouble with the difficult borderline cases of immaterial capital and the "heritage of improvement" described by Edwin Cannan, or those encountered in British controversies over the capital levy twenty-five years ago.

Now as to depletion. It is almost always neglected. Indeed, I do not know of any country where it is treated seriously in social accounts, though Bowley, Pigou, and Lindahl show an interest in it, and Stanley Jevons and Sir Leo Chiozza Money stressed it in Britain many years ago. Since that time depletion has become more evident in areas where extractive industries are carried on, and is probably the largest factor in depressing certain incomes over the long term as many parts of North America have learned to their cost.

As to estimates of depletion, they lie in the fields where forestry, geology, and agronomy overlap economics. Many studies have been attempted and the first task of the economist-statistician is to appraise from his own standpoint the methods employed and indicate how they can be improved. I do not believe that it will ever be wise to offer full and detailed estimates; the pitfalls are so many that such a thing would be wholly meretricious unless as a stage in "estimating by parts." What I envisage is rather the gathering of information in terms of units for a few industries where exhaustion of resources is most serious. Valuations could then be applied from time to time if advisable.

Accretion must also be recognized, as in the case of growing forests or resources rendered economical by changes in prices or techniques or transport facilities. From an historical standpoint accretion may be important enough to warrant neglect of depletion in some cases, but not in all.

For these reasons I feel that Professor Shirras' argument would have been more convincing if he had stressed the importance of partial estimates of wealth for particular purposes.

CONTRIBUTED PAPERS: I

Monday, September 15, at 2:00 p.m.

CHAIRMAN:

Leonid Hurwicz
Associate Professor of Economics, Iowa State College (United States)

STRUCTURAL MATRICES OF NATIONAL ECONOMIES

by Wassily Leontief
Professor of Economics, Harvard University (United States)

This paper is presented in the form of a general report on a new method of quantitative analysis of the national economy, a method which passed more or less certain preliminary laboratory tests and even what we call in the United States the stage of a pilot-plant production. I will make an attempt, in this half an hour, to present to you the basic procedure without trying at the same time to anticipate possible comments and criticisms. These can be brought out much better in the course of the discussion.

The first, simplest approach to the study of the quantitative relationship within a national economy is obviously a partial one: an approach in which we center our attention on some small sector—it might be the wheat market or the production function of the steel industry or the shipbuilding cycle—and using the hypothesis of the partial-equilibrium theory try to fill in the necessary numerical constants on the basis of certain more or less sophisticated statistical procedures. However, it is quite clear that most of the most interesting and significant phenomena which we as econometricians want to study are related not to partial equilibria but to the general, over-all equilibrium, which involves more or less simultaneously all parts of an economic system, the type of the relationship which traditionally the general-equilibrium theory deals with.

Up to a relatively recent time it was considered to be too difficult, and certainly not very promising, to indulge in a real empirical general-equilibrium analysis. What was used instead, particularly since the

advent of Keynesian theory, was the shortcut device of aggregative analysis. It is an attempt to deal with a general-equilibrium problem involving implicitly all parts of the economic system, but at the same time to keep down the number of the variables by using extremely broad averages, *i.e.*, by dealing with such composite variables as the "total level of production," or the "general price level," "all exports," "total employment," or "average productivity," all of which are obviously very broad index numbers.

Anybody who was concerned with the practical application of econometric analysis, I think, is conscious of the fact that in a large number of instances, these aggregative measures are not very useful. Particularly in connection with many problems of policy-making and of economic planning of any kind, aggregative concepts are very limited in their application, because in this type of question we have to deal with concrete, separate industries, with individual prices, or at least outputs and prices of small commodity groups.

Sooner or later, the econometricians will have to devise some method of dealing with the national economy in rather differentiated terms comparable to those which he uses in partial analysis of a wheat or a steel market; but at the same time taking account of the general interrelationships between the separate parts of the national economy. The method that I shall now associate with the concept of a structural matrix of the national economy enables us to approach, very hesitantly and imperfectly, the solution of the problem of combining the general-equilibrium analysis with preservation of a differentiated classification of all individual aspects of the economic phenomena.

Let me first acquaint you with the basic factual material, which by its form of presentation suggests the type of theoretical handling that I shall tell you about subsequently. Table 1 is not a fictitious table; it contains actual statistics pertaining to the United States in the year 1939. The interpretation of these figures is very simple. The first row, for example, shows the amounts of the products of Agriculture and Fishing absorbed by various industries, totalling $12,575 million in the last column. Similarly, the first column shows the amounts of the products of various industries absorbed by Agriculture and Fishing. These figures represent the basic factual data on which all the subsequent analysis is based. Actually, we often use much more detailed figures; we have data for nearly one hundred groupings of American industries. There is no limit but the practical one — no conceptual limit to the amount of detail that can be put into this presentation.

Obviously these figures are not independent of each other. A planner could not possibly assign arbitrary values to all these figures. There is a necessary relationship between magnitudes in certain parts of this

table and magnitudes in certain other parts. There is, for example, a clear-cut relationship between the total output of a given industry and the total input that it absorbs of commodities and services from other industries. This is the relationship that Walras describes in terms of his production function, his coefficients of production, each coefficient describing the amount of any particular input necessary to produce one unit of the final output.

The figures are given in dollars, but may be considered as representing physical quantities if the physical unit is taken as one dollar's worth of each commodity in the given year. This is a very convenient way of using the data.

Now we can pass to the theoretical manipulation of these figures; and to do that I will present a few very simple, very elementary equations:

$$
\text{(I)} \quad
\begin{array}{ll}
X_1 - x_{21} - x_{31} - \dots - x_{m1} & = x_{n1}, \\
-x_{12} + X_2 - x_{32} - \dots - x_{m2} & = x_{n2}, \\
-x_{13} - x_{23} + X_3 - \dots - x_{m3} & = x_{n3}, \\
\dots\dots\dots\dots\dots\dots\dots\dots & \\
-x_{1m} - x_{2m} - x_{3m} - \dots + X_m & = x_{nm}, \\
-x_{1n} - x_{2n} - x_{3n} \quad \dots - x_{mn} + X_n & = 0.
\end{array}
$$

The system (I) of equations represents the equilibrium conditions of production and consumption (use) of each commodity. Each of the x_{i1} in the first equation represents the amount of the commodity of industry 1 used in industry $i = 2, 3, \dots$. The X_1 shows the total output of the industry 1 and hence the combined use. The total production equals the total use, *i.e.*, all uses combined equal output. But it is one particular use x_{n1}, namely the so-called final use in households or government, that I transfer to the right-hand side of the equation, since subsequently it is treated as an independent variable. In a similar way we can set up a separate equation to describe the balance between the production and consumption of the output of each other industry. The last of these equations describes the balance between the separate labor inputs absorbed by various industries and the total combined labor input X_n.

Now, using the coefficients of production we pass to the system (II) of equations which shows how to define certain constants a_{ik}:

$$\text{(II)} \qquad x_{ik} = a_{ik}X_i,\ i = 1, 2, \dots, m;\ k = 1, 2, \dots, m, n;\ i \neq k.$$

You can interpret these constants as representing the ratio between the input of a particular good in a given industry divided by the total output of that industry; in other words, it is input per unit of output.

TABLE

Allocation of Goods and Services by

[*All figures in*

INDUSTRY PRODUCING	INDUSTRY (1) Agriculture and fishing	(2) Food tobacco, and kindred products	(3) Ferrous metals	(4) Motor vehicles, industrial and heating equipment	(5) Metal fabricating	(6) Nonferrous metals and their products	(7) Nonmetallic minerals and their products	(8) Fuel and power	(9) Chemicals
1. Agriculture and fishing. . . .	950	4,998	....	....	....	....	....	....	176
2. Food, tobacco and kindred products	645	1,530	....	....	....	....	....	....	47
3. Ferrous metals. .	24	....	1,188	479	861	43	....	....	....
4. Motor vehicles, industrial and heating equipment. .	188	72	4	1,645	7	9	19	109	7
5. Metal fabricating.	433	306	37	611	717	12	5	137	40
6. Nonferrous metals and their products	5	23	109	117	221	1,325	4	51	89
7. Nonmetallic minerals and their products.	14	137	29	70	64	6	280	6	127
8. Fuel and power. .	474	168	318	102	164	65	185	2,452	197
9. Chemicals	357	133	36	34	108	3	17	13	828
10. Lumber, paper, and their products, printing and publishing. .	94	260	1	35	63	6	46	4	69
11. Textiles and leather.	66	43	....	105	8	1	2	....	13
12. Rubber.	54	3	....	195	22	....	....	1	4
13. All other manufacturing.	2	....	....	13	23	....	....	....	1
14. Construction. . .	342	70	41	24	42	8	18	821	18
15. Transportation. .	793	392	266	108	135	75	295	2,200	222
16. Trade.	1,446	4,052	78	1,260	1,254	25	394	1,892	800
17. Foreign countries (imports from). .	337	824	22	10	17	331	63	81	161
18. Business and consumer services. .	550	376	13	85	77	4	12	39	183
19. Households and Government. . .	5,624	3,584	1,043	2,362	3,078	721	779	4,683	1,126
20. Unallocated and stocks.	1,347	1,952	536	724	2,241	599	622	1,383	819
Total gross outlays.	13,745	18,923	3,721	7,979	9,102	3,233	2,741	13,872	4,927

1

Industry of Origin and Destination, 1939
millions of dollars]

PURCHASING											
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	
Lumber, paper, and their products printing, and publishing	Textiles and leather	Rubber	All other manufacturing	Construction	Transportation	Trade	Foreign countries (exports to)	Business and consumer services	Households and Government	Unallocated and stocks	Total gross output
185	583	....	7	167	....	....	453	....	4,495	461	12,475
7	156	....	2	....	....	....	269	1	15,751	391	18,799
64	....	....	....	592	69	....	171	1	12	383	3,887
96	102	16	....	423	77	76	409	3	2,819	1,591	7,672
63	16	....	12	1,301	380	108	544	195	1,657	2,118	8,692
5	1	....	50	144	....	....	159	6	58	589	2,956
51	2	5	8	1,401	13	2	65	2	282	170	2,734
202	138	33	44	127	768	712	519	111	4,990	1,823	13,592
120	326	31	42	426	16	....	203	38	1,508	672	4,911
2,152	56	5	29	910	9	270	142	1,406	1,961	1,375	8,893
84	3,122	58	15	2	....	8	168	29	7,879	429	12,032
1	30	20	2	4	32	20	41	33	348	360	1,170
10	117	....	180	16	30	55	71	373	1,075	394	2,360
42	22	4	11	....	828	189	....	251	7,358	....	10,089
387	52	31	8	....	138	4	103	....	1,919	493	7,621
995	2,928	270	550	....	....	....	....	....	....	2,618	18,562
259	381	196	61	....	286	....	....	....	....	92	3,121
63	83	41	16	4	73	686	2	404	15,709	2,151	20,571
2,366	2,821	339	812	3,940	5,467	11,023	....	14,516	17,285	7,407	88,976
1,777	1,634	164	564	632	....	5,409	....	3,442	....	....	23,845
8,929	12,570	1,213	2,413	10,089	8,186	18,562	3,319	20,811	85,106	23,517	

We have, of course, as many of these equations of type (II)—defining the Walrasian technical coefficients—as there are separate kinds of inputs in all the different industries.

Substituting each of these equations from system (II) into system (I), we can eliminate the x_{ik} and represent each separate input in one industry as a function of the total output X_i of that particular industry. System (III) is the resulting set of equations:

$$
\text{(III)}\quad
\begin{array}{llllll}
X_1 - a_{21} X_2 - a_{31} X_3 - \dots - a_{m1} X_m & & = x_{n1}, \\
-a_{12} X_1 + X_2 - a_{32} X_3 - \dots - a_{m2} X_m & & = x_{n2}, \\
-a_{13} X_1 - a_{23} X_2 + X_3 - \dots - a_{m3} X_m & & = a_{n3}, \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & & \\
-a_{1m} X_1 - a_{2m} X_2 - a_{3m} X_3 - \dots + X_m & & = x_{nm}, \\
-a_{1n} X_1 - a_{2n} X_2 - a_{3n} X_3 - \dots - a_{mn} X_m + X_n & & = 0.
\end{array}
$$

It contains $m+1$ equations, based on the system of $(m+1)n$ coefficients, the a_{ik}. The variables in this case are the X_i which represent total outputs; with m industries you have m outputs plus X_n representing the total labor input.

The x_{ni} on the right-hand side—representing final demand—remain on the outside as independent variables. Thus we can solve this system of equations, expressing all the X's as functions of the final demands. In other words, if we know the magnitudes of all the a's, and if our general theoretical assumptions are correct, we can predict or compute the outputs of all individual industries, one by one, as a function of a final demand. In the same way, of course, we can compute the employment (if we know the employment coefficients) industry by industry as a function of the final demand.

The actual solution is written out in a general form in equation (IV):

$$
\text{(IV)}\quad X_i = A_{i1}x_{n1} + A_{i2}x_{n2} + \dots + A_{im}x_{nm}, \qquad i = 1, 2, 3, \dots, m, n.
$$

Here the total output of an industry, X_i, is expressed as a function of the demand for the separate commodities, the A being fixed coefficients which are computed from the determinant of the matrix coefficients of all the technical coefficients. This is what I referred to as the structural matrix of the economy. (For those of you who want more detail, it can be said that each A can be written out in determinantal form: $A_{ik} = |M_{ik}|/|M|$. M is the determinant of the matrix $|\,a_{ik}\,|$ and $|\,M_{ik}\,|$ is a complement of element a_{ik} in this matrix.).

If we want to compute the dependence of each type of output upon each kind of final demand, we simply have to find the inverse of the structural matrix. Now, of course, if this matrix is three by three, three rows by three columns, it is easy to perform such inversion with a pencil in hand; if it is ten by ten, we need a computing machine; and

if it is 40 by 40, the computation becomes pretty complicated. Fortunately, the modern computing machines, for example the one we have at Harvard, do it without difficulty. So there are no serious computational difficulties even with systems as large as 90 by 90.

Let us consider now another problem, that of price relationships. The same basic method of analysis can be applied here as that used in the study of the physical structure of the economic system. Total value, *i.e.*, quantity times selling price, of a finished commodity, can be equated, by definition, to the quantity of all cost factors purchased from other industries, multiplied by their respective prices, plus the quantity of labor hired multiplied by the wage rate, plus profits π earned per unit of output.

As an economic theorist, I certainly would like to have a complete theory of profits that would enable me to explain the profits earned in an industry as a function of some other magnitudes. But we do not have such a theory yet, and in the following analyses the profits earned per unit of output in an industry are considered as a parameter, as an independent variable. In the course of future work we—let us hope—will be able to expand our theoretical system, add additional equations, and cease to consider this to be a parameter, but rather explain it as one of the dependent variables.

We have m prices, $P_1, P_2, \ldots, P_m$, one for each particular kind of output. We have the wage rate, let us call it P_n (I reserve the subscript n for price of labor) and m profit ratios π, one for each industry: in system (V)

$$
\text{(V)}\quad
\begin{array}{l}
P_1 - a_{12}P_2 - a_{13}P_3 - \ldots - a_{1m}P_m - a_{1n}P_n - \pi_1 = 0, \\
-a_{21}P_1 + P_2 - a_{23}P_3 - \ldots - a_{2m}P_m - a_{2n}P_n - \pi_2 = 0, \\
-a_{31}P_1 - a_{32}P_2 + P_3 - \ldots - a_{3m}P_m - a_{3n}P_n - \pi_3 = 0, \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
-a_{m1}P_1 - a_{m2}P_2 - a_{m3}P_3 - \ldots + P_m - a_{mn}P_n - \pi_m = 0.
\end{array}
$$

There are m equations, and $2m + 1$ variables, obviously fewer equations than unknowns, but what we can do is to determine certain limits within which the system must lie. For example, if some central planning board were to fix all the prices, and enforce certain wage levels, there would be only one profit system—one column of profit rates—which could be earned in different industries and be consistent with the given prices and wages. Or, putting it in a different way, even the most powerful central planning board, if it were to fix the wage rate and the profit rate in all industries, could prescribe only one price system that would be consistent with these previous decisions related to the wages and the profit rate. So from any two sets of these three sets of variables, prices, wages, and profits, the remaining third can be computed.

On what basis? Again on the basis of the same matrix of technical coefficients. As a matter of fact, the system of equations corresponding to the value relationship has the same matrix as that of the quantity relationship, only turned around in the sense that its rows and columns are interchanged.

There are some quite interesting applications, if this has an empirical validity, for the explanation of the relation between the prices, wages, and profits within our system. This type of approach, for example, would enable us (and that has been done) to answer the following question: Imagine we keep the profit rate constant. We decide to increase the wage rate by ten percent. By how much will the prices increase in all the different industries? The effects, as the computation (and of course the experience without any computation) shows, are very differentiated; and when we compute them through, we get a result that is even for practical purposes certainly much more significant than some index of an "average" price increase for the system as a whole.

Now let me make some remarks on the further work in the same direction. Our system as described above is a static system, because it deals entirely with flows of commodities, or rather with the rates of flow, and as such has a validity only for relatively short-run analysis in which the stocks of commodities, at least a dependent element in the system, can be neglected. The analysis for the long run must take in account the fact that in order to produce say, steel or cotton cloth, you need not only flows of certain cost factors, but also stocks, inventories. The stock of machinery is the difference between the rate of purchase and wearing-out of machinery. The same thing applies to buildings. Theoretically or mathematically, this means that we have to introduce technical coefficients of stocks which show the relation between the integral of the difference between the inflows and the rate of use of separate factors in a given industry on the one hand and its rate of output on the other. This transforms our system of ordinary linear equations into a system of linear differential equations. Theoretically, of course, it is not difficult to handle; there is only the problem of irreversibility. When output is increased, capital has to be accumulated; when output is reduced, the full capacity of the previously accumulated plant can not be used. Fortunately, the modern computing machines can perform the difficult operation of integrating differently upwards and downwards. So this difficulty can also be overcome. The second problem is that of getting sufficient empirical data in order to replace the algebraic letters in our formulae with actual figures. The work in this direction has progressed considerably, and for many industries we have already compiled rather detailed sets of investment coefficients.

Résumé

L'article donne un aperçu général d'une méthode nouvelle d'analyse quantitative de l'économie nationale. Un secteur étroit de l'économie nationale peut être étudié sur la base d'équilibres partiels. Il est évident; toutefois, que la plupart des phénomènes les plus intéressants et significatifs sont en rapport, non avec des équilibres partiels, mais avec l'équilibre général, compréhensif, qui englobe plus ou moins simultanément toutes les parties d'un système économique. En raison des difficultés que cela entraine, les économistes ont employé le procédé simplifié d'une analyse d'ensemble, mais ceci n'est pas suffisamment détaillé pour être utile, spécialement dans les problèmes de politique économique et de planification économique.

La méthode exposée ici permet de résoudre le problème de combiner l'analyse d'équilibre général avec le maintien d'une classification différentiée de tous les aspects individuels des phénomènes économiques.

Le tableau donne des statistiques concernant les Etats-Unis pour l'année 1939. Chaque ligne montre comment les produits d'une industrie sont distribués et chaque colonne montre la valeur en dollars des produits fournis à l'industrie. Il y a 20 groupes d'industries dans le tableau; toutefois, l'on dispose de chiffres plus dètaillés se rapportant à une centaine d'industries.

La manipulation théorique de ces chiffres est facilitée par quelques formules simples. Dans le système (I) chacune des x_{ik} indique le montant du produit de l'industrie k utilisé dans l'industrie i, tandis que x_k est la production totale de l'industrie k. Dans le système (II) les constantes a_{ik} indiquent le rapport entre le montant du produit k utilisé dans l'industrie i et la production totale de l'industrie i. Le système (III) est déduit en éliminant les x_{ik} des systèmes (I) et (II). Au côté droit, il reste les x_{ik} représentant les montants consommés finalement par les ménages et l'administration publique. Alors, si l'on connait la valeur de tous les a_{ik} et si nos hypothèses théoriques générales sont exactes, nous sommes à même de prédire ou de calculer la production de chaque industrie individuelle, en fonction de la demande finale, suivant le système (IV). Les A sont des coefficients fixes calculés sur la base du déterminant de la matrice de tous les coefficients techniques. Cette dernière est appelée la matrice structurelle de l'économie, et la matrice des A est son inverse.

Si nous désirons calculer la relation de chaque type de demande finale, il faut simplement trouver l'inverse de la matrice structurelle, ce qui est facile à faire à l'aide des machines à calculer modernes telles que le calculateur "Aiken Relay" de l'Université Harvard. Dans le système

V les prix, P_1——P_m, de chaque type de marchandise produite, P_n, le taux des salaires, et le pourcentage des bénéfices, π_1——π_m pour chaque industrie, sont introduits. Il y a n équations, évidemment moins nombreuses que le nombre d'inconnues, mais il est possible de déterminer, pour ces trois groupes d'inconnues, la dépendance d'un groupe quelconque par rapport aux deux autres. C'est-à-dire, à exprimer chaque prix en fonction des salaires et des bénéfice, ou par exemple, déterminer les bénéfices dans une certaine industrie en fonction des salaires et prix donnés.

L'analyse exposée ci-dessus a un caractère statique, parce qu'elle traite de courants de marchandises ou de leurs taux de circulation, et néglige les stocks. L'analyse à longue échéance doit tenir compte des stocks, ce qui transforme notre système d'équations linéaires ordinaires en un système d'équations différentielles linéaires. Les problèmes de calcul numérique y relatifs peuvent être résolus à l'aide de machines modernes; le travail pour obtenir les données statistiques nécessaires fait des progrès satisfaisants.

Mr. Leontief's paper was discussed by Messrs. Tjalling C. Koopmans, François Divisia, Michal Kalecki, Leonid Hurwicz, Donald C. MacGregor, and the speaker.

LA METASTATIQUE

par Georges Lutfalla

Administrateur Général de l'Ecole Nationale de l'Organisation Economique et Sociale, Ministere de l'Economie Nationale (France)

Il est habituel aujourd'hui de distinguer les *théories* économiques en théories statiques et théories dynamiques.

Cette classification *binaire* est empruntée à Auguste Comte. Il avait observé que la division de la mécanique "statique-dynamique" trouvait son analogie en biologie avec le binome "anatomie-physiologie." Il avait été ainsi conduit, par une généralisation peut-être hâtive, à la transposer dans le domaine de la sociologie qui, à son tour, l'a léguée à l'économique.

Quoiqu'une très abondante littérature traite de ce sujet, il ne semble pas que, à notre connaissance, elle ait été soumise à une analyse systématique (sauf par François Perroux). Une critique est pourtant nécessaire. Les récents développements de l'économique conduisent à penser que: a) le binome statique-dynamique est insuffisant; b) une division tripartite est nécessaire. A titre d'exemple on peut indiquer que la théorie Keynésienne n'est ni statique ni dynamique. Encore faut-il donner de ces deux derniers qualificatifs une définition rigoureuse.

1. *La théorie de l'équilibre général et sa double qualification statique.*—Il y a une quinzaine d'années déjà que l'examen comparé de la théorie de l'équilibre général et de modèles dynamiques a conduit R. Frisch et J. Tinbergen à donner cette définition très simple, adoptée par J. R. Hicks: est statique une théorie qui ne date pas ses variables, ou encore une théorie d'où le temps, en tant que *paramètre significatif*, est absent. C'est là un *critère formel.*

Par ailleurs, P. M. Sweezy (*Review of Economic Studies*, juin 1938) a montré que, avec le postulat du comportement rationnel du sujet économique, la prévision parfaite équivaut à l'absence du temps comme paramètre significatif dans le système étudié. C'est la un *critère implicite.* L'équivalence logique en question permet donc de définir la théorie de l'équilibre général soit par le critère formel soit par le critère implicite.

Par définition, le caractère statique d'une théorie sera acquis sitôt satisfait l'un de ces deux critères.

2. *Dichotomie des critères.*—La remarque de Sweezy ne peut être inversée: tout système d'où la variable "temps" est absente n'est pas nécessairement un modèle où la prévision est parfaite. Cela revient

à dire qu'un tel système n'est pas nécessairement statique au sens précis donné plus haut. Le critère formel (absence ou présence du paramètre "temps") et le critère implicite (prévision parfaite ou prévision imparfaite) peuvent ainsi se conjuguer et donner lieu à l'une des quatre éventualités suivantes:

1. absence "temps" + prévison parfaite,
2. absence "temps" + prévision imparfaite,
3. présence "temps" + prévision imparfaite,
4. présence "temps" + prévision parfaite.

La première éventualité est celle qui a été précédemment analysée: les théories qui y correspondent sont *statiques.*—La note de P. A. Sweezy montre également, dans la quatrième éventualité, que le paramètre "temps" quoique présent, n'est pas significatif; qu'il s'agit au total d'une théorie statique camouflée.—Il est facile de reconnaître dans la troisième tous les traits que l'on accorde généralement à un modèle dynamique.

Seule la deuxième éventualité semble n'avoir pas été nettement dégagée jusqu'ici. Son trait caractéristique est *l'émergence* du concept "prévision imparfaite." Le type d'une telle théorie est fourni par l'essentiel de la théorie Keynésienne: elle ne *date* pas les variables du modèle et cependant elle ne fait appel, ni explicitement, ni implicitement, à l'hypothèse de la prévision parfaite. Ce seul exemple montre l'importance de ce deuxième type que je propose d'appeler MÉTASTATIQUE pour le distinguer du premier et du troisième.

3. L'exposé est complété par diverses remarques relatives à l'importance pédagogique de cette distinction et aux traits caractéristiques des équilibres métastatiques.

Résumé

Economic theories are nowadays divided into static and dynamic theories. This bifurcation stems from Auguste Comte, who had noted that the static-dynamic division in mechanics was analogous to the distinction between anatomy and physiology in biology. He was thus led by perhaps an over-hasty generalization to carry over into sociology these classifications which were then bequeathed to economics.

Although the literature on this subject is very rich, it has not been the object, so far as I know, of systematic analysis outside of the work of

François P e r r o u x. A critical examination is needed. Recent developments in economics suggest that (a) the static-dynamic classification is inadequate, (b) a tripartite division is necessary. For example, the Keynesian theory is neither static nor dynamic, as is seen when we define these terms rigorously.

1. *The theory of general equilibrium considered as stàtic from two points of view.*—Fifteen years ago a comparative study of general equilibrium and of dynamic models led R. F r i s c h and J. T i n b e r g e n to formulate the following simple definition, adopted by J. R. H i c k s: A static theory is one which does not date its variables, or perhaps one in which time plays no part as a *significant* parameter. This is a *formal criterion.*

P. M. S w e e z y, on the other hand, has shown (in the *Review of Economic Studies,* June, 1938) that, assuming rational conduct on the part of the economic subject, perfect foresight is tantamount to the absence of time as a *significant parameter* in the system considered. This is an *implicit criterion.* Thanks to the logical equivalence of the two criteria it is possible to define the theory of general equilibrium in terms of the one or the other.

By definition, the static character of a theory is given as soon as one or the other of these criteria is present.

2. *Classificatory Dichotomy.*—Sweezy's remark cannot be reversed: a system from which the parameter "time" is absent is not necessarily static in the exact sense defined above. The formal criterion (absence or presence of the parameter "time") and the implicit criterion (perfect or imperfect foresight) when brought together give rise to one of the following possibilities:

1. absence of time + perfect foresight,
2. absence of time + imperfect foresight,
3. presence of time + imperfect foresight,
4. presence of time + perfect foresight.

The first possibility is that which has just been analyzed, theories of this character are static.—Sweezy's note points to the fourth possibility in which the parameter "time," though present, is not significant. This is a case of disguised static theory. It is easy to perceive that the third possibility possesses all the characteristics of what is generally conceded to be a dynamic.

Only the second possibility has so far remained undefined. Its characteristic trait is emergence of the concept of imperfect foresight. A typical example is found in the essential parts of Keynes' theory when the variables of the model are not dated, but in which the hypothesis of perfect foresight is not referred to either explicitly or implicitly.

This single example reveals the importance of the second type which I prefer to call *Metastatic*, in order to distinguish it from the first and third possibilities.

This discussion will be supplemented by various remarks dealing with the pedagogical importance of the distinction and of the characteristics of metastatic equilibria.

Mr. Lutfalla's paper was discussed by Messrs. Tjalling C. Koopmans, François Divisia, René Roy, Leonid Hurwicz, and the speaker.

OFFRE ET DEMANDE D'EFFORT DE L'OUVRIER AU TRAVAIL

par Robert Hénon

Professeur a l'Institut de Statistique a l'Université de Paris (France)

1. Introduction

a) Dans le cas du *travail remunéré aux pièces ou à la prime*, l'ouvrier règle sa cadence en faisant un arbitrage plus ou moins conscient entre le désir de gagner plus par une production accrue et une sensation de fatigue qui augmente très rapidement avec son rendement. Cet arbitrage se traduit par une stabilité relative assez étroite pour un même individu et entre individus d'un même groupe.

Autrement dit il existe *un point d'équilibre* théorique dont les points observés constituent les éléments aléatoires d'un phénomène statistique.

Si les paramètres du salaire varient, les points d'équilibre varient eux aussi. Il était admis, avant la guerre, qu'une augmentation du taux de pièces entrainait une augmentation du rendement et inversement mais on ne savait pas quelle relation numérique pouvait exister entre ces variations (flexibilité positive du rendement).

Or il a été constaté pendant les années 1941 à 1944 une augmentation continue du rendement dans les entreprises où le personnel était rémunéré à la production. Si l'on rapproche de ce fait l'existence d'une augmentation continue du coût de la vie avec des salaires "bloqués" tout se passait comme si la formule de salaire exprimée en valeur *réelle* voyait ses paramètres diminuer progressivement. Le phénomène était donc contraire à celui admis couramment puisqu'il présentait une flexibilité négative du rendement.

C'est cette observation qui nous a conduit à rechercher une explication.

b) *Le rendement* x, consituant la variable importante de toutes les observations, précisons que ce rendement est mesuré par le rapport de la production réalisée à une production de base fixée par la Direction.

Cette production de base est aujourd'hui fixée avec minutie après analyse des mouvements élémentaires, stabilisation de ceux-ci quant à leur régularité et chronométrage avec jugement de vitesse sur une échelle de rendement. La cote 100 de cette échelle précise le rendement de base.[1]

[1] A titre indicatif le rendement dans les ateliers où les temps ne sont pas contrôlés est de l'ordre de 30% à 40%. Si les temps sont contrôlés et si le salaire est à la production, le rendement peut monter au voisinage de 130%.

c) La *sensation de fatigue* est une donnée immédiate de la conscience, sa relation "instantanée" avec la fatigue physiologique est très imprécise.

d) La fatigue physiologique est repérable par des tests: modification des échanges respiratoires, allongement des temps de réaction, etc. Le niveau de fatigue pour un même travail accompli dans un même temps dépend de la manière dont le travail a été exécuté. Autrement dit, c'est une *fonction de ligne* qui dépend du programme d'effort (exemple des compétitions sportives). Ceci conduit à admettre l'existence d'une infinité de niveaux de fatigue pour un même rendement moyen x. Cependant il existe un cas où la *correspondance est univoque*: c'est quand le programme d'effort conduit au niveau de fatigue minimum (condition isopérimètrique). Nous admettons que dans le travail en usine cette condition fondamentale est réalisée.

2. La demande d'effort

Dans tout ce qui suit le rendement moyen x est le rendement journalier dans le cas d'un horaire de travail invariable.

Etant donné la faible dispersion des rendements constatés il est possible de *réduire la formule de salaire* à prime à une droite moyenne:

$$\rho = mx + n, \tag{1}$$

où ρ est le salaire *réel* journalier et $d\rho/dx = m$ le *stimuli*.

La demande d'effort repéré sur l'échelle des rendements peut donc s'écrire sous la forme différentielle:

$$\frac{dx}{d\rho} = \frac{1}{m}, \tag{2}$$

qui mesure la *demande de rendement final* en revenu.

3. L'offre d'effort

En contre-partie de la pénibilité de son effort, l'ouvrier au travail se représente toutes les commodités qu'il pourra tirer de son salaire. Il est donc en présence d'un champ de choix caractérisé en chaque point par une *fonction de satisfaction* correspondante:

$$S(\rho, x).$$

En désignant par $\Omega(\rho)$ la fonction d'utilité croissante et de courbure négative du revenu et par $F(x)$ la fonction, de sensation de fatigue croissante et de courbure positive, la forme générale de $S(\rho, x)$ est[2]

$$S(\rho, x) = K_1\Omega(\rho) - K_2F(x) + K_3.$$

[2] En admettant avec M. Allais l'extension aux échelles de jugement de l'équivalence des échelons minimum perceptibles, équivalence qui se trouve vérifiée en psycho-physiologie par la loi de Weber-Fechner.

Ici les notions d'utilité et de fatigue sont des *données subjectives*, et les trois constantes sont spécifiques d'un individu à un moment donné.

Comme celui-ci cherche à maximer sa satisfaction, on doit avoir $dS = 0$ ou

$$\omega(\rho)d\rho = Kf(x)dx. \tag{3}$$

Cette formule exprime la tendance à l'égalisation des deux termes. *C'est l'espérance mathématique de la situation* qui résulte d'un arbitrage plus ou moins conscient chez l'homme au travail.

La constante individuelle K s'interprète facilement en admettant qu'elle traduit l'incidence de l'aptitude ou de l'entrainement de l'ouvrier à son travail.

En résumé; la satisfaction est maximum quand l'equation differentielle liant ρ et x,

$$\frac{dx}{d\rho} = \frac{\omega(\rho)}{Kf(x)}, \tag{4}$$

est vérifiée. Le lien (4) ainsi défini décrit donc une courbe *d'offre de rendement final* en revenu.

4. Equilibre entre l'offre et la demande d'effort

a) Les paramètres du salaire étant fixés, le point d'équilibre (x, ρ) doit satisfaire aux deux relations:

$$\begin{cases} \omega(\rho)d\rho = Kf(x)dx, \\ \rho = mx + n. \end{cases} \tag{5}$$

Le lieu de ces points dans le plan (ρ, x) quand le stimuli m varie s'obtient en éliminant m d'où:

$$(\rho - n)\omega(\rho) = Kxf(x). \tag{6}$$

b) Pour étudier les propriétés générales de cette courbe, donnons à la fonction $\omega(\rho)$ une forme plus appropriée aux calculs en posant

$$\omega(\rho) = \frac{1}{R(u)-R_0}, \tag{7}$$

avec le changement de variable $u = \log \rho$.

$R(u)$ est une fonction positive croissante telle que pour $\rho = \rho_0$, $R(u_0)=R_0$. La fonction d'utilité finale est donc bien décroissante. D'autre part, elle devient infinie, quand ρ atteint le "minimum vital"$= \rho$.

La dérivée est:

$$\frac{d\omega}{d\rho} = \frac{d\omega}{du}\frac{du}{d\rho} = -\frac{1}{\rho}\cdot\frac{R'}{(R-R_0)^2}.$$

La fonction $R(u)$ étant croissante, $R' > 0$ et la dérivée $d\omega/d\rho$ est bien négative.

La dérivée seconde est:

$$\frac{d^2\omega}{d\rho^2} = \frac{1}{\rho^2}\left[-\frac{R''}{(R-R_0)^2} + \frac{2R'^2}{(R-R_0)^3} + \frac{R'}{(R-R_0)^2}\right].$$

Si donc on s'impose la condition $R'' < 0$ la dérivée seconde est toujours positive. Ainsi, en prenant pour la fonction $R(u)$ une fonction croissante à courbure négative, les propriétés générales de la fonction d'utilité finale sont bien respectées, celle-ci est décroissante, toujours positive et à courbure toujours positive.

Si nous calculons la flexibilité de la fonction nous trouvons:

$$\varphi_\omega = \rho\frac{\omega'}{\omega} = -\frac{R'}{R-R_0}.$$

La flexibilité est toujours négative et, comme R' décroit avec u elle tend vers zéro quand u augmente indéfiniment. De plus en calculant la dérivée $d\varphi/d\rho$ on constaterait qu'elle est nécessairement positive.

c) Ces conditions préliminaires posées, on recherche la forme du lieu d'équilibre dans le plan (ρ, x) *des données directement saisissables par l'observation.*

Pour cela, calculons la dérivée $dx/d\rho$, on a d'abord:

$$\omega'(\rho)(\rho-n) + \omega(\rho) = K\Big[xf'(x) + f(x)\Big]\frac{dx}{d\rho},$$

d'où

$$\frac{dx}{d\rho} = \frac{x}{\rho}\frac{1}{1+\varphi_f}\left[\frac{\rho}{\rho-n} + \varphi_\omega\right]$$

qui peut encore s'écrire:

$$\varphi_{x\rho} = \frac{1/k + \varphi\omega}{1+\varphi_f}.$$

La flexibilité de la fatigue finale en rendement étant désignée par φ_f et celles du rendement et de la demande en salaire par $\varphi_{x\rho}$ et k.

Le signe de la dérivée ne dépend que du terme entre crochets. Celui-ci s'annule pour:

$$-\frac{\rho}{\rho-n} = \varphi_\omega.$$

Il est bien évident que si l'on trace les courbes des fonctions représentant les deux membres de l'égalité on est certain d'avoir toujours une solution $\rho = \rho_m$ et une seule si $n < \rho$.

On arrive ainsi à trouver une caractéristique importante de la courbe d'équilibre dans le plan (ρ, x) quand le stimuli m varie de 0 à $+\infty$, le paramètre n étant constant et inférieur à ρ_0.

De plus, la solution ρ_m ne dépend plus du facteur individuel K ce qui est encore une propriété intéressante.

d) L'existence d'un minimum dans le cas réalisé en pratique où $n < \rho_0$ conduit à admettre deux régimes de travail. L'un de *travail libre*, l'autre de *travail forcé*.

Dans le deuxiène régime, la flexibilité du rendement est négative et l'ouvrier arbitre son effort final avec les biens qui lui sont indispensables pour assurer son existance. *C'est précisément le cas qui a été observé dans l'industrie pendant les années 1941 à 1944.*

Dans ce cas le lieu présente un minimum.

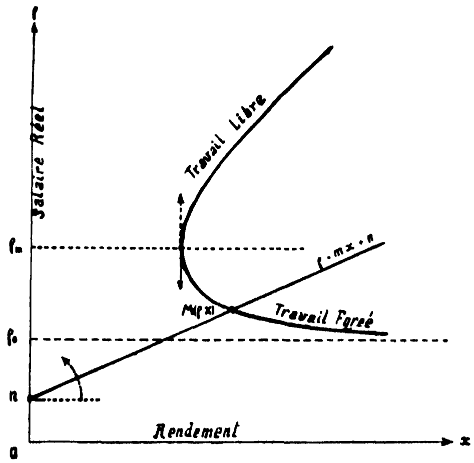

Figure 1. Lieu d'équilibre pour un stimuli m variable ($n < \rho_0$)

e) Le point d'équilibre individuel dépend de deux paramètres ρ_0 et K. Ces deux paramètres confèrent un caractère de rigidité aux solutions analytiques proposées. Pour éviter cet inconvénient il suffirait de faire correspondre à tout point d'équilibre une *densité de probabilité*:

$$p(\rho_0)p(K),$$

les valeurs ρ_0 et K aléatoires étant soumises à des lois de distributions indépendantes.

5. Courbes d'indifférence

Si la formule de salaire est telle qu'en tout point l'offre de rendement final égale la demande de rendement, on a

$$\frac{d\rho}{dx} = \frac{Kf(x)}{\omega(\rho)},$$

et le point d'équilibre se place indifféramment sur cette courbe. Cette expression donne la forme du *salaire idéal.* Cette courbe est très rapidement croissante.[3]

6. Flexibilité des facteurs

a) *Paramètre m variable:*

Désignons par k la flexibilité de la demande de salaire en rendement d'après la formule $\rho = mx+n$. On a

$$k = \frac{mx}{mx+n},$$

d'où le système:

$$\begin{cases} \dfrac{d\rho}{\rho} = k\left(\dfrac{dx}{x} + \dfrac{dm}{m}\right), \\ \dfrac{d\omega}{\omega} + \dfrac{dm}{m} = \dfrac{df}{f}, \end{cases}$$

et par division, on en déduit

$$\varphi_f = \frac{k\varphi_\omega(1 + \varphi_{x,\,m}) + 1}{\varphi_{x,\,m}}$$

avec

$$\varphi_{x,\,m} = \frac{\Delta x}{\Delta m}\cdot\frac{m}{x} \text{ (flexibilité du rendement en } m).$$

b) *Paramètre n variable:*

On trouve

$$\varphi_f = \varphi_\omega\left(k + \frac{1-k}{\varphi_{x,\,n}}\right) \tag{9}$$

avec

$$\varphi_{x\cdot n} = \frac{\Delta x}{\Delta n}\cdot\frac{n}{x} \text{ (flexibilité du rendement en } n).$$

c) *Flexibilitè de l'effort et de l'utilité du revenu:*

Par élimination on déduit deux formules fondamentales:

$$\varphi_f = \frac{1 + \dfrac{mx}{n}\varphi_{x,\,n}}{\varphi_{x,\,m} - \dfrac{mx}{n}\varphi_{x,\,n}}, \tag{10}$$

[3] Lallemand, Inspecteur général de Mines a fait appliquer avec succes aux Travaux Publics en 1883 la formule $S = S + \frac{1}{2}kx^2$. Celle-ci était fondée sur l'hypothese simple $ds/dx = kx$, c'est-a-dire que l'intensité de l'effort elémentaire xdx était proportionnel au travail antérieur x.

$$\varphi_\omega = \frac{1}{(1-k)^{\frac{\varphi_{x,m}}{\varphi_{x,n}}} - k} \tag{11}$$

en fonction des données mesurables.

Une nouvelle expression de la flexibilité du revenu est ainsi établie indépendamment de la théorie des choix de l'individu en tant que consommateur.

Conclusions[4]

Cette étude devrait être généralisée. L'horaire d'utilisation devrait être pris comme deuxième variable en mutuelle dépendance avec l'intensité d'effort x.

La rémunération matérielle étant parfaitement saisie il faudrait aussi faire intervenir la *remunération immatérielle*, c'est-à-dire l'influence du revenu psychologique.

Ceci ouvre un champ d'investigation étendu aux observations statistiques.

Cependant ces observations seraient coûteuses étant donné la nécessité inhérente à toutes les *batteries de tests d'étalonner* par chronométrages les tâches observées pour estimer le rendement.

L'avantage de pouvoir calculer un niveau de vie sans faire appel à l'ensemble des biens de consommation entrant dans la composition d'un complexe adapté ouvre la voie à une théorie plus générale de l'utilité et donne des éléments rationnels pour établir des formules de salaires.

De plus, l'équivalence de complexes adaptés très différents prend un sens plus intuitif quand on passe par la notion d'effort, et on y découvre l'aspect à la fois social et évolutif qui fait dépendre cette équivalence du *lieu* et du *moment* ou une collectivité est observée.

Résumé

Our study intends to show how the rate of output x of a working man is fixed when the wage is a function of output.

Effort demand: This demand is nothing else than the constraint effect by the *real* wage formula which can always be expressed in linear form: $\rho = mx + n$, in the labor range.

Effort supply: The feeling of residual weariness associated with the objective measure of output x is an unknown increasing function $F(x)$ that we may call with R. Frisch, the function of *disutility*. All

[4] La place ne m'a permis de parler, ni des travaux de Frisch sur le problème de l'offre de travail en durée, ni des suggestions de Mr. Roy sur la notion d'utilité. Qu'ils trouvent ici l'expression de ma gratitude pour les méditations qu'ils ont provoquées dans mon esprit et qui m'ont permis d'exposer brièvement ce travail.

the derivatives of $F(x)$ are positive. In particular we shall describe $f(x) = dF(x)/dx$ as the final degree of disutility in the output.

If the living standard for the wage is defined by the utility function, the final degree of utility in terms of wage is: $\omega(\rho) = d\Omega(\rho)/d\rho$. If we admit that each individual is able to make a choice between the different levels of disutility and utility, he will reach the highest satisfaction by equalizing the final disutility and utility, so as to obtain the fundamental relation:

$$\omega(\rho)d\rho = Kf(x)dx,$$

K being a constant appropriate to the individual. The integration of the latter relation allows us to specify the *law of effort demand* as a function of utility.

Equilibrium equation: Adding the demand constraint we have the system:

$$\begin{cases} \omega(\rho)\dfrac{d\rho}{dx} = Kj(x) \\ \rho = mx + n. \end{cases}$$

The elimination of n gives the curve of the equilibrium points when m is varying. If we adopt for $\omega(\rho)$ the general form

$$\omega(\rho) = \frac{C}{R(u) - R_0}, \text{ with : } u = \operatorname{Log}\rho \text{ and } R > 0\,, \ \frac{dR}{du} > 0\,, \ \frac{d^2R}{d^2u} < 0,$$

where $R_0 = R\,(\operatorname{Log}\rho_0)$ corresponding to the lowest living standard, and C is an individual constant, we find finally:

$$(\rho - n)\omega(\rho) = kxf(x).$$

If $n < \rho_0$ we obtain the curve in Figure 1. We notice two branches, one in the area of *free labor* (superior branch), the other in the area of *forced labor.*

Considering such concrete elements, a statistical study would allow us to find:

Effort flexibility: $$\varphi_f = \frac{1 + \dfrac{mx}{n}\varphi_{xn}}{\varphi_{xm} - \dfrac{mx}{n}\varphi_{xn}},$$

Income flexibility: $$\varphi_\omega = \frac{1}{(1-k)\dfrac{\varphi_{xm}}{\varphi_{xn}} - k},$$

with: $$\varphi_f = \frac{d\operatorname{Log}f(x)}{d\operatorname{Log}x},\ \varphi_\omega = \frac{d\operatorname{Log}\omega(\rho)}{d\operatorname{Log}\rho},$$

$$\varphi_{xn} = \frac{\Delta x}{\Delta n}\cdot\frac{n}{x},\ \varphi_{xm} = \frac{\Delta x}{\Delta m}\cdot\frac{m}{x},\ k = \frac{mx}{mx+n}.$$

Yet, great care should be taken in a field where phenomena of collective unconsciousness may prevail.

ECONOMETRICS AND THE PREVENTION OF INFLATION AND UNEMPLOYMENT: I

Tuesday, September 16, at 2:00 p.m.

CHAIRMAN :

Colin Clark

Government Statistician; Director, Bureau of Industry; Financial Adviser to the State Government; Under Secretary for State, Department of Labour and Industry, Queensland (Australia)

SUR LA THEORIE QUANTITATIVE ET LE PHENOMENE DE REGULATION MONETAIRE[1]

par Jacques Rueff

Président de l'Agence Interalliée des Réparations (France)

1. Définition des indices du niveau général des prix et de la vente totale[2]

Des indices du niveau général des prix et de la vente totale seront de bons indices s'ils donnent une expression quantitative aux notions intuitives de niveau général des prix et de vente totale.

On peut montrer que cette condition est satisfaite par les indices P et Q définis, relativement aux indices de la période de base P_0 et Q_0, par les formules suivantes:[3]

$$\frac{dP}{P} = \frac{\Sigma q\, dp}{\Sigma qp}, \qquad \frac{dQ}{Q} = \frac{\Sigma p\, dq}{\Sigma qp}.$$

Nous appellerons dépense totale D, la valeur totale, Σpq, des articles vendus pendant l'unité de temps de la période considérée.

[1] La substance du présent article fut présentée à Washington sous le titre "La notion d'inflation à la lumière d'une théorie des variations du niveau général des prix."

[2] Voir *L'Ordre Social*, (Librairie du Recueil Sirey, 1946), tome I, chap. III.

[3] Cette formule a été proposée en 1925 par M. Divisia, "L'indice monétaire et la théorie de la monnaie," *Revue d'Economie Politique.*

On a évidemment:

$$\frac{dD}{D} = \frac{\Sigma q dp}{\Sigma qp} + \frac{\Sigma p dq}{\Sigma qp} = \frac{dP}{P} + \frac{dQ}{Q}, \tag{1}$$

ce qui montre que l'on peut écrire, en représentant par k une constante:

$$D = kPQ.$$

La dépense totale est donc égale, à une constante près, au produit de l'indice du niveau général des prix par l'indice de la vente totale.

Par ailleurs, l'égalité (1) peut s'écrire:

$$\frac{dP}{P} = \frac{dD}{D} - \frac{dQ}{Q}. \tag{2}$$

D'où l'on peut déduire que lorsque l'indice de la vente totale ne varie pas, l'indice du niveau général des prix ne peut varier que lorsque la dépense totale varie. En ce cas, les variations de l'indice ne dépendent que des variations de la dépense totale et sont indifférentes à sa répartition entre les divers articles du marché.

On peut énoncer, *mutatis mutandis*, une conclusion analogue, en ce qui concerne les variations de l'indice de la vente totale.

Lorsque les prix varient librement sur le marché, la dépense totale est égale à la demande totale, la vente totale à l'offre totale. Toutes les propositions précédentes restent vraies, lorsque l'on substitue les mots de demande à dépense, d'offre à vente. La principale d'entre elles s'énonce sous la forme suivante: *lorsque les prix varient librement, la variation relative de l'indice du niveau général des prix entre les instants t et $t+dt$ est égale à la différence entre les variations relatives de la demande totale et de l'offre totale entre les mêmes instants.*

2. Théorie des variations du niveau général des prix sur un marché ou les prix varient librement

Si l'on tient compte du fait que $D = kPQ$, on a:

$$dP = \frac{dD - kPdQ}{kQ}. \tag{3}$$

Ainsi, le niveau général des prix ne variera que s'il y a différence entre les variations de la demande totale et de la valeur totale des richesses offertes pendant l'unité de temps, aux instants considérés.

Mais ces deux grandeurs sont étroitement solidaires. C'est le lien qui les unit que nous allons maintenant dégager.

Pour y réussir, observons en premier lieu que, sur un marché ou la quantité de monnaie en circulation et le montant des encaisses que chaque individu désire détenir ne varieraient pas, chaque individu demanderait, en chaque séance du marché, à concurrence de la valeur des offres

qu'il y aurait formulées. Les variations de la demande seraient toujours égales à celles de la valeur des offres, quelles qu'elles soient.

C'est la situation que l'on peut observer sur tout marché de campagne, trop court en durée ou trop éloigné des centres bancaires pour qu'il s'y produise des variations dans la quantité de monnaie existante. Si chaque individu participant au marché désire partir avec l'encaisse qu'il avait en arrivant, sa demande est toujours au niveau de la valeur de ses offres. Toute variation de l'offre totale entraîne variation de même montant de la demande totale. L'égalité (3)—et le bon sens—montrent que le niveau général des prix ne varie pas.

Mais la quantité de monnaie en circulation, donc le montant total des encaisses effectives, peuvent varier. De même, le montant total des encaisses que les individus désirent détenir.

Si la variation des premières est égale à celle des secondes, il n'y a toujours demande qu'à concurrence de la valeur des offres; le niveau général des prix reste immuable.

Mais si, par exemple, la variation des encaisses effectives pendant la période considérée excède la variation des encaisses désirées, les titulaires d'encaisses indésirées n'auront d'autre solution que de demander sans offrir. Tout se passera comme si leur désir de substituer des richesses proprement dites à leurs encaisses indésirées majorait à due concurrence leur demande sur le marché, antérieurement limitée à la valeur des offres.

Au contraire, si la variation des encaisses désirées excédait la variation simultanée des encaisses effectives, certains individus seraient amenés à offrir sans demander; les variations de la valeur totale des offres excéderaient à due concurrence celles de la demande totale.

Ainsi, en chaque séance du marché, la différence entre la variation de la demande totale et de la valeur totale des offres est identiquement égale à la différence entre les variations des montants globaux des encaisses effectives et des encaisses désirées.

Si l'on représente par E_e et E_d les montants globaux des encaisses effectives et désirées, l'équation (3) s'écrit;

$$dP = \frac{dE_e - dE_d}{kQ}.$$

Une analyse plus complète permettrait de montrer que cette variation du niveau général des prix rend nécessaire au règlement des transactions les excédants d'encaisses qui l'ont suscitée et, par là, les rend désirés.

Compte tenu des observations précédentes, la théorie quantitative de la monnaie prend la forme suivante: *le niveau général des prix ne*

varie qu'en fonction de la différence entre les variations simultanées du montant global des encaisses effectives et désirées.

Tant que les premières varient comme les secondes, le niveau général des prix est indifférent à l'offre et à la demande, car si l'offre augmente, la demande augmente du même montant.

De même, le niveau général des prix est indifférent à l'augmentation de la quantité de monnaie en circulation tant que celle-ci est désirée.

3. La régulation monétaire en régime de vraie monnaie

D'aucuns penseront que les énoncés précédents règlent le problème de la théorie quantitative et que l'action sur la différence entre le montant de la circulation monétaire et celui des encaisses désirées permet de faire varier à son gré le niveau général des prix. C'est la position des théoriciens pour qui les autorités monétaires gouvernent le niveau des prix en "faisant varier" la quantité de monnaie en circulation.

Or, ceux-là se trompent entièrement, car ils omettent le lien fondamental qui tend à unir la quantité de monnaie en circulation au montant des encaisses désirées.

Depuis l'élaboration de mon *Ordre Social*, je suis convaincu que toute théorie quantitative qui néglige ce lien est aussi fausse dans les règles d'action qu'elle suggère, qu'erronée dans les principes sur lesquels elle repose.

Au cours du présent paragraphe, j'étudierai le phénomène en supposant que toute la monnaie existante a une contrepartie d'égale valeur—or ou effets de commerce—dans le bilan de l'Institution émettrice. On est en régime de "vraie monnaie." Je dis qu'en pareil régime la quantité de monnaie en circulation est liée au montant des encaisses désirées par un mécanisme de régulation dont je vais exposer le principe, mais dont l'analyse minutieuse devrait constituer l'une des tâches urgentes des économistes soucieux de théorie monétaire.

En régime uniquement métallique, aucune difficulté. L'existence du mécanisme est généralement admise, bien que pas toujours très consciemment.

Si le montant des encaisses effectives est inférieur à celui des encaisses désirées, les personnes qui désirent majorer leur encaisse n'ont d'autre solution que d'offrir sans demander. Ce faisant, elles suscitent, dans un régime où les prix varient librement, une tendance à la baisse du niveau général des prix.

Mais, puisque l'on est en régime de monnaie convertible, un prix demeure stable parmi tous les prix décroissants: le prix de l'or, maintenu au niveau de la parité légale par les achats de l'Institut d'Emission,

De ce fait, des facultés de production sont transférées des richesses aux prix déprimés vers l'or. La production des premières tend à diminuer, celle du métal jaune tend à augmenter.

Mais la Banque d'émission ou le service de frappe prennent tout l'or offert et non demandé, et par là augmentent la quantité de monnaie en circulation.

Comme la baisse des prix, donc le transfert de facultés de production, se poursuivent tant que n'a pas disparu la cause qui les a provoqués—à savoir l'insuffisance des encaisses effectives relativement aux encaisses désirées—ils ne peuvent pas ne pas avoir pour effet de porter les encaisses effectives au niveau des encaisses désirées, et, en rétablissant entre le prix de l'or, maintenu immobile au niveau de la parité légale, et les autres prix du marché leurs rapports antérieurs, de ramener à sa valeur primitive l'indice du niveau général des prix.

Il va de soi qu'un excès d'encaisses effectives aurait provoqué, *mutatis mutandis*, un phénomène analogue.

Dans un univers sans résistance ni frottement, le phénomène assurerait la fixité rigoureuse du niveau général des prix. Dans un univers réel, il fait naître une force qui tend à ramener ce niveau à sa position d'équilibre, avec d'autant plus d'intensité que l'écart à corriger est plus grand.

On voit ainsi combien est fausse la thèse qui tend à faire croire qu'en régime métallique, on peut, à son gré, faire varier la circulation monétaire, et, par elle, le niveau des prix.

Il est vrai que l'existence de ce mécanisme, familier à beaucoup d'économistes, sera facilement admise. Mais l'affirmation qu'il existe un mécanisme de même nature en régime de monnaie non convertible sera moins aisément acceptée.

En pareil régime, le phénomène est fondé sur le lien—moins connu—qui unit le taux d'intérêt à court terme aux mouvements du niveau général des prix.

Comme dans le cas précédent, toute insuffisence du montant des encaisses effectives relativement à celui des encaisses désirées conduit les personnes qui veulent augmenter leur encaisse à offrir sans demander.

L'excédent d'offres peut porter, soit sur des richesses proprement dites, soit sur des créances. Dans le premier cas, il provoque baisse des prix; dans le second, hausse de taux. Si l'on admet que l'excédent d'offres se produit "toutes conditions égales," il affecte richesses proprement dites et créances dans la proportion où les unes et les autres entrent dans l'offre totale. Il entraîne donc, à la fois baisse de prix et hausse de taux.

Ceci suffit à faire sentir que les deux mouvements contraires sont étroitement liés. J'ai montré plus en détail dans mon *Ordre Social*

(tome 1, pages 61 et 228) que l'un des deux mouvements tendait,toutes conditions égales, à susciter l'autre.[4]

Si, à la lumière de cette relation, on suit le déroulement des phénomènes consécutifs à une insuffisance d'encaisse, on observe que, dans la première phase, la baisse du niveau général des prix fournit, par réduction des encaisses nécessaires au règlement des transactions, les suppléments d'encaisse désirés.

Mais le taux d'escompte de la Banque d'émission est toujours très proche du taux du marché. Dès que, dans son ascension, le taux du marché bute contre le taux d'escompte, il cesse de croître, puisque, à ce taux, la Banque prend tous les effets offerts et non demandés.

A partir de ce moment, la circulation effective est portée au niveau du montant global des encaisses désirées par monétisation d'effets, donc création de suppléments de monnaie, sans nouvelle variation du niveau général des prix.

Dans le cas où, au lieu d'insuffisance, il y a excès d'encaisses effectives relativement au montant des encaisses désirées, le phénomène est à peu près l'inverse du précédent.

Les titulaires d'encaisses indésirées demandent sans offrir. Leur demande peut porter, soit sur des créances, soit sur des richesses proprement dites.

Dans la mesure ou elle porte sur des créances, elle tend à provoquer baisse de taux sur le marché monétaire. Elle incite les débiteurs d'effets antérieurement escomptés venant à échéance à offrir sur le marché les effets de renouvellement au lieu de les remettre à la Banque. En remboursant la Banque, ils résorbent, à due concurrence, des monétisations antérieures, et, par là, tendent à ramener, sans variation du niveau général des prix, les encaisses effectives au niveau des encaisses désirées.

Dans la mesure où le supplément de demandes porte sur des richesses proprement dites, il tend à provoquer hausse des prix au comptant. Il incite donc des personnes qui, antérieurement, vendaient à terme, à transférer leurs offres sur le marché au comptant. Si, par exemple, les effets offerts sur le marché représentaient des ventes de sucre du grossiste au détaillant, la hausse des prix au comptant incite le grossiste à vendre au comptant ce qu'il vendait à terme, alors que l'existence

[4] En gros, la démonstration est la suivante: si la majoration d'offre ou la diminution de demande porte sur des richesses proprement dites et non sur des créances, elles provoquent directement baisse des prix au comptant. Cette baisse rend profitable l'opération consistant à acheter au comptant pour revendre à terme, en se procurant par escompte des créances tirées de la deuxième transaction les fonds nécessaires au réglement de la première. Cette opération étant profitable est accomplie par les arbitragistes. Elle suscite hausse du taux de l'intérêt sur le marché.

d'encaisses indésirées fournit au détaillant le moyen de payer au comptant ce qu'il payait à terme.

Ainsi, la tendance à la hausse des prix diminue la quantité d'effets créés, donc offerts sur le marché monétaire. Or, la Banque prend tous les effets offerts et non demandés.[5] La diminution de l'offre d'effets réduit le montant des entrées dans le portefeuille d'escompte à un niveau inférieur à celui des sorties provoquées par la venue à échéance d'escomptes antérieurs. Par là, elle réduit le montant global des effets escomptés, donc la quantité de monnaie en circulation, qu'elle tend à ramener au niveau des encaisses désirées.

Cependant, ce double processus de résorption ne peut jouer qu'à concurrence des possibilités de remboursement d'escomptes antérieurs, puisque c'est seulement à concurrence de ce montant que les effets de renouvellement ou les valeurs qu'ils représentent peuvent être déroutés de la Banque vers le marché. Si, au cours d'une certaine séance, le montant des encaisses à résorber dépasse celui des échéances d'escomptes antérieurs, l'excédent de demandes ne peut susciter d'offre compensatrice. Il provoque donc les deux mouvements liés de hausse de prix et baisse de taux. La première se poursuit jusqu'au moment où elle a absorbé, par majoration des encaisses nécessaires au réglement des transactions, le solde des encaisses indésirées.

Dans un univers réel, les divergences entre encaisses effectives et encaisses désirées pourront n'affecter le marché monétaire qu'avec certains délais, variables d'ailleurs avec l'objet particulier de la demande émanant des personnes désirant augmenter ou diminuer leur encaisse. Mais, dès que le mouvement de taux tendra à se produire, le phénomène de régulation jouera et entraînera, par monétisation ou démonétisation, l'adaptation de la circulation effective au montant des encaisses désirées.

On voit l'erreur que l'on commettrait en admettant que l'on peut, même en régime de monnaie inconvertible, faire varier à son gré le niveau des prix en créant ou détruisant de la monnaie.

La quantité de monnaie en circulation tend à s'adapter, à chaque instant, au montant des encaisses désirées. Le phénomène joue plus ou moins vite, suivant le marché sur lequel il prend naissance et suivant que le système économique envisagé est plus ou moins rigide. Mais, dès qu'il joue, il met un terme au mouvement de prix en comblant, par variation de la quantité de monnaie en circulation, l'écart qui peut exister entre les montants globaux des encaisses effectives et désirées.

[5] Sur les marchés anglo-saxons, l'intervention de courtiers modifie la forme, non la substance, du phénomène.

4. La régulation monétaire en régime de fausse monnaie

Au cours du paragraphe précédent, nous avons considéré que la Banque ou le marché n'hésiteraient jamais, en raison de doutes sur la bonne fin des effets offerts, à opérer les achats auxquels les mouvements de prix où de taux les inciteraient.

Mais il va de soi qu'il n'en sera pas ainsi lorsque certains de ces effets, émis par des entreprises déficitaires, ne seront pas susceptibles de remboursement à l'échéance.

Le déficit de Trésorerie, en particulier, conduit les Etats qui se refusent à suspendre le paiement des dépenses publiques, à offrir sur le marché des effets—généralement des bons du Trésor—qui n'ont pas de contrepartie dans l'actif du débiteur, et qui, faute de dispositions spéciales, ne pourraient être remboursés.

Mais dès qu'il serait informé de cette situation, le marché se refuserait à les acheter. Pour éviter la fermeture de ses guichets, le Gouvernement déficitaire impose à une Banque, généralement sa Banque d'émission, l'achat de ces "fausses créances" en l'autorisant à les porter pour leur valeur nominale, nonobstant leur caractère fallacieux, à l'actif de son bilan.

Cette disposition donne à tout détenteur des Bons du Trésor émis, la certitude de pouvoir les échanger, à son gré, sous réserve de l'agio d'escompte, contre leur valeur en monnaie. Elle assure donc à ces bons un débouché certain.

Dès lors, lorsque le montant des bons offerts dépassera le montant désiré—et l'analyse du mécanisme du circuit permet de démontrer que presque toujours il le dépassera—les bons qui ne seront pas demandés par le marché seront achetés par la Banque, qui créera en contrepartie de la monnaie. Celle-ci majorera de leur montant la quantité de monnaie en circulation. Mais la monnaie ainsi émise sera sans contrepartie de même valeur dans le bilan de l'Institution émettrice. Nous la qualifierons de "fausse monnaie."

Tant que les encaisses nouvellement créées seront désirées, elles n'affecteront pas le niveau des prix. Mais il arrivera nécessairement un moment où elles cesseront de l'être. A partir de ce moment, toute nouvelle création de monnaie tendra à susciter le phénomène de démonétisation précédemment analysé.

Les titulaires d'encaisses indésirées demanderont sans offrir. Toutefois, leur demande ne pourra porter sur les créances qui sont la contrepartie de ces encaisses, car si elle portait sur ces créances, celles-ci auraient été absorbées par le marché et n'auraient pas suscité la création d'encaisses nouvelles.

La demande des titulaires d'encaisses indésirées portera donc sur des richesses proprement dites. Elle tendra à provoquer hausse de prix au comptant.

Par là, conformément au processus décrit au paragraphe précédant, elle incitera les débiteurs d'escomptes antérieurs à vendre au comptant plutôt qu'à terme les richesses qui, en état de régime, eussent été la contrepartie des effets de renouvellement destinés à se substituer aux effets échus.

L'expérience confirme dans tous les cas, avec une extrême netteté, cette première partie de notre analyse à laquelle elle apporte ainsi une très sérieuse confirmation. On observe, en effet, toujours, en régime de déficit, la diminution du portefeuille commercial, qui, lorsque le déficit est prolongé, se trouve ramené au minimum répondant à de simples besoins d'encaissement.

Ainsi, les titulaires d'encaisses indésirées trouvent les richesses proprement dites qu'ils souhaitent, ou leur équivalent, et en les demandant, résorbent, à due concurrence, la monnaie qu'ils ne souhaitent pas.

Mais, lorsque le portefeuille commercial est réduit au minimum au dessous duquel il ne peut descendre, ce processus cesse de jouer. Le Trésor, débiteur d'effets qui n'ont pas, dans son actif, de contrepartie de vraie richesse, ne dispose pas, comme en disposerait le débiteur d'une vraie créance, de l'option entre vente au comptant et vente à terme.

Si, par exemple, ces effets représentaient simplement l'anticipation de la recette afférente à la vente de services publics, le Trésor pourrait, s'il y avait un intérêt important, vendre ces services au comptant plutôt qu'à terme. Mais, ici, cette contrepartie n'existe pas. Il n'y a aucune valeur susceptible d'être vendue derrière les effets que le Trésor escompte.

La hausse des prix ne peut donc démonétiser les encaisses indésirées émanant de ces escomptes. Elle s'exerce en vain et, faute de pouvoir susciter une offre compensatrice, se poursuit jusqu'au moment où, par majoration des encaisses nécessaires au règlement des transactions, elle a fait désirer les suppléments d'encaisse qu'au niveau des prix ancien personne ne souhaitait détenir.

Ainsi, lorsque des créances sans contrepartie, c'est-à-dire des fausses créances, ont été rendues éligibles à l'escompte, le processus de régulation continue à jouer dans le sens de l'augmentation de la circulation monétaire, qu'il y ait insuffisance d'encaisse ou excès d'offre de créances sur le marché, mais il ne peut plus jouer dans le sens de la réduction de la circulation qu'à concurrence des échéances de vraies créances antérieurement escomptées. Dès que le portefeuille de vraies créances a été réduit au minimum au-dessous duquel il ne peut descendre, la monnaie indésirée ne peut être résorbée. Elle reste en circulation et suscite

la hausse de prix nécessaire pour la faire désirer comme instrument de règlement sur le marché.

5. La notion d'inflation et les deux types de sociétés

Aux termes de l'analyse précédente, il y a inflation lorsqu'une quantité de monnaie existante et non résorbable, faute d'un matelas de vraies valeurs dans le bilan de l'Institution émettrice, devient indésirèe.

Dès qu'une pareille situation existe, la monnaie indésirée, ne pouvant disparaître, provoque hausse des prix jusqu'au moment ou l'augmentation de la valeur des transactions la rend nécessaire en tant qu'instrument de règlement.

Cette distinction entre monnaie résorbable et non résorbable, qui paraît purement technique, conduit à distinguer deux types de Sociétés, aux caractères entièrement différents, suivant que les fausses créances y sont ou non éligibles à l'escompte.

Dans les premières, personne ne peut demander sur le marché sans y susciter ou tendre à y susciter une offre d'égale valeur.

De ce fait les déterminations individuelles, quelles qu'elles soient, ne peuvent affecter d'une façon importante le niveau général des prix. L'entière liberté des individus ne peut entraîner aucune perturbation importante. Ils peuvent produire ou ne pas produire, demander ou offrir, réduire ou augmenter leurs encaisses, toujours la valeur globale de l'offre tendra être au niveau de la demande. S'il y a peu à offrir, la demande sera faible, si beaucoup à offrir, elle sera forte. Des variations de prix particuliers pourront survenir, mais pas de variation importante ou continue du niveau général des prix.

Dans une Société à vraie monnaie, la liberté est possible parce qu'elle ne peut entraîner aucun désordre grave.

Lorsque des fausses créances ont été rendues éligibles à l'escompte, le phénomène de régulation peut devenir inefficace, en ce qui concerne la résorption des éléments d'encaisse indésirés.

Dès que la part de circulation émise contre vraies créances a été réduite au minimum au-dessous duquel elle ne peut descendre—parce que répondant, par exemple, à des commodités d'encaissement—toute réduction des encaisses désirées ou toute augmentation de la circulation effective entraînent une hausse du niveau général des prix à laquelle aucune augmentation de l'offre ne tend à s'opposer.

Dans pareille Société, l'usage que les hommes font des droits qu'ils détiennent affecte directement le niveau général des prix. Tant qu'ils accepteront de conserver les fausses créances qui leur ont été attribuées ou la monnaie par laquelle ces fausses créances peuvent être représentées, la création de fausses créances, donc le déficit, n'entraîneront aucune hausse du niveau général des prix. Mais s'ils entendent substituer de

vraies richesses à ces fausses créances ou à la monnaie qui peut les représentor, leur demande s'exercera sans contrepartie sur le marché et les mouvements des prix particuliers qui en seront la conséquence seront tels qu'ils entraînent hausse du niveau général des prix.

Ainsi, dans une Société où de fausses créances ont été admises à l'escompte, la liberté n'est plus sans danger. Si l'on veut èviter toute possibilité de hausse du niveau général des prix, il faut, à partir du moment où la quantité de vraie monnaie en circulation a été réduite au minimum au-dessous duquel elle ne peut descendre, limiter l'usage que les hommes peuvent faire de leur pouvoir d'achat en leur interdisant, par le rationnement généralisé, de tenter de substituer de vraies richesses aux fausses créances qu'ils détiennent ou à la monnaie par laquelle celles-ci ont pu être représentées.

C'est donc seulement en régime de vraie monnaie que l'entière liberté des hommes ne peut pas être une cause de désordre social.

La véracité de la monnaie est la condition de la liberté humaine.

Résumé

1. *Definitions of indices of general level of prices and of global supply.* If p and q represent individual prices and quantities of goods exchanged during the period under consideration, the indices P and Q are defined by the formula[1]

$$\frac{dP}{P} = \frac{\Sigma q dp}{\Sigma qp}, \quad \frac{dQ}{Q} = \frac{\Sigma p dq}{\Sigma qp}.$$

2. *Theory of variations in the general price level.* If the total value, Σpq, of goods exchanged during the same period, called total demand, is represented by D, we have:

$$\frac{dP}{P} = \frac{dD}{D} - \frac{dQ}{Q},$$

which is equivalent to

$$dP = \frac{dD - kPdQ}{kQ}.$$

But in any period $dD-kPdQ$ is equal to the difference between the variations, in the same period, of the total amount of cash balances in hand and the total amount of cash balances desired.[2]

[1] For these definitions see *L'Ordre Social*, (Sirey, Paris), chap. III.

[2] *Ibid.*, tome I, p. 311.

Therefore the quantitative theory must be expressed as follows: The general price level varies only as a function of the difference between the total amount of cash balances in hand and cash balances desired. The general price level does not depend on supply and demand. If supply is large, demand will also be large if or insofar as there is no difference between the total amount of cash balances in hand and cash balances desired. Inversely, the general price level will not be affected by the total amount of money in circulation, so long as this total amount is desired by the people using it.

3. *Monetary regulation in a real-money regime.* Real-money is any money obtained by monetization of assets saleable on the market at the price for which they have been bought by the Bank for monetary units.

Under a real-money regime, the quantity of money in circulation, whether convertible or unconvertible, tends always to adapt itself to the total amount of cash balances desired. This mechanism tends to maintain stability of the general price level.[3]

4. *Monetary regulation under false-money regime.* Monetary regulation implies the monetization of cash in demand and the demonetization of undemanded cash. But in order that undemanded cash may be demonetized, it is necessary that it has been made through monetization of an asset saleable on the market for a value of the same amount. As soon as all the money obtained through monetization of real assets, *e.g.*, commercial bills or gold, has been demonetized, the remaining amount of money is no longer capable of demonetization. If some part of this amount of money remains undesired, it causes an increase in the price level until such time as—because of a related increase in the cash necessary for the performance of business transactions—the surplus cash becomes desired. When that happens, there is inflation.

5. *The Concept of inflation.* According to the previous analysis, inflation occurs when one part of the existing cash in circulation is not desired and cannot be demonetized for lack of a counterpart of real value in the balance sheet of the issuing bank.

In such circumstances the economic troubles can only be avoided by rationing, which means suppression of freedom in economic choices.

Reality of money is the condition of human freedom.

Mr. Rueff's paper was discussed by Messrs. Arthur Smithies, Michal Kalecki, N. L. J. Van Buttingha Wichers, and the speaker.

[3] *Ibid.*, chaps. 17 and 18.

CAPITAL ACCUMULATION AND THE END OF PROSPERITY*

by Evsey D. Domar

The University of Chicago (United States)

A. The Problem

Over the business cycle, investment plays a double role: on the one hand income generated by it raises the level of total national income and employment and creates prosperity; on the other, the resulting rapid accumulation of capital uses up existing investment outlets and eventually causes a breakdown and a depression.

This is the essence of several business-cycle theories developed by a number of economists, such as Marx, Hobson, more recently Keynes, Hansen, Kalecki, Kaldor, Harrod, Paul Sweezy, and others.[1] The purpose of this paper is to examine some of these theories by means of a few simple economic models and to develop some of their ideas somewhat further. Owing to limitation of space, we shall devote most of our attention to the logic of the argument, its economic implications having been published elsewhere.[2]

The discussion will refer to a private capitalist society in which the government plays a minor part. Therefore we can disregard the difference between national income and national product, the tax collections being very small. It is also assumed that the general price level is kept constant.

Notations: $Y =$ national income; $C =$ consumption; $S =$ saving (including corporate saving); $I =$ investment; $P =$ productive capacity of

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1 Marx's writings are well known; J. A. Hobson, *Economics of Unemployment* (London, 1922), and *Rationalization and Unemployment* (New York, 1930); J. M. Keynes, *The General Theory of Employment, Interest and Money* (New York, 1936); A. H. Hansen, *Fiscal Policy and Business Cycles* (New York, 1941), and *Economic Policy and Full Employment* (New York, 1947); Michal Kalecki, *Essays in the Theory of Economic Fluctuations* (New York, 1939), and "Full Employment by Stimulating Private Investment?," *Oxford Economic Papers*, No. 7 (March, 1945), pp. 83–92; Nicholas Kaldor, "Stability and Full Employment," *Economic Journal*, Vol. 48 (December, 1938), pp. 642–57; R. F. Harrod, *The Trade Cycle* (Oxford, 1936); P. M. Sweezy, *The Theory of Capitalist Development* (New York, 1942).

2 See my papers "Capital Expansion, Rate of Growth, and Employment," *Econometrica*, Vol. 14 (1946), pp. 137-147; "Expansion and Employment," *American Economic Review*, Vol. 37 (1947), pp. 34-35; and particularly "The Problem of Capital Accumulation," *American Economic Review*, Vol. 39 (1948), pp. 777-794.

the economy at full employment; K = capital; E = exogenous causes; t = time; s, a, α, β, r = constants. Y, C, S, I, P, s are in net terms (over and above depreciation) per unit of time.

B. A Simplified Keynesian System[8]

(1.1) $C + S = Y,$

(1.2) $S = F(Y),$

(1.3) $I = G(Y, E),$

(1.4) $I = S,$

(1.5) $Y = P$ (P constant).

The stock of capital (as an integral of investment) does not enter Keynes's system explicitly. The possibility of a breakdown or of chronic unemployment is due to the contradiction between the value of Y given by the first four equations and the requirement that income equal productive capacity expressed in (1.5). It is a static system, and in the form presented here cannot be used as an explanation of the cycle.

C. A Simplified Hobson-Marx System

This system represents an attempt to express the theories of these two writers by an explicit model. Such an undertaking is always hazardous, particularly in the case of Marx whose work is so complex. There is no doubt that an alternative system, or even several alternative systems, could be derived from his writings with equal accuracy.

Capital accumulation, absent in Keynes's formal system, appears here explicitly, but its role is still not entirely clear. So long as the stock of capital remains below some vaguely defined "critical" magnitude K_n, the system takes the following form:

CASE I. For $K < K_n$:

(2.1*a*) $C + S = Y,$

(2.2*a*) $S = F(Y),$

(2.3*a*) No investment function,

(2.4*a*) $I \equiv S,$

(2.5*a*) $Y = P$ (P constant),

(2.6*a*) $\dfrac{dK}{dt} = I.$

This is essentially a restatement of Say's Law. The saving-investment problem, so prominent in Keynesian literature, is entirely absent here. Investment and saving are identically equal, and full employment is automatically maintained.

[8] The interest-rate and money equations are omitted because they are not essential for our purposes.

As soon, however, as K reaches its critical magnitude K_n, the profitability of further additions to capital stock (*i.e.*, investment) drops radically, and prosperity abruptly ends. We have then the second case.

CASE II. For $K \geqslant K_n$:

(2.1*b*) $C + S = Y,$

(2.2*b*) $S = F(Y),$

(2.3*b*) $I = a,$

(2.4*b*) $I = S,$

(2.5*b*) $Y = P$ (P constant),

(2.6*b*) $\frac{dK}{dt} = I,$

where a is some small constant. The system is overdetermined; the magnitude of I given by (2.3*b*) is presumably much too small to yield the Y required by (2.5*b*), and a depression comes in. The two cases could be combined by making $I = \Phi(K)$ such that $I = S$ for $K < K_n$, and $I = a$ for $K \geqslant K_n$. The essence of the Hobson-Marx system lies in the idea that capital accumulation has a strong negative effect on profitability of investment, an idea which is accepted, explicitly or otherwise, by Keynes and his followers, as well as by some non-Keynesians. But the character and magnitude of K_n requires further exploration.

D. An Expanded System

This system contains some of the characteristics of the preceding ones; it emphasizes the relation between capital and productive capacity.

I. No Contradictions

(3.1) $C + S = Y,$

(3.2) $S = F(Y),$

(3.3) No investment function,

(3.4) $I \equiv S,$

(3.5) $Y = P,$

(3.6) $\frac{dK}{dt} = I,$

(3.7) $P = H(K, s),$

where s is some constant to be discussed presently. We should also note that P is no longer constant.

The system is not overdetermined and is capable of a solution. If we take the simplest possible case[4]

[4] It would be more general and realistic to take $S = \beta + \alpha Y$ ($\beta \leqslant 0$). The particular assumption that $\beta = 0$ simplifies the argument to such an extent, as to make this assumption worthwhile even at the expense of generality.

(3.2) $S = \alpha Y \quad (0 < \alpha < 1)$, and

(3.7) $P = Ks$,

where s is the ratio between productive capacity and capital required by existing technological and other conditions — a concept similar to that (or rather to its inverse) used in connection with the acceleration principle, the solution takes the form of

(3.8) $Y = Y_0 e^{\alpha st}$.

II. Possible Contradictions

(a) If the investment function is reintroduced as

(3.3) $I = G(Y, E)$

and (3.4) is changed to $I = S$, we have the familiar Keynesian case. The latter, however, would not make use of all possibilities of this system.

(b) A more interesting case arises when (3.7) is made subject to the restriction that

(3.9) $P = Ks$ only for $\alpha s \leqslant r$,

where r is the maximum rate of growth the economy can achieve.

If $\alpha s = r$, then, with investment and saving functions being identical, the economy is in equilibrium in the sense that not only full employment (of labor) is preserved, but that no excess accumulation of capital develops as well. If we introduce an additional assumption that capital income (profits and interest) remains a constant fraction of national income, then (with s remaining constant), it can be shown that the average yield from capital remains unchanged in spite of continuous capital accumulation. As far as our system is concerned, this state of affairs can continue indefinitely. On the other hand, if the magnitude of s gradually diminishes (the so-called "deepening" of capital), full employment and full utilization of capital are maintained, but the profitability of investment will gradually fall off. It appears to me, however, that in the absence of additional assumptions, such as lags, the cyclical significance of this gradually diminishing rate of profit should not be great.

The restriction (3.9) implies that the required rate of growth αs may not be achieved owing to limited quantities, or more exactly, owing to insufficient growth of factors of production other than capital; of these labor is probably the most important. If, in the presence of this restriction, national income grows at the maximum rate r, full employment of *labor* is maintained by definition of r, but a part of capital stock becomes idle. If we define $\sigma = dP/dK$, then $\sigma < s$. This case has interesting cyclical implications. If (3.4) becomes an equation, and

if in addition (3.3) is changed to

(3.10) $I = \theta(Y, E, \gamma)$,

where $\gamma = s/\sigma$ (or possibly $\gamma = s - \sigma$) and naturally $\theta_\gamma < 0$, the system acquires a strong cumulative force. If for some reason (not necessarily even due to labor shortage) income fails to grow at the αs rate, idle capital develops (γ increases). This reduces the profitability of investment; its rate of growth slackens (or becomes even negative), and the corresponding fall in the rate of growth of income makes an additional part of the capital stock idle. This in turn depresses the profitability of investment, and so on. A dynamic model along these lines could easily be constructed, though it should be indicated that it could only be used as a partial explanation of the downturn (there may be other causes as well), and, as presented here, does not explain the upturn at all.

It should be made clear that the presence of idle capital observed in our economy so often does not prove by itself that the restriction (3.9) has been operative. Idle capital will develop whenever income fails to grow at the αs rate, irrespective of the cause. There is nothing in the investment function used here or in the one that could be obtained empirically, that indicates that investment and income will grow at the required rate. Thus besides the obvious hypothesis that the αs rate is impossible physically, another one can be suggested—that even in the absence of this limitation the required growth will not materialize owing to institutional factors, or more precisely, to the peculiarities of the investment process in a capitalist society. An empirical verification of the two hypotheses is highly desirable.

III. Possible Objections

(a) Rejection of the whole approach on the grounds that the parametric treatment of s is illegitimate. The existence of a relatively rigid ratio (or even a reasonably small range) between output and the stock of capital needed for its production is by no means an established fact, though it has been assumed in economic literature many a time. The rejection of s will deprive our system of most of its cyclical significance, because a slowly declining rate of profit resulting from $d^2Y/dK^2 < 0$ will hardly exert strong depressing effects over some five or seven years, and few, if any, of our prosperities lasted longer. Yet there are reasons to believe that the relation of capital stock to its productive capacity over relatively short periods is less flexible than it is usually supposed in the traditional theory. The current investigations of Professor W. W. Leontief of Harvard should shed more light on this problem.

(b) Idle capital is not likely to be distributed evenly over all industries and firms. Its presence in some sectors of the economy need not deter others from expanding their capital facilities.

(c) If exogenous factors, such as development of new methods of production or new products, changes in tastes and habits, population growth, aggressive competition, etc., play an important role in the investment function, the past accumulation of capital, idle or otherwise, will have little effect on profitability of investment. Professor J. A. Schumpeter is likely to take this point of view.

The processes described in (a) and (b) may involve heavy capital losses. And if the economy is to continue expanding in spite of them, it is necessary that those on whom they are inflicted be unable to prevent the construction of new capital. The institutional structure of the economy, the presence of powerful business combinations and financial interconnections between firms is of utmost importance here.

IV. Policy

The purpose of policy here is to bridge the gap between the required rate of growth of income and the actual one.

(a) If (3.9) is effective, there are three possibilities:

i. Reduction of α.

ii. Reduction of s by developing industries requiring large capital outlays per unit of output.

iii. A continued rise in the general price level (excluded from consideration here).

(b) If (3.9) is not effective, *i.e.*, if the required rate of growth is physically possible, there are other possibilities in addition to those listed in (a). They consist in various methods of encouraging private investment, such as low interest rates, incentive taxation, liberal loss offsets for income-tax purposes, etc. A guaranteed growth of income as a method of creating investment opportunities should be explored in this connection. And there is no inherent reason why public investment should not play a more important role as well.

Résumé

A. Le Problème

La prospérité est caractérisée par l'accumulation rapide de capital. L'accumulation de capital comme telle met-elle fin à la prospérité?

Signification des symboles: Y = revenu national; C = consommation; S = épargne; I = investissements; P = capacité productive de l'économie à plein emploi; K = capital; E = causes exogènes; t = temps;

s, a, α, β = constantes. Y, C, S, I, P, s sont considérés comme étant nets par unité de temps; le niveau de prix est constant.

B. Schéma Keynesien simplifié

Le système (1.1)—(1.5) du texte anglais est sur-déterminé. Son essence consiste en la contradiction entre la valeur de Y dans les équations (1.1)—(1.4) et la condition imposée dans l'équation (1.5).

C. Schéma de Hobson-Marx simplifié

Deux schémas sont à distinguer:

I. Pour $K < K_n$, où la nature de K_n, grandeur "critique," est plutôt vague, nous avons les équations (2.1*a*)—(2.6*a*) du texte anglais. C'est ici essentiellement un exposé de la loi de S a y. Le plein emploi est maintenu.

II. Pour $K > K_n$, nous avons les équations (2.1*b*)—(2.6b) du texte anglais, où a est une constante minime. Le système est sur-déterminé; la grandeur de I, exprimée dans (2.3*b*), est probablement trop petite pour rendre le Y requis par (2.5*b*).

D. Schéma combiné

I. Pas de contradictions: nous avons les équations (3.1)—(3.7) du texte anglais, où s est le rapport constant du revenu au capital. P n'est plus constant. Le système n'est pas sur-déterminé. Si on écrit (3.2) et (3.7) on aura la solution (3.8).

II. Contradictions possibles:

(a) Si la fonction d'investissement est réintroduite comme (3.3), et si (3.4) devient $I = S$, nous aurons le cas Keynesien.

(b) Si (3.7) est soumis à la restriction (3.9), où r représente le taux de croissance maximum d'investissement, alors une "sur-accumulation" de capital aura lieu, dans le sens que $dY/dK = \sigma < s$.

(c) Pour les buts cycliques, il peut être utile de combiner les cas (a) et (b) en changeant (3.3) à (3.10).

III. Objections possibles:

(a) Rejet de toute la théorie, s pouvant n'être pas un concept significatif.

(b) Si des facteurs exogènes jouent un rôle important dans la fonction d'investissement, l'accumulation de capital n'aura qu'une importance minime.

(c) Les effets d'une accumulation excessive de capital dépendent de sa répartition.

(d) Nous n'avons guère de preuve empirique que la restriction (3.9) a été conforme à la réalité, spécialement au cours d'une courte période de temps, telle que cinq ans.

IV. Solutions:

(a) Si (3.9) est applicable—

i. Réduction de α.

ii. Réduction de s, en développant les industries qui requièrent de fortes dépenses de capital par unité de production.

iii. Accroissement du niveau général des prix (non considéré ici).

(b) Si (3.9) n'est pas applicable, le plein emploi peut être maintenu par une garantie du gouvernement que Y croîtra au taux nécessaire.

Mr. Domar's paper was discussed by Messrs. Arthur Smithies, E. F. Lundberg, Donald C. MacGregor, and the speaker.

CONTRIBUTED PAPERS : II

Wednesday, September 17, at 9:30 a.m.

CHAIRMAN :

Herman O. A. Wold

Director, Institute of Statistics, University of Uppsala (Sweden)

SUR LA DIVISION PRAGMATIQUE

par H. Steinhaus

Professeur a l'Université de Wroclaw (Pologne)

Le principe de division en deux moitiés, que l'on peut désigner par la règle "l'un divise, l'autre choisit," est bien connu; on prétend même qu'il existe dans la tradition juridique anglaise. Je me suis demandé si l'on peut étendre ce principe à n partenaires, pour $n > 2$. Ayant résolu ce problème pour $n = 3$, je crus d'abord qu'il n'y a pas de solution pour n dépassant 3 et je communiquai cette question à S. Banach par l'intermédiaire de M. B. Knaster; ces deux mathématiciens l'ont résolue pour un n quelconque.

La division pragmatique peut être considérée comme un jeu; or, dans la théorie d'un jeu il faut distinguer les règles et les méthodes. Le problème de la division en deux moitiés est un jeu dont la règle a été donnée par: "l'un divise, l'autre choisit." La méthode, qui montre que ce jeu peut servir à une division équitable est, pour le premier, de diviser en deux parties qu'il considère égales en valeur, pour le second, de choisir entre ces deux parties, celle-là qu'il considère comme la plus grande en valeur ou, du moins, comme égale à l'autre. Ainsi, chacun des deux partenaires devient indépendant de l'autre, parce qu'il est sûr d'obtenir au moins ce qu'il considère être la moitié de l'objet à diviser sans se soucier de l'autre: il n'a qu'à suivre la méthode qui vient d'être décrite.

La solution du problème pour n partenaires, donnée par Banach et Knaster, est la suivante: en désignant les partenaires par $A, B, C, \ldots, N$, on laisse A choisir ce qu'il considère comme la $1/n$-ième partie qui lui est due; il découpe donc de l'objet, que' l'on peut s'imaginer être un gâteau, une partie quelconque. B a maintenant le droit de diminuer cette partie, en découpant un morceau et en le restituant au gâteau; C peut, à son

tour, diminuer encore la partie découpée par A et diminuée (ou non) par B; ainsi tous les partenaires peuvent, l'un après l'autre, exercer leur droit de diminution, sans y être contraints. La règle force le dernier qui a exercé ce droit, d'accepter le morceau restant, comme la partie qui lui revient. Ce partenaire sort du jeu, qui recommence de la même manière alors avec les $n—1$ partenaires restants. Il est facile à voir que la méthode qui consiste à diminuer toujours un morceau que l'on considère comme plus grand que le $1/n$-ième du total, en le réduisant au $1/n$-ième, a pour résultat que chacun reçoit un lot qui est au moins égal à la partie à laquelle il a droit, suivant sa propre estimation des valeurs; l'ignorance, la malice, ou l'avarice des autres partenaires ne peut empêcher cela, même s'ils forment un complot dans le seul but de le désavantager.

La méthode décrite ici peut servir pour une division d'objets tels que champs, jardins, en général tous les objets pour lesquels on peut considérer la valeur comme égale à la somme de la valeur des parties et tels qu'une partie quelconque de valeur v contient toujours, pour tout v' moindre que v, une partie de valeur v'. Elle peut servir aussi quand il s'agit de diviser un total entre plusieurs partenaires qui y participent en vertu de droits exprimés par des fractions inégales.

L'importance pratique de la méthode pragmatique consiste en ce que l'acte de la division dispense d'une appréciation objective de la valeur des différentes parties de l'objet considéré; ainsi une expertise imposée par le juge aux parties devient superflue. La méthode pragmatique permet à chaque partenaire de s'assurer une partie du total dont la valeur, appréciée par lui-même, est au moins égale à la fraction de la valeur du total (qui est aussi définie par lui seul), qui lui est due *a priori*, en vertu d'un testament ou d'un autre acte légal qui précède la division physique et qui sert comme point de départ au problème. Cette méthode est donc en accord parfait avec les théories proposées au XIX siècle par les Gossen, K. Menger, Stanley Jevons et Léon Walras, économistes éminents qui considèrent la valeur subjective comme une notion fondamentale pour la théorie des prix.

La division pragmatique ne garantit pas que le partenaire, qui est sûr d'acquérir au moins ce qui lui est dû, considère les parties acquises par les autres partenaires comme étant égales entre elles et égales à la sienne. Or on peut démontrer, en se servant d'une méthode exposée par MM. A. H. Stone et J. W. Tukey (*Duke Mathematical Journal*, Vol.. 9, 1942, pp. 356-359), basée sur un théorème de Borsuk, qu'il existe une division d'un objet T en n parties P_i $(i = 1, 2, \ldots, n)$ correspondant a n nombres q_i $(0 \leqslant q_i \leqslant 1)$ telle que P_i ait une valeur égale à $q_i \times$ valeur de T et que ces n équations subsistent pour les estimations de tous les partenaires : $E_j(P_i) = q_i E_j(T)$ $(i, j = 1, 2, \ldots, n)$;

E_j signifie la valeur attribuée à X par le j-ième partenaire. On peut donc réaliser une division équitable du point de vue de tous intéressés, en respectant en même temps leurs estimations subjectives.

M. Bęczkowski m'a demandé d'appliquer la division pragmatique au problème pratique suivant: il y a quatre secteurs qu'il faut distribuer entre quatre équipes; il y a une somme fixe (p.e. 20 millions) devant servir à l'ensemble du travail à accomplir. La solution est la suivante: chaque équipe nomme un montant quil lui semble suffisant pour exécuter les travaux du secteur I; on attribue le secteur à l'équipe qui a nommé le chiffre minimum; elle doit se charger du secteur I; le procédé recommence avec le secteur II et les trois équipes libres, après qu'on a attribué de la même manière le secteur III, il reste encore une équipe libre et un certain montant disponible; l'équipe est obligée de se charger du secteur IV pour ce montant. La méthode est donc, pour chaque équipe, de fixer des chiffres qui correspondent autant que possible aux frais relatifs des travaux à exécuter; en supposant que le total de 20 millions suffise pour la totalité du travail, cette méthode garantit à chaque équipe de n'être pas chargée d'un travail dont les frais, suivant sa propre appréciation, sont plus éléves que le montant qui lui sera adjugé.

M. Knaster a donné la méthode suivante pour la division d'héritages composés d'entités telles que maisons, objets d'art, etc., qui ne peuvent être divisés; cette méthode est d'une grande importance pour la pratique. La règle est la suivante: supposons qu'il y ait n héritiers participant à l'héritage, leurs droits respectifs étant représentés par les nombres $p_1, p_2, \ldots, p_n$, avec $\Sigma p_i = 1$. Il y a plusieurs objets $U, V, W, \ldots$, qui constituent la masse à diviser. Chaque héritier donne son évaluation de chaque objet. On obtient ainsi un tableau dont les lignes correspondent aux objets et les colonnes aux héritiers. Sur chaque ligne on enregistre la côte maxima; l'objet sera adjugé à la personne qui a donné cette côte. Pour évaluer les droits de chaque héritier, on multiplie la somme de tous les nombres de cette colonne par le p_i correspondant. La différence entre ce produit et valeur des objets adjugés à l'héritier en question doit lui être payée en argent comptant. Or, ces différences seront positives pour certains héritiers, négatives pour les autres; leur somme sera positive ou zéro; le cas de nullité ne se présente qui si toutes les évaluations sont identiques pour tous les objets. Il y aura donc, en général une somme positive qui constitue un bonus; on repartit ce bonus entre les héritiers proportionnellement aux nombres p_i, en réalisant ainsi ce que les économistes appellent "consumers' surplus." L'arbitre, qui n'a que la tâche formelle d'enregistrer les côtes et de faire les calculs pour déterminer les payements à verser par certains héritiers aux autres, pourra donc garantir à chaque héritier qu'il obtiendra la partie à laquelle il a droit, soit en objets, soit en argent comptant, la

valeur de la masse étant celle de sa propre évaluation, et la valeur des objets qui lui seront adjugés étant aussi appreciée d'après sa propre évaluation; l'arbitre pourra même lui promettre un bonus. La même méthode peut être appliquée au cas où les nombres p_i dépendent des objets. Il n'est pas nécessaire d'adjuger les objets aux personnes dont l'estimation est la plus élevée; il suffit de les distribuer de manière que le "surplus" résultant soit positif; entre toutes les distributions obéissant à cette condition, on choisit celle-là qui rend minimum la somme de paiements mutuels en valeur absolue.

Quand on applique ce procédé pour la répartition entre n personnes, dont chacune participe en $1/n$-ième part, dans le cas qu'il y ait quelques objets identiques, p.e. des coupons du même tissu ne différant ni par la grandeur ni par le dessin, on peut attribuer ces objets à des personnes différentes, en choisissant sur chaque ligne du tableau k côtes maxima, k étant le nombre de pièces identiques. Cette remarque est à M. Bęczkowski. Elle n'est pas applicable quand les droits des partenaires sont inégaux.

Une étude sur le même sujet, plus complète au point de vue mathématique, sera présentée par M. Knaster à la Société des Sciences et des Lettres de Wrocław.

Résumé

There is an old custom of dividing an object fairly between two partners by letting one partner divide it into two parts and letting the other partner choose his part.

This procedure is a *fair game*, and has its *rules* and its *methods*. The rule for the first partner allows him to cut the object as he pleases; the rule for the second partner gives him a free choice of the two parts. The methods are: (1) for the first partner to cut in such a manner as to give the parts equal value in his own estimation, and (2) for the second partner to choose the part that he considers more valuable.

Having found during the war a solution for three partners, I proposed the problem of n partners to B. Knaster and S. Banach. Their solution is as follows:

The partners being ranged $A, B, C, \ldots, N$, A cuts from the object, say a cake, an arbitrary slice. B now has the right, but is not obliged, to diminish the slice cut off. Whatever he does, C has the right (without

obligation) to diminish the already diminished (or not diminished) slice, and so forth up to N. The rule obliges the last diminisher to take as his share the slice he was last to touch. The remaining $n-1$ persons continue in the same way with the remainder of the cake.

The above procedure applies to continuous objects that can be divided in any ratio. The method can be extended to a collection of individual objects, by having each partner write down his estimate of the value of each object, and in effect awarding objects to the highest bidders, with differences between the values of objects awarded and the receivers' proportional claims adjusted in cash.

Mr. Steinhaus' paper was discussed by Mr. Tjalling C. Koopmans and the speaker.

LE CONCEPT OPERATIONNEL EN ECONOMIE

par Jacques Dumontier
Administrateur de l'Institut de Conjoncture (France)

I

L'esprit scientifique que les économistes s'efforcent d'introduire en économie ne doit pas consister uniquement dans l'utilisation d'un appareil mathématique complexe. Il est en effet plus important encore d'introduire en économie les notions logiques qui sont à la base de toute source expérimentale. Parmi celles-ci figure en bonne place la notion de concept opérationnel.

Qu'est-ce qu'un concept opérationnel:

Parmi les nombreux concepts qui se rapportent à l'être humain, les uns sont une construction logique de notre esprit. Ils ne s'appliquent à aucun être observable par nous dans le monde. Les autres sont l'expression pure et simple de l'expérience. A de tels concepts, Bridgman a donné le nom d'opérationnels. Un concept opérationnel est équivalent à l'opération ou à la série d'opérations que l'on doit faire pour l'acquérir. En effet, toute connaissance positive dépend de l'emploi d'une certaine technique. Quand on dit qu'un objet a une longueur d'un mètre, cela signifie que cet objet a la même longueur qu'une baguette de bois ou de métal dont la longueur est égale à celle de l'étalon du mètre conservé à Paris au bureau international des Poids et Mesures. Il est bien évident que nous ne savons réellement que ce que nous pouvons observer. Dans ce cas, le concept de longueur est synonyme de la mesure de cette longueur. Les concepts qui se rapportent à des choses placées en dehors du champ de l'expérience sont, d'après Bridgman, dépourvus de sens. De même, une question ne possède aucune signification s'il est impossible de trouver des opérations qui permettraient de lui donner une réponse. (Carrel, *L'homme, cet inconnu.*)

D'après ce que nous venons de voir, il y aura autant de concepts opérationnels que de techniques expérimentales en économie. Quelles sont donc celles-ci?

Il y a en économie trois types d'expériences, de valeur inégale. Ce sont, non dans l'ordre dans lequel ils ont été historiquement utilisés, mais dans l'ordre d'importance, suivant lequel ils se rapprochent de la condition essentielle de l'expérimentation, d'abord, celui de la confrontation de la prévision avec la réalité. Ce premier type d'expérience est le plus poussé, c'est celui où l'on prévoit les résultats à l'avance et où l'on voit les évènements confirmer ou infirmer ces prévisions. Le deuxième type, moins près de la technique expérimentale des sciences, est celui où l'on va fouiller dans les documents, dans le passé et où on essaye quand même de se mettre dans les conditions d'une expérience, à la condition de satisfaire aux principes de la prévision. Enfin, le troisième

type, c'est ce que les économistes appellent la méthode abstraite, que l'on peut encore considérer avec une certaine valeur scientifique, mais dont il faut se méfier terriblement.

A. Premier type d'expérience: la confrontation de la prévision et de la réalité

a) Il faut d'abord que ce soit une prévision conditionnée. Considérons par exemple la prévision des conséquences d'une dévaluation du franc; une demi-douzaine de facteurs sont en jeu: majorité du gouvernement, réaction de l'étranger, est-ce-que l'activité économique continuera de croître, etc. Pour faire une prévision de valeur de l'expérimentation scientifique, il faut isoler les variations de chaque facteur. C'est quelque chose qui est assez difficile à faire; et en général cette chose n'est pas faite dans la pratique parce que les gens qui font des prévisions ne les font pas dans un but descr ptif. En physique, on place le corps ou le phénomène à étudier dans certaines conditions de manière à voir seulement le déroulement des phénomènes dans ces conditions. En économie, on n'a pas le choix des conditions et il faut prévoir ce qui va se passer dans chaque éventualité: il faut faire toute une gamme de prévisions.

Il y a évidemment une question de mesure: si la précision n'est pas conditionnée, la réalisation des évènements ne prouvera rien. Mais si elle l'est trop, on n'aura rien prouvé non plus, parce qu'on aura souvent fait passer l'aléa dans les termes de la rédaction d'une hypothèse.

b) La deuxième condition qui se relie à celle-ci, est la nécessité de l'expérience écrite. Beaucoup de gens font des expériences et ne consignent pas les résultats par écrit; alors, lorsque l'expérience est réalisée, ils oublient la conclusion même à laquelle ils étaient arrivés, si elle n'a pas été confirmée. Et ils ont l'impression, après, d'avoir prévu ce qui est arrivé. Ce phénomène est absolument général et tient à la sélection des impressions.

Mais, par contre, si on a prévu les diverses éventualités par écrit, l'expérience que l'on veut faire a toutes les qualités d'une expérience et d'une vérification. Cela ne veut pas dire qu'une loi sera établie à la suite d'une seule expérience; il faut la vérifier plusieurs fois, ce qui rétrécira peut-être le domaine dans lequel la loi sera valable.

B. Deuxieme type d'expérience

Le deuxième type d'expérience est ce que l'on appelle quelquefois la méthode statistique qui consiste à rassembler des documents statistiques pour en tirer des conclusions.

Cette manière de faire tient encore en partie à la méthode expérimentale, mais à deux conditions:

a) que l'on ait une loi, même non expérimentée, même grossière, à vérifier,

b) et surtout ce qui est plus important, c'est qu'on se place dans les conditions d'une expérience. Ceci mérite un développement.

Si quelqu'un veut vérifier une loi entre le salaire et le rendement, il prend les chiffres de la documentation minière, peut-être parce qu'ils sont les plus sûrs, mais aussi peut-être parce qu'il sait confusément qu'ils donneront le résultat attendu.

Autrement dit, il faudrait que celui qui rassemble les données ignore le sens de la loi à vérifier. Cette ignorance des faits remplace l'ignorance de l'avenir dans la prédiction et c'est pourquoi j'ai parlé de la prévision du passé.

La grosse difficulté est dans l' impartialité dans le rassemblement des sources. En outre, le lecteur est d'autant moins mis en garde contre la partialité possible que le total des sources possibles n'est pas un tout bien défini, et que les arguments de l'expérimentateur pour écarter certaines sources paraissent généralement convaincantes. C'est là souvent que se trouve la "faille" logique de cette méthode expérimentale qui en permet les abus et qu'il faut savoir dépister.

C. La troisieme méthode

La troisième méthode, qui est plutôt une méthode de remplacement, est la méthode qui fait intervenir la psychologie humaine.

Ce qu'il faut bien voir, c'est en quoi consiste cette méthode. Certains croient que c'est l'application des deux règles de la concurrence parfaite et de l'*homo oeconomicus*. Le fait de supposer une concurrence parfaite et un *homo oecomomicus* ne constitue pas par lui même la méthode abstraite: on peut supposer une concurrence imparfaite et n'importe quel individu, cela ne changerait pas le sens de la méthode; ce serait encore de la méthode abstraite. Celle-ci consiste à déduire dans des cas particuliers le comportement d'un homme d'après une psychologie simplifiée (*homo oeconomicus* ou homme de V e r l e n) dans un système d'échanges (concurrence imparfaite ou parfaite, etc.).

Quel est donc le principe de la méthode abstraite? En réalité, il est double. L'opération a deux caractères dont il faut étudier séparément la valeur scientifique. La première caractéristique est de démonter les rouages d'une grande loi en éléments partiels. Le second caractère réside dans la nature de ces rouages, qui sont psychologiques.

a) Démonter les rouages d'une loi consiste à dire, par exemple; je veux savoir pourquoi les gens partent de la campagne. Il ne suffit

pas d'avoir constaté que les gens partent, mais il faut se demander pourquoi ils le font. Mais il s'agit d'un stade ultérieur de la science. Personne n'a compris le pourquoi de la pesanteur; on l'a ramenée à l'attraction universelle, mais on n'a pas compris le pourquoi. On l'a établi par expérience; et pourtant la loi de la pesanteur est une loi scientifique.

Une loi n'est pas valable parce qu'on a démonté le mécanisme, car, au fond, alors on trouve une autre loi, une ou plusieurs. Une loi est un tout; il y a les hypothèses et la conclusion et non des intermédiaires. Il y a quelque chose qui commence, quelque chose qui lui suit; si l'expérience a montré que la première chose entrainait la seconde, la véritable induction scientifique se passe d'explications intermédiaires. Dans cette volonté de démonter le mécanisme par les rouages, à côté d'un désir de pousser plus loin, il y a un manque de confiance dans l'expérience. Ce manque de confiance est en partie justifié, parce que l'économiste n'a pas un matériel d'expérimentation suffisant. On essaye de trouver des procédés de remplacement, mais il faut bien voir que le seul procédé scientifique est l'expérimentation.

Deux autres objections logiques peuvent être faites au principe de décomposition d'une loi. La première est relative à l'échelle. On peut étudier un phénomène à diverses échelles d'observation. Etudier le comportement d'un individu pour savoir comment se comporte un marché, c'est comme si l'on étudiait le comportement d'un atome pour connaître la solidité d'une table. Les individus entre eux ne réagissent pas comme des individus seuls. Une foule ne se comporte pas comme un millier de fois un individu. L'échelle n'est pas la même si on démonte un rouage en éléments trop nombreux. La question reste la même pour le démontage d'une loi: cette dernière peut n'être pas vraie à toutes les échelles.

La deuxième objection est relative à la loi des grands nombres. En physique on tombe quelquefois dans un domaine où les lois des grands nombres ne peuvent plus s'appliquer avec rigueur, mais où les fluctuations ont un rôle prépondérant. En économie, on ne peut faire toujours jouer la loi des grands nombres sur le comportement des individus, parce que ceux qui agissent sur le marché ne sont pas assez nombreux. Or, considérer qu'un individu a une psychologie-type, c'est faire jouer la loi des grands nombres. C'est souvent plausible, parce qu'il y a bien des phénomènes économiques où tout le monde joue le même rôle, par exemple, le rôle d'acheteur. Mais pour l'entrepreneur, c'est déjà beaucoup moins évident: le nombre en est trop petit.

b) Pour caractériser la méthode psychologique, il ne suffit pas de dire que l'on démonte les rouages des lois, mais il faut préciser que ce sont des rouages psychologiques. Il y a deux sortes de psychologies: celle de l'introspection et celle du comportement qui est la psychologie

vue de l'extérieur èt qui correspond, si vous voulez, à l'étude objective de la science. La mesure des phénomènes s'obtient ainsi par l'observation et non par le raisonnement.

L'explication que donne des faits l'introspection ne saurait être admise sans réserve. En effet, une difficulté peut se présenter: plusieurs enchaînements peuvent être concevables; ils peuvent aussi s'exclure, ou au contraire se combiner entre eux. Mais, s'il peut y avoir plusieurs enseignements contradictoires, cela prouve que l'enchaînement n'est pas une démonstration. Car, si deux enchaînements psychologiques peuvent donner des résultats opposés, leur valeur est petite. Et devant un enchaînement donné on se demandera toujours s'il n'en existe pas un autre caché qui conduirait à une conclusion opposée et qui serait le seul valable.

Rien ne saurait mieux illustrer la déficience de la méthode psychologique que cette constatation de Simiand: "on a soutenu que les salaires sont plus ou moins élevés suivant que les métiers sont plus ou moins agréables [il est bien évident que c'est l'application de la psychologie élementaire], alors que le dépouillement d'une enquête sur le même sujet fait apparaître autant de cas contraires que de cas conformes à cette thèse."

La méthode abstraite n'a donc aucune valeur de démonstration, elle ne saurait en aucun cas remplacer l'expérimentation. Elle peut exciter l'imagination et à ce titre conduire à un projet de loi; mais ce projet devra toujours recevoir l'investiture de l'expérience. L'erreur est lourde de voir une rigueur dans l'explication psychologique. C'est au mieux un enchaînement possible.

Seulement, la méthode abstraite est souvent en l'absence d'autres observations, la seule possible. L'outil est très mauvais, mais on n'en a pas d'autres. Il est donc naturel de s'en servir. L'économie abstraite a son utilité, et ses projets de lois peuvent n'être pas sans intérêt.

II

De ces trois voies expérimentales, trois sortes de concepts opérationnels sont naturellement dégagés. Nous ne suivrons pas ici l'ordre d'excellence des méthodes expérimentées comme au paragraphe précédent, mais bien plutôt l'ordre chronologique dans lequel ces concepts ont été élaborés.

A. La valeur, concept basé sur le choix

A la base de toute analyse psychologique se trouve la possibilité de réaliser l'expérience de choix. Celui-ci est bien un comportement de l'individu, mis dans une alternative et dans l'obligation de choisir.

Mais cette expérience est soumise à certaines conditions; en particulier, elle est instantanée. La question se pose de savoir alors si la notion de valeur a sens dans le temps. Pour comprendre cette question, pensons à cette autre expérience instantanée qui est la pesée où l'on donne le poids ou la masse d'un objet relativement à un autre; seulement en physique, l'étalon de poids peut se garder dans le temps, tout au moins en première approximation, ce qui fait que l'on peut dire que le poids d'un autre objet ou plus exactement sa masse, a varié ou n'a pas varié dans le temps, en faisant deux pesées à des moments differents. L'on peut donc parler du concept de masse ou de poids dans le déroulement du temps. Mais, il n'y a, en économie, rien de semblable à la conservation du poids dans le temps. Ce qui correspondrait au poids, c'est un peu le comportement d'un individu, mais ce comportement varie, peut varier dans le temps. Donc, se servir de l'expérience du choix pour discuter de la valeur dans le temps est quelque chose qui n'est pas opérationnel, pour parler comme Bridgman "qui est dépourvu de sens."

Les travaux de l'école de Lausanne sont, en la matière, très significatifs. Comme ce sont des déductions qui ne sont rattachées à l'expérience que par la possibilité de choix, ils ne permettent, en toute logique scientifique, que de conclure à des relations instantanées et sont impuissants à analyser des évolutions.

A. Marshall avait bien saisi cette infirmité logique qui limitait le domaine des conclusions de Walras et de Pareto. D'où son appel à l'analogie biologique. Mais cet appel est resté lettre morte, car lui-même ne s'est pas rendu compte de ce qu'étaient les concepts à la base du raisonnement qui étaient insuffisants et qu'il fallait en changer.

C'est également pour une raison de logique bien simple qu'il faut rejeter dans le fatras des discussions philosophiques les digressions sur l'imputation de la valuer. Parler de l'imputation de la valeur, c'est précisément vouloir maintenir la notion de valeur dans l'écoulement du temps puisqu'elle est un concept instantané. C'est donc résoudre une impossibilité logique par un artifice verbal.

B. Les concepts expérimentaux de longue durée

A l'autre extrémité des concepts sont ceux qui se sont établis sur des renseignements de longue durée—notamment ceux de progrès techniques, d'expansion démographique et d'évolution de la consommation globale dans le long terme. Les travaux de M. Colin Clark ont pu dégager le concept de progrès technique et le concept d'évolution de la consommation globale qui servent maintenant à établir des théories de l'évolution économique logiquement fondées comme celle de M. Jean Fourastie.

C. Les concepts opérationnels pour les mouvements de courte durée

En toute logique, aucun concept opérationnel d'une des séries précédentes ne peut être à la base d'une étude d'un mouvement de courte durée. Valeur et progrès technique sont prohibés dans ce domaine. Qu'on nous comprenne bien: une étude des mouvements de courte durée peut comprendre les mots de valeur et de progrès technique. Mais l'explication même du mouvement doit être basée sur d'autres concepts.

Ces concepts ne peuvent être dégagés que de l'expérience. Or, nous avons vu précisément que c'est en matière de mouvements de courte durée que les conditions scientifiques d'une bonne expérience sont les mieux réalisés par la prévision conditionnée et sa vérification. Là est l'avenir de la recherche économique.

On peut voir certaines notions se dégager dans le court terme. D'abord la différenciation des allures de l'économie en économie d'offre et économie de demande. En outre, les notions de goulots d'étranglement dans le premier cas et de variations de l'emploi dans le second.

Mais, ces notions doivent être basées sur des expériences à court terme et non pas ramenées à des substituts de la valeur telles que la propension à consommer ou l'utilité, qui ne se conçoivent que par l'éventualité d'un choix. La théorie de Keynes est un des derniers essais de ce genre. L'expérience permettra bientôt de dégager des concepts appropriés.

Résumé

Introduction

Not only should economics make use of mathematical apparatus, but it should have the strictness of thought and the logical processes of other experimental sciences, being one of them. The notion of the operational concept is one of these methods of reasoning.

The operational concept is equivalent to the series of measures and operations necessary to obtain it. In economics, it will arise only from experimental possibilities.

1. *The three experimental ways in economics:*

a) Prediction subjected to the trial of events (the only scientific way); t should be: really conditional and written out in full before the experiment.

b) Verification of a law by statistical documents (imperfect process), must be impartial in the choice of the sources.

c) Abstract analysis: explicative method based on two principles: taking to pieces the mechanism of a law (it does not increase the face-value of the law) and searching for a psychological mechanism, which is but the substitution of the psychology of introspection for that of behavior (inefficient method, but it can excite imagination).

2. *Different operational concepts:*

a) *Choice.* Every psychological analysis in economics is based on the possibility of making a choice. Choice is instantaneous and can be put to no use for reasoning with the factor time. Marshall and J. Schumpeter realized that it was compulsory to create an economy not depending on the instantaneous, but were wrong not to change their operational concept of choice.

b) *Long-trend concepts.* Technical progress, demographical pressure, etc. The experimental method of research in statistical documents might make the operational concepts bearing on the long trend of economics clear. The evolution theories have been based on them.

c) *Concepts of a short-time trend.* Choice can not, logically, be used to study movement, and the above concepts can not be used in the short-trend such as crisis, full employment, or even interest rate. New concepts will be needed for studying economic phenomena of short duration. An *a posteriori* study of the realization of predictions is actually needed.

Actually, economists are inclined to specialize and to work with only one section of this series of concepts.

Even in political economy, that division of work has its consequences. Liberalism is influenced by the instantaneous concept of choice that dominates its philosophy. On the other hand, interventionism that tries to give its due share to investment should be at the basis of the second or third category.

Mr. Dumontier's paper was discussed by Messrs. Jacques Rueff, François Perroux, Michel J. J. Verhulst, Leonid Hurwicz, René Roy, Maurice Fréchet, and the speaker.

LES MODELES ECONOMIQUES DYNAMIQUES

by Y. Rocard

Professeur dans l'Ecole Normale Supérieure, Paris (France)

Résumé

Sans se faire d'illusion sur la valeur pratique des modèles d'économie dynamique, bâtis sur des hypothèses toujours trop simples, il a semblé possible de reconnaître, grâce à leur emploi, des propriétés mathématiques générales qui peuvent servir ensuite de principes de base dans les raisonnements.

Sur un exemple arbitrairement simplifié, l'auteur établit une équation:

$$\frac{dy}{dt} + \alpha y_{(t-\tau)} = 0 \tag{1}$$

donnant l'évolution d'un prix ou d'une production, on peut alors montrer que *l'augmentation du temps de délai τ finit toujours par rendre le système instable.* Il est clair que cette propriété existe pour la généralité des équations économétriques à temps de délai, l'auteur en cite pourtant qui sont toujours stables mais elles sont de types très restreints.

On peut ensuite envisager une distribution de temps de délai, ce qui donne des équations du genre:

$$\frac{dy}{dt} + \alpha \int_0^\infty f(\tau)\, y_{(t-\tau)}\, d\tau = 0. \tag{2}$$

On établit alors que *plus la distribution $f(\tau)$ est large autour d'un temps τ_0 moyen, plus a τ_0 constant, on retrouve de stabilité.* Cette proposition est vraie pour tous les cas pratiques. Du point de vue mathématique, il y a quelque difficulté à lui donner de la généralité, à cause de la difficulté de définir une distribution plus "large" qu'une autre si les formes de $f(\tau)$ sont compliquées.

L'auteur montre l'usage que l'on peut faire en économétrie de ces principes qui semblent très généraux.

Résumé

Without illusions as to the practical value of dynamic economic models, based on very simple assumptions, it seems possible to survey by their help the general mathematical properties that may serve as basic principles of reasoning.

As an arbitrarily simplified example, the author set up equation (1) giving the evolution of a price or of production, from which it can be shown that *the increase of the lag always ends in making the system unstable.* It is clear that this property exists for lagged econometric equations in general; the author cites some, however, that are always stable but they are of very restricted types.

We may next consider a distribution of the lag, which gives equations of the form (2). We may then show that the wider the distribution of $f(\tau)$ about a constant mean lag τ_0, the more we regain stability. This proposition is true for all practical cases. From the mathematical point of view, there is some difficulty in generalizing this, because of the difficulty of defining one distribution as "wider" than another if the forms of $f(\tau)$ are complicated.

The author showed the use that can be made in econometrics of these principles that seem very general.

Mr. Rocard's paper was discussed by Messrs. Jacques Rueff, Tjallign C. Koopmans, Leonid Hurwicz, and the speaker.

ECONOMETRICS AND THE PREVENTION OF INFLATION AND UNEMPLOYMENT: II

Wednesday, September 17, at 2:00 p.m.

CHAIRMAN :

François Divisia

Professeur au Conservatoire des Arts et Métiers, a l'Ecole Polytechnique, et a l'Ecole des Ponts et Chaussées (France)

A FISCAL AND MONETARY FRAMEWORK FOR ECONOMIC STABILITY

by Milton Friedman

The University of Chicago (United States)

Résumé

The paper outlines a suggested monetary and fiscal framework designed to promote economic stability and, at the same time, to be entirely automatic and to involve no discretionary action by governmental authorities. The particular proposal presented is not original; it is an appropriate selection and combination of elements from existing proposals. It contains four main elements:

(1) A reform of the monetary and banking system to eliminate both the private creation or destruction of money and discretionary control of the quantity of money by central-bank authority. This could probably best be done by introducing a 100-per-cent reserve system, and eliminating the present discretionary authorities of the reserve system. With these changes, the supply of money would be increased only to cover government deficits and decreased only when the government has a surplus, and the system would not engage in open-market operations or take any other actions that involve discretion with respect to timing or magnitude.

(2) A cyclically stable volume of government expenditures on goods and services—defined to exclude transfer expenditures of all kinds, for example, relief and public assistance. The level of expenditure should be determined by long-term considerations and no attempt should be made to vary expenditures in response to cyclical fluctuations.

(3) A stable predetermined *program* of transfer expenditures, consisting of a statement of the conditions and terms under which relief and assistance and other transfer payments will be granted. Actual government expenditures under this program will vary automatically with changes in business conditions and employment.

(4) A stable and progressive tax structure, collected at source so far as possible. Rates, exemptions, etc., should be set entirely in light of the expected yield at a level of income corresponding to reasonably full utilization of resources at a predetermined price level. The principle might either be that the hypothetical yield should approximately balance government expenditures (at the same hypothetical level of income) or that it should lead to a deficit of a fixed amount in order to provide a secular increase in the quantity of money. Thus, the principle of balancing outlays and income at a hypothetical income (the stable budget) would be substituted for the principle of balancing actual outlays (the actual budget). The tax structure should not be varied in response to cyclical fluctuations, though actual receipts will, of course, vary automatically.

The remainder of the paper discusses how this proposal would operate, its implications in a regime of flexible prices and minor lags in response, and the effect of price rigidities and lags in response. It is emphasized that the proposal would not automatically promote "full employment" unless prices, including wages, were flexible, so that flexibility of prices would have to be an important part of a program aimed to promote not only economic stability but also a high level of employment.

Résumé

La communication esquisse un cadre monétaire et fiscal en vue de promouvoir la stabilité économique, tout en étant entièrement automatique et en dehors de l'action discrétionnaire des autorités gouverne mentales. La proposition ici présentée n'est pas originale; elle resulte d'une selection et d'un combinaison appropriées d'éléments de théories connues. Elle contient quatre éléments principaux:

1) Une réforme du système monétaire et bancaire en vue d'abolir à la fois la création et la destruction privée de monnaie et le contrôle discrétionnaire de la quantité de monnaie par l'autorité bancaire centrale. Ceci pourrait probablement être le mieux réalisé par l'application du système de la couverture à 100%, et par l'abolition des pouvoirs discrétionnaires du système central. A la suite de ces réformes, l'offre de monnaie ne pourrait être accrue qu'en vue de couvrir un deficit budgétaire de l'état; elle ne pourrait être diminuée qu'à la suite d'un boni

budgétaire; le système central ne pourrait effectuer des opérations d'"open market" ni aucune autre opération comportant une décision discrétionnaire quant à sa date ou son ampleur.

2) Un volume cycliquement stable de dépenses gouvernementales pour biens et services—à l'exclusion des dépenses de transfert de toute espèce, par exemple, d'assistance publique et de secours. Le niveau de dépenses devrait être déterminé par des considérations à long terme et l'on ne devrait pas s'efforcer de le faire varier en fonction des fluctuations cycliques.

3) Un *programme* détermine a l'avance et stable de dépenses de transfert; il serait un énoncé des conditions auxquelles les indemnités de secours, d'assistance et autres seront accordées. En vertu de ce programme, les dépenses gouvernementales effectives varieraient automatiquement à la suite des fluctuations de la conjoncture et de l'emploi.

4) Un système stable et progressif d'impôts. Les impôts devraient autant que possible être perçus à la source. Les taux, exemptions, etc., devraient être fixés entièrement à la lumière du rendement prévu à un niveau du revenu national correspondant à une utilisation raisonnablement complète des ressources à un niveau de prix prédéterminé. Le rendement hyopthétique devrait soit balancer approximativement les dépenses gouvernementales (au même niveau hypothétique de revenu), soit leur être inférieur d'un certain montant afin de réaliser une expansion séculaire de la quantité de monnaie. Ainsi, le principe de l'égalité des dépenses et des recettes à un niveau de revenu hypothétique (le budget stable) se substituerait au principe de l'égalité des dépenses et recettes effectives (le budget effectif). La structure des impôts ne devrait pas être modifiée en fonction des fluctuations cycliques, quoique les recettes réelles varieront, évidemment, automatiquement.

Dans le reste de la communication il est montré comment la proposition ci-dessus peut être appliquée, quelles sont ses conséquences dans un système à prix flexibles et de légers retards d'adaption, et les effets de rigidité de prix et de retards d'adaption. Il est souligné que la proposition ne conduit pas automatiquement au plein emploi, à moins que les prix, y compris les salaires, ne soient flexibles, de sorte que la flexibilité des prix devrait être un élément important d'un programme visant non seulement à la stabilité économique mais aussi à un niveau élevé de l'emploi.

Mr. Friedman's paper was discussed by Messrs. M. Allais, Jacques Rueff, François Divisia, and the speaker.

REMARKS ON INFLATION AND UNEMPLOYMENT

by M. Kalecki (*Poland*)

Assistant Director, Division of Economic Stability and Development, United Nations

(No résumé of Mr. Kalecki's paper is available).

Mr. Kalecki's paper was discussed by Messrs. Jacques Rueff, W. S. Woytinsky, M. Allais, Leonid Hurwicz, and the speaker.

PRICES AND MONEY

by Luigi Amoroso*

Professor of Economics and Finance, University of Rome (Italy)

1. Empirical Description of the Tendencies of the Market

Let us consider a market of any kind, be it the flower market, or the fish market, or the stock exchange, or a territory—a square, a section of a town, a whole town, a province, a country, or the whole world—in which a given group of goods or services are bought and sold. An empirical judgment regarding the tendencies of the market will be expressed by means of statements that can be ascribed to either one of the two following types:

(a) the market is strong, steady, weak, wavering; or

(b) roses increase in price compared to carnations; mackerel is getting cheaper than cod; electric stock loses ground compared to textiles; gwas do not keep pace with the cost of living; and so on.

The expression listed under (a) include a judgment about all prices and quotations—they state what is *the tendency prevailing on the average and for the time being* among the prices of the group of goods and services under consideration which, by abstraction, is treated as a unit; they state whether there is a trend towards a rise or a decline in prices.

On the other hand, the expressions listed under (b) refer to *phenomena and changes occurring inside the group*, and point out contrasts among the different elements of the latter.

2. Theoretical Formulae Translating the Empirical Statements

Let n be the total number of goods and services, and let

$$p_1, p_2, \ldots, p_n \tag{1}$$

be their respective prices.

The internal structure of the group is then determined by the ratios of any two individual prices to each other. Such ratios number $\frac{1}{2}n(n-1)$; but evidently it is possible to express them all as functions of $n-1$ among them; for example the ratios of $p_2, p_3, \ldots, p_n$ to p_1. The internal structure of the group determines therefore all prices except for a common proportionality factor.

Usually the prices $p_1, p_2, \ldots, p_n$ vary continually with the time, and hence it is possible to consider them as continuous and differentiable functions of the time. Let $p'_1, p'_2, \ldots, p'_n$ be their derivatives.

* The speaker was not able to be present. This paper was read by title.

Let us now introduce the variable p as defined by the equation

$$(2) \qquad \frac{p'}{p} = \alpha_1 \frac{p'_1}{p_1} + \alpha_2 \frac{p'_2}{p_2} + \dots + \alpha_n \frac{p'_n}{p_n},$$

where p' is the derivative of p with respect to t, and $\alpha_1, \alpha_2, \dots, \alpha_n$ are positive constants connected by the relation

$$(3) \qquad \alpha_1 + \alpha_2 + \dots + \alpha_n = 1$$

and expressing the weights—or, in other words, the importance—of the individual prices $p_1, p_2, \dots, p_n$ from the commercial viewpoint. We shall call p the *average level of prices of the group*—or, more briefly, the *average level of the group*.

Now we are in a position to express the internal structure of the group in a more symmetrical way by means of the ratios

$$(4) \qquad \frac{p'_1}{p}, \frac{p'_2}{p}, \dots, \frac{p'_n}{p},$$

which evidently are functions of the time t. Then the function $p(t)$ —namely the average level of the group—is the proportionality factor which, together with the internal structure of the group, determines the individual elements—or the individual prices (1).

3. Monetary Actions and Structural Actions

If the market is a territory (a town, or a nation, or the whole world), and if the group is wide enough to be representative of all goods and services that are bought and sold in that territory, then the variable p can be considered as an index of the average level of prices, and it gives an expression of the value of the money in that market.

If this is the case, it is possible to distinguish among all forces that at any moment determine the fluctuations of prices those forces which *directly tend to affect the function $p(t)$*—namely the value of money—from those other actions which instead *directly tend to modify the ratios (4)*.

The former forces have their epicenter in the sector of money and credit where they take at first the form of an expansion or a contraction of currency circulation, or, more generally, of financial media. Let us call them *monetary actions*.

On the other hand, the other actions—namely those which directly tend to modify the ratios (4)—have their epicenter in the sectors of agriculture, industry, and trade in which they operate by modifying the internal structure of the economic system and which for this reason we shall call *structural actions*.

Monetary forces and structural forces influence each other mutually—insofar as a variation of the price level brings about changes in the ratios of individual prices to each other, and on the other hand, variations in the ratios between individual prices bring about sooner or later a change in the general level of prices.

In the first case the deformation of the social structure expressing itself in the variation of price ratios is the *effect* resulting from a monetary cause. In the second case that deformation of the social structure is a *cause* provoking a monetary effect as a reaction.

4. Inflation as a Monetary Phenomenon

Inflation is a typical example of a structural deformation following, as a reaction, a monetary cause. The deformation has its origin in the fact that the various sectors of the economic universe react with different intensities to the same stimulus, namely the expansion of credit and of currency in circulation. As a consequence some prices increase more than others, some do not undergo any change at all, and some may even decrease. Differences are made more acute by speculation which discounts immediately variations that are foreseen for the future and acts with different intensities in different sectors.

There is a wide category of prices that do not undergo any change *nominally*; it includes all those prices which are fixed by contractual stipulation—among them financial contracts essentially amounting to mere relations of debit and credit. Unless clauses to the contrary are included in the contract, relations of debit and credit remain unchanged however the value of money may change. From this situation conjunctural revenues arise, constituting a loss for creditors in case of a decrease, and a profit in case of an increase of the value of money. If the action originating the process goes on for a certain length of time and and with a certain intensity, these conjunctural revenues progressively accumulate, and the whole internal set-up of the social body undergoes a deformation.

5. Deflation as a Structural Phenomenon

Deflation (of prices) is not the opposite term of inflation. If inflation can be looked at as a fever, an infectious disease or, in extreme cases, bubonic plague, deflation appears instead as an organic disease like a heart condition or an atrophy in the functioning of the liver or kidneys. Inflation has its roots in an outer event—the expansion of currency circulation, and in general of means of payment; and this is the reason why it can be compared to an infectious disease. If infection spreads,

in the long run it damages the arteries and poisons the social body. But this is an effect and not a cause: it is the structural effect of a monetary cause.

On the other hand, the cause of deflation (deflation of prices, I mean) is only rarely and in the least serious cases a monetary one. Usually deflation of prices is a monetary *effect* brought about by causes that go deeper and attack the very inner structure of the social system. Generally deflation has its origin in an excess of immobilization which although it may be confined at first to a few sectors, eventually spreads to ever more distant sectors in consequence of the interdependence of the various parts of the economic system. Excess of immobilization means lack of equilibrium between the fixed and the circulating portions of the invested capital; it means inability to sustain the effort that is necessary to harmonize the whole productive cycle with the hypertrophic development of some of its parts. Hence a paralysis which although confined at first to the sectors where the lack of equilibrium originated, will, if it is not immediately overcome, spread subsequently to the other section of production, and from these to the sectors of consumption and eventually bring about a general decrease in prices.

Deflation cannot be overcome by monetary measures just because it is the expression of a deformation whose cause is structural. An outstanding example for this is the economic policy of the United States in the years from 1922 to 1928 which, while successful in keeping the price level stable, could not succeed in eliminating the difficulties that were gradually piling up in the sector of production. That policy succeeded in concealing for a certain time the outward symptoms of the disease, but not in eliminating its cause; and the fire that had been burning under the ashes burst out with a menacing flare in the great depression of 1929.

6. The Quantitative Equation of Exchange

Inflation is hence a typically monetary phenomenon, and its cause must be looked for among the factors that determine the general price level p.

Deflation, instead, is a typical structural phenomenon, and its cause must be looked for among the factors that determine the ratios of the individual prices to each other or, what amounts to the same, the ratios of the individual prices to the general price level p, as determined in formula (4).

The classical quantitative equation of exchange in the masterly presentation by Irving Fisher, while providing the key for the

explanation of the process of inflation, *proves inadequate to provide an equally good explanation for the process of deflation.*

The reason for the difference lies in the fact that *in a period of inflation* the quantity of money in circulation becomes the most important of the different variables on which the general price level depends, *and it can be considered as an independent variable* expressing in a direct way the monetary policy of the Treasury, and consequently of the Central Bank.

In a period of deflation, instead, the greatest importance falls upon the quantity representing the volume of commercial transactions, *which cannot be considered as an independent variable* since it is the resultant of a very large number of movements and countermovements depending on the conduct of all the actors on the economic scene.

7. The Structural Equation

In order to find our way among the actions and reactions that determine the conduct of the individuals, let us consider the *differential equations* of supply and demand. These express the quantities (of supply and demand) as functions of prices (1) and of their derivatives with respect to the time. Expressing by way of mathematical symbols the fact that supply and demand are in equilibrium, we obtain a system of differential equations of the first order which, if solved with respect to the derivatives and with the help of the symbols introduced in section 2 of this paper, can be written as follows:

$$\begin{aligned} p'_1 &= X_1(p_1, p_2, \ldots, p_n), \\ p'_2 &= X_2(p_1, p_2, \ldots, p_n), \\ \ldots & \quad \ldots \quad \ldots \quad \ldots \\ p'_n &= X_n(p_1, p_2, \ldots, p_n), \end{aligned} \tag{5}$$

where $X_1, X_2, \ldots, X_n$ are functions of the prices $p_1, p_2, \ldots, p_n$ which, we assume, do not contain the time explicitly.

The time enters hence only in the first term of the equation (5) and can be eliminated by writing the above system as follows:

$$\frac{dp_1}{X_1} = \frac{dp_2}{X_2} = \ldots = \frac{dp_n}{X_n}. \tag{6}$$

In this form the system determines $p_2, p_3, \ldots, p_n$ as functions of p_1 and of arbitrary constants that may be considered as expressions of the values which $p_2, p_3, \ldots, p_n$ assume for a given value of p_1.

Let

$$
(7) \qquad \begin{aligned}
u_1(p_1, p_2, \ldots, p_n) &= H_1, \\
u_2(p_1, p_2, \ldots, p_n) &= H_2, \\
\ldots \quad \ldots \quad &\ldots \quad \ldots \\
u_{n-1}(p_1, p_2, \ldots, p_n) &= H_{n-1},
\end{aligned}
$$

be the general integral of the system (6), $H_1, H_2, \ldots, H_{n-1}$ being arbitrary constants, and $u_1, u_2, \ldots, u_{n-1}$ being known functions of $p_1, p_2, \ldots, p_n$. These functions remain unchanged as prices vary. They are, hence, *invariants of the system of prices* $p_1, p_2, \ldots, p_n$. The theory of linear differential equations tells us that any invariant of the system of prices must be a function of $u_1, u_2, \ldots, u_{n-1}$.

But any and all functions of $u_1, u_2, \ldots, u_{n-1}$ are characterized by being integrals of the following partial differential equation of first order:

$$
(8) \qquad X_1 \frac{\partial \phi}{\partial p_1} + X_2 \frac{\partial \phi}{\partial p_2} + \ldots + X_n \frac{\partial \phi}{\partial p_n} = 0.
$$

This equation represents therefore all and only those relations between individual prices which do not include time or money explicitly. And this is why we call it a *structural equation or equation of the connections between prices.*

8. Connection with Money and Time

The connection with money can be obtained by combining (8) with the equation that defines the general price level p. This equation can be arrived at by integrating (2).

Assuming that the basis of the index has been chosen in such a manner that $p = 1$ for $p_1 = p_2 = \ldots = p_n = 1$, the integral of (2) is

$$
(9) \qquad \log p = \alpha_1 \log p_1 + \alpha_2 \log p_2 + \ldots \alpha_n \log p_n.
$$

The system consisting of equations (8) and (9) determines the individual prices as functions of the general price level p. And since the price level may be assumed to be an expression of the value of money, the above-mentioned equations give the system of prices as determined by the social structure *on the hypothesis that the value of money be known.* It follows that if we leave this value undetermined we obtain all price systems that are compatible with the social structure.

Up to this point time has not been considered in this presentation. Time can be introduced by considering the general price level not as an independent variable but as a function of t.

As long as this function remains undetermined the equations represent *all virtual price movements*—that is to say all price movements that are

compatible with the social structure. Among these is included the *real price movement* that is obtained by assigning to $p(t)$ its empirical value. In both cases the price movement is decomposed into the product of two components: a monetary component expressed by the variation of p as a function of time, and a structural component expressed by the ratios (4) as represented in equations (8) and (9).

Résumé

Les fluctuations des prix sont des symptômes de perturbations se produisant dans le système économique et qui peuvent être divisées en deux catégories:

(a) perturbations ayant leur origine dans le secteur monétaire, où elles se manifestent par un changement de la valeur de la monnaie, lequel détermine à son tour une perturbation structurelle, c'est-à-dire une perturbation des relations d'équilibre existant entre les différentes parties de la vie économique;

(b) perturbations ayant leur origine dans les secteurs de l'agriculture, de l'industrie, ou du commerce et se manifestant directement par une perturbation structurelle qui provoque à son tour un changement de la valeur de la monnaie.

Les perturbations structurelles se manifestent par des variations des rapports entre les différents prix. L'auteur démontre que ces rapports dépendent d'une équation linéaire aux dérivées partielles du premier ordre, qu'il appelle *équation structurelle*. Cette équation admet comme intégrales toutes les fonctions (et point d'autres) des différents prix qui restent invariables par rapport aux fluctuations de ces prix (*invariants des prix*).

Ni le temps ni la monnaie ne figurent dans l'équation structurelle. On peut obtenir la connexion avec la monnaie en associant les invariants des prix (intégrales de l'équation structurelle) avec l'équation qui définit le niveau général des prix. On obtient ainsi un système d'équations représentant le système des prix, tel qu'il est déterminé par la structure sociale, supposé que la valeur de la monnaie soit connue. En laissant indéterminée la valeur de la monnaie, on obtient tous les systèmes de prix qui sont compatibles avec la structure sociale.

La variable "temps" peut être introduite dans le système en considérant le niveau général des prix non comme variable indépendante, mais comme fonction du temps. Par ce moyen, on obtient une représentation de tous les *mouvements virtuels*—c'est-à-dire les mouvements qui sont compatibles avec la structure sociale et parmi lesquels se trouve aussi le mouvement réel.